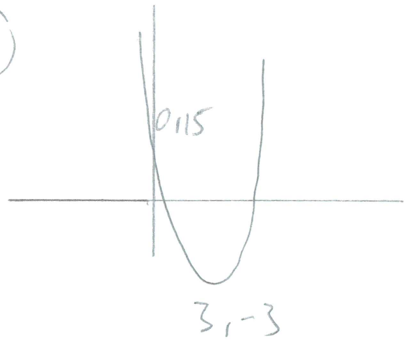
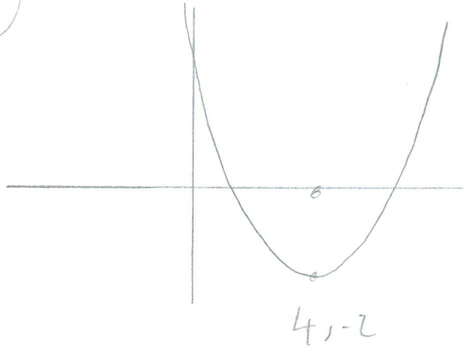
Year 2, 11 Combining Transformations

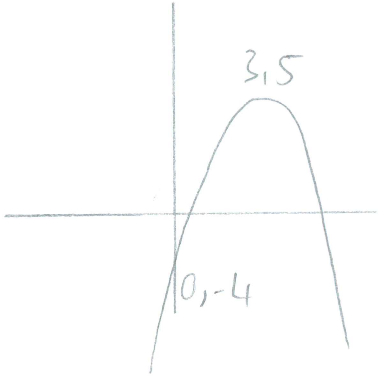
1 (a)



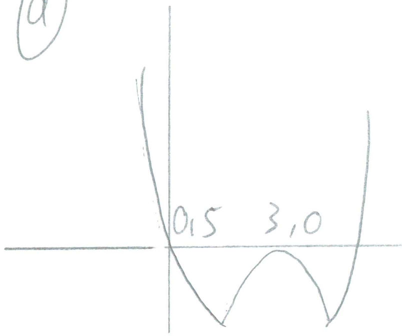
(b)



(c)



(d)



(e)

2 Translation $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and a reflection in the x axis

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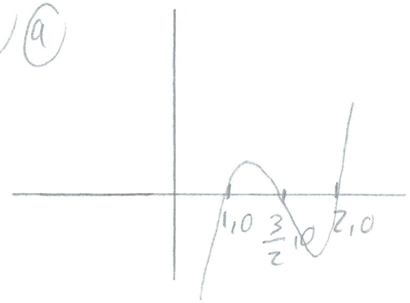
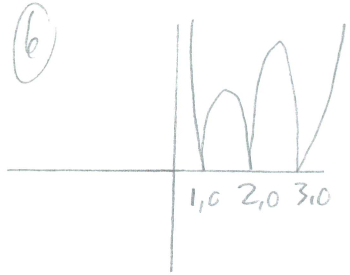
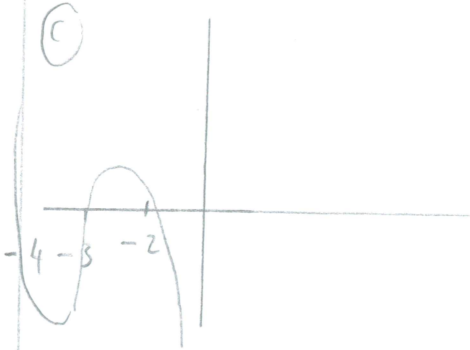
1 (a)



(b)

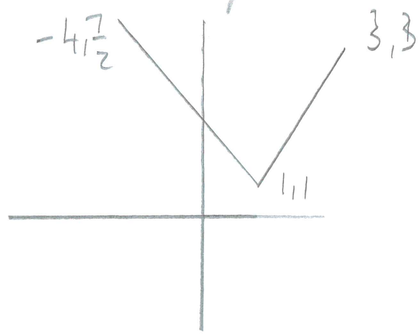


(c)

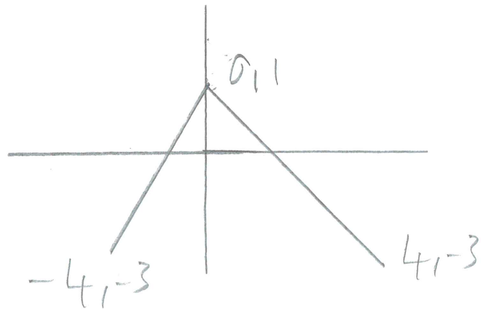


(d) Nothing. They remain invariant points

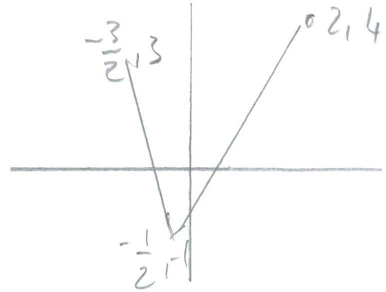
2 (a)



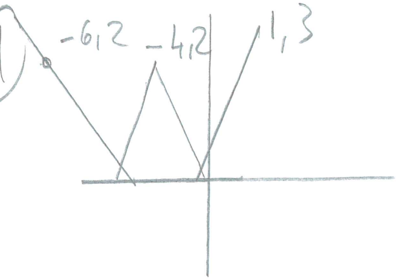
(b)



(c)



(d)



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1 If min point is $(-4,-18)$ then $f(x)$ has minimum point $(-1,-9)$

$$\therefore f(x) = (x+1)^2 - 9$$
$$= x^2 + 2x + 1 - 9$$
$$= x^2 + 2x - 8$$

2 $f(-1) = (-1) - 4(1) + (-1) + 6$
$$= -1 - 4 - 1 + 6$$
$$= -6 + 6$$
$$= 0 \checkmark$$

11 Continued

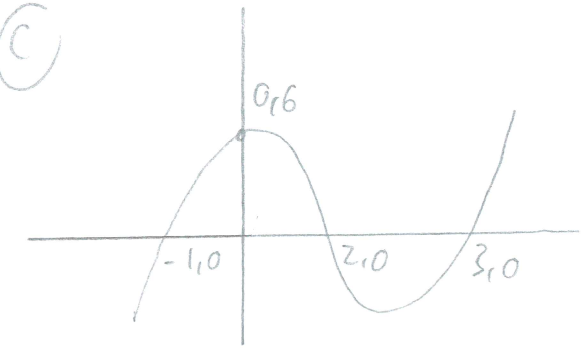
2(b)

$$\begin{array}{r|l} & x^2-5x+6 \\ \hline x+1 & x^3-4x^2+x+6 \\ & \underline{x^3+x^2} \\ & 0\ -5x^2+x \\ & \underline{-5x^2-5x} \\ & 0\ +6x+6 \\ & \underline{+6x+6} \\ & 0+0 \end{array}$$

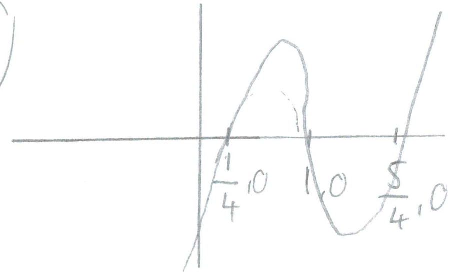
$(x+1)(x^2-5x+6)$

$(x+1)(x-3)(x-2)$ ✓

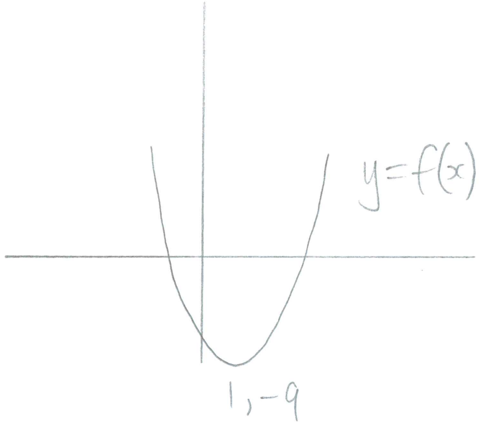
(c)

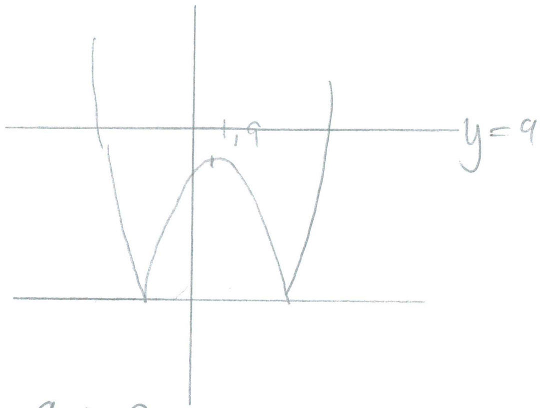


(d)



3 $f(x) = (x-1)^2 - 9$





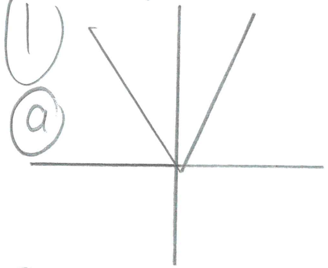
$a > 9$

12 Year 2 Solving Modulus Equations and Inequalities

D/E

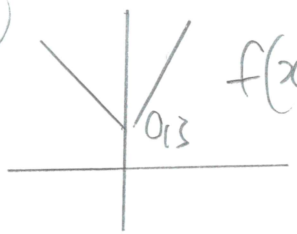
1

a



b $f(x) \geq 0$

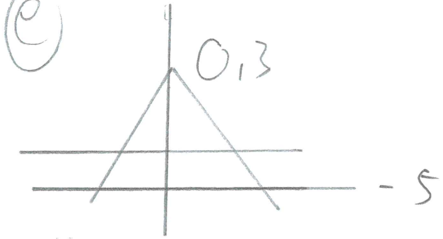
c



$f(x)+3$

d $y \geq 3$

e



$-x+3=-5$

$x=8$

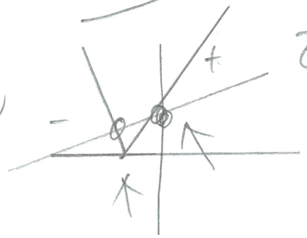
f

$x+3=-5$

$x=-8$

2

a



+

$2x+6=x+4$

$x=2$

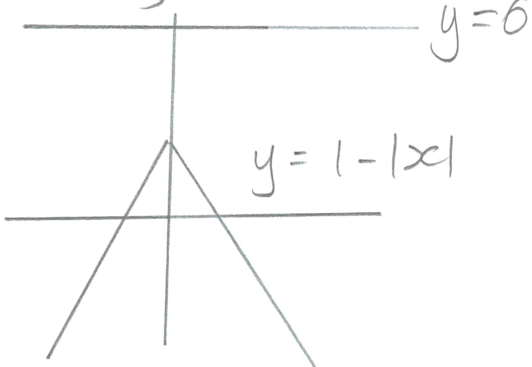
−

$-2x-6=x+4$

$3x=-10$

$x=-10/3$

b $\therefore -\frac{10}{3} \leq x \leq 2$

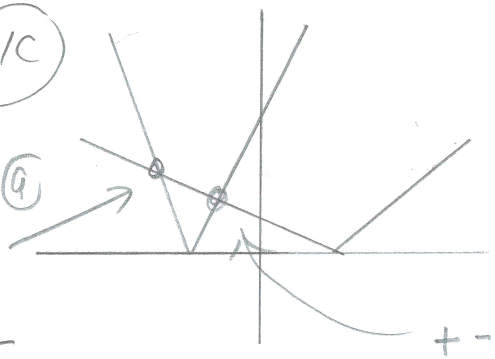


3

No points of intersection

3/C

1 a



− −

$-3x-7=-x+3$

$-10=2x$

$-5=x$

+ −

$3x+7=-x+3$

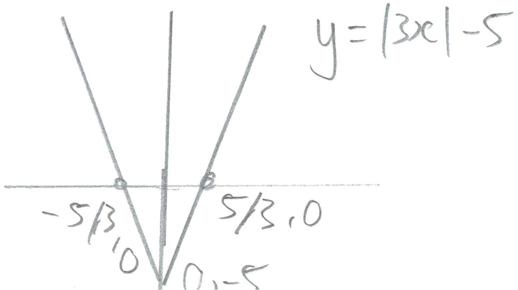
$4x=-4$

$x=-1$

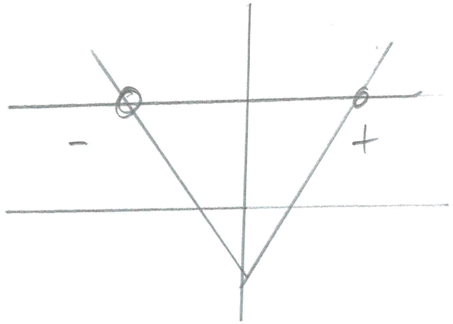
b $-5<x<-1$

2

a



b



$-3x-5=6$

$3x=-11$

$x=-\frac{11}{3}$

$3x-5=6$

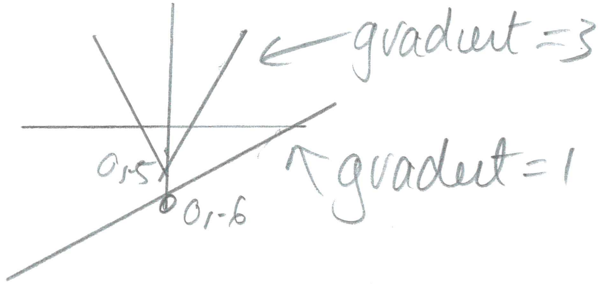
$3x=11$

$x=\frac{11}{3}$

$\therefore \quad x<-\frac{11}{3}$ or $x>\frac{11}{3}$

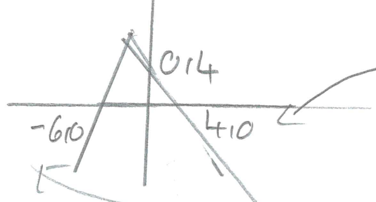
c Not 1-2-1 function.

d



No points of intersection $\therefore$ no solutions

3



$0=5-(x+1)$

$x=4$

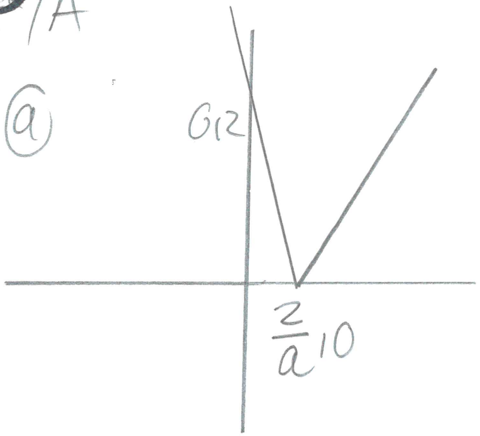
$0=5+x+1$

$x=-6$

(12) Continued.

A*/A

1 (a)



(b) $ax - 2 \geq a$

$ax \geq a + 2$

$x \geq \dfrac{a+2}{a}$

$-ax + 2 \geq a$

$2 - a \geq ax$

$\dfrac{2-a}{a} \geq x$

$\therefore$

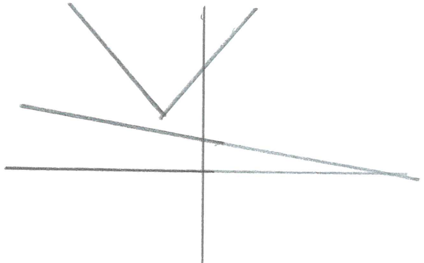
$x \geq \dfrac{a+2}{a}$

or $x \leq \dfrac{2-a}{a}$

(c) $b = \dfrac{2}{a}$, $a > 1$

2 $a = 1$, $b = 4$

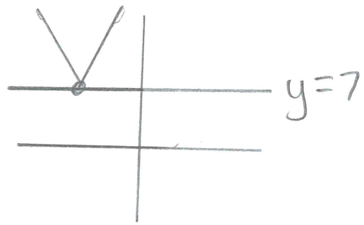
(b) $|x+1| + 4 < 3 - \frac{1}{2}x$

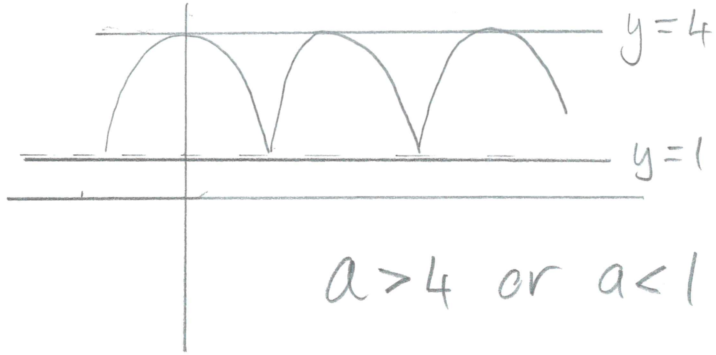


No real solutions as no points of intersection.

(c) $|x+1| + 4 + c = 7$

$\therefore c = 3$





$a > 4$ or $a < 1$

Pure 13 Arithmetic Sequences

1 (a) 7, 9, 11

(b) $2n+5=91$
$2n=86$
$n=43$
$\therefore$ 43rd term

E/D

(c) $2a+1=47$
$2a=46$
$a=23$

2 (a) (i) $24-3n>0$
$n<8$
$\therefore n=7$

(ii) $24-3(20)$
$=-36$

(b) (i) $n=1$ $\therefore$ $12+8=20$

(ii) $d=8$

(iii) $12+8n>100$
$8n>88$
$n>11$
$\therefore$ 12^{th} term
$12+8(12)=108$

3 $u_n=-4n-6$

1 (i) $a+3d=18$ ①
$a+11d=34$ ②

$\therefore$ $8d=16$
$d=2$
and $a=12$

C/B

$\therefore$ $U_n=12+(n-1)2$
$=10+2n$

(b) $10+2n=40-5n$
$5n=30$
$n=6$
$\therefore$ the term is 22

2 $4+(n-1)3=241$
(a) (i) $3(n-1)=237$
$n-1=79$
$n=80$

(ii) $5+(n-1)(-2)=-121$
$-2(n-1)=-126$
$n-1=63$
$n=64$

(b) $40+-4(n-1)=-236$
$-4(n-1)=-276$
$n-1=69$
$n=70$

70 terms in the sequence 10 are positive 1 is 0 and 59 are negative.

3 Subtracting: $U_2-U_1=U_3-U_2$
$p-2-(2p-1)=4p+9-(p-2)$
(a) $-p-1=3p+11$

(b) $-1-11=4p$
$p=-3$

(c) $-7, -5, -3$
$U_n=-2n+9$

(d) 4

1 $3p+10-(p^2+1)=5$
(a) $3p+10-p^2-1=5$
$p^2-3p-4=0$
$(p-4)(p+1)=0$
$p=4$ $p\neq-1$

(b) 17, 22, 27

(c) $U_n=17+5n$
$17+5n<100$
$n<\frac{83}{5}$

$\therefore$ $n=16$
$17+5(16)=87$

(d) 2, 7, 12, 17 ... when $n=-1$
$U_r=2+5r$

(e) 2, 7, 12

(f) If arithmetic $U_2-U_1=U_3-U_2$
$4p-p^2=2p+10-4$
$p^2-2p+6=0$
$b^2-4ac<0$
$(-2)^2-4(1)(6)=-20$
$\therefore$ no real roots.

(14) Arithmetic Series

1 (a) $S_n=\frac{n}{2}(2a+(n-1)d)$

$a=4$, $n=80$, $d=6$, $S=?$, $l=$

$S_{80}=\frac{80}{2}(2(4)+79\times6)$

$=19280$

(b) $a=-3$, $n=80$, $d=4$, $S=?$, $l=$

$=\frac{80}{2}(-6+79\times4)$

$=12400$

(c) $a=-1$, $n=80$, $d=3$, $S=$, $l=$

$=\frac{80}{2}(-2+79\times3)$

$=9400$

(d) $a=1$, $n=80$, $d=-1$, $S=$, $l=$

$\frac{80}{2}(2+79(-1))$

$=-3080$

2 $a=6$, $n=n$, $d=5$, $S=$, $l=611$

$611=6+(n-1)5$

$605=5(n-1)$

$121=n-1$

$122=n$

$\therefore S_n=\frac{n}{2}(2a+(n-1)d)$

$S_{122}=\frac{122}{2}(12+(121)5)$

$=61(617)$

$=37637$

or use

$\frac{n}{2}(a+l)=\frac{122}{2}(6+611)$

3 (a) $a=-2$, $n=n$, $d=-3$, $S=-5430$, $l=$

$S_n=\frac{n}{2}(2a+(n-1)d)$

$-5430=\frac{n}{2}(-4+(n-1)(-3))$

$-10860=n(-3n-1)$

$-10860=-3n^2-n$

$3n^2+n-10860=0$

(b) $n=60$ $n\neq-\frac{181}{3}$

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1 $a=2$, $n=40$, $d=2$, $S=$, l

$S_{40}=\frac{40}{2}(4+39(2))$

$=20(82)$

$=1640$

2 (a) $S_n=\frac{n}{2}(a+l)$

$a=9$, $n=$, $d=$, $S=14541$, $l=384$

$\therefore 14541=\frac{n}{2}(9+384)$

$29082=393n$

$n=9694$

(b) $a_n=a+(n-1)d$

$a=9$, $n=$, $d=$, $S=$, $l=384$

$\therefore 384=9+(9693)d$

$d=\frac{375}{9693}$

$d=\frac{125}{3231}$

$\therefore$ 12th term is $a+11d$

$=9+11\left(\frac{125}{3231}\right)$

$=\frac{4782}{73}$

3 (a) $2p+p^2-11+p+5=-6$

$p^2+3p=0$

$p(p+3)=0$

$p=0$, $p=-3$

$p\neq0$ as not arithmetic

(b) $a=-6$, $n=14$, $d=3$, $S=$, $l=$

$a+(n-1)d$

$\therefore -6+13(3)$

$=33$

(c) $a=-6$, $n=60$, $d=3$, $S=$, $l=$

$S_n=\frac{n}{2}(2a+(n-1)d)$

$\therefore S_{60}=\frac{60}{2}(-12+59(3))$

$=4950$

---

1 $a+7d=28$

(a) $a+13d=64$

$6d=36$

$d=6$ $\therefore a=-14$

$a=-14$, $n=100$, $d=6$, $S=$

$50(-28+99(6))$

$=28300$

(b) $3000\geq\frac{r}{2}(-28+6(r-1))$

$6000\geq-34r+6r^2$

$3r^2-17r-3000\leq0$

$r\leq34.5\ldots$

$\therefore r=34$

(c) $a=-14$, $n=k$, $d=6$, $S>0$, l

$-14, -8, -2, 4, 10, \cancel{16}$

$\therefore k=5$

2 Write the sum forward, write the sum backwards. Add them, collect like terms and factorise

$S_n=a+a+d+a+2d+\ldots a+(n-1)d$

$S_n=a+(n-1)d+\ldots a+2d+a+d+a$

adding

$2S_n=n(a+a+(n-1)d)$

$2S_n=n(a+L)$

as $L=a+(n-1)d$

$\therefore S_n=\frac{n}{2}(2a+(n-1)d)$

3 Even | Odd

Even: $a=2$, $n=100$, $d=2$

Odd: $a=1$, $n=100$, $d=2$

$\frac{100}{2}(4+99(2))-\frac{100}{2}(2+99(2))$

$10100-10000=100$

15 Geometric Sequences

1 (a) $a_n = ar^{n-1}$

$a = 3$
$r = 1.8$
$S =$
$n = 5$

$\therefore a_5 = 3 \times 1.8^4$

(i) $= 31.4928$

$a_{12} = 3 \times 1.8^{11}$
$= 1928.052302$

(ii) $a_n = ar^{n-1}$

$a = 1$
$r = 5$
$S =$
$n = 5$

$\therefore a_5 = 1 \times 5^4$
$= 625$

$a_{12} = 1 \times 5^{11}$
$= 48828125$

(b) $6.08 \div 1.9 = 3.2$
$19.456 \div 6.08 = 3.2$
$60.3136 \div 19.456 = 3.1$

$\therefore$ not geometric as no common ratio.

(2) $a_n = ar^{n-1}$

$a = 2$
$r = r$
$S =$
$n = 5$

$\therefore 0.0512 = 2 \times r^4$

$\frac{16}{625} = r^4$

$\pm \frac{2}{5} = r$

(3) $a_n = ar^{n-1}$

$a = 2$
$r =$
$S =$
$n = 7$
$l = \frac{2}{15625}$

$\therefore \frac{2}{15625} = 2 \times r^6$

$\frac{1}{15625} = r^6$

$\left(\frac{1}{15625}\right)^{\frac{1}{6}} = r$

$r = \pm \frac{1}{5}$

(1) $ar^4 = 3.1104$
$ar^6 = 4.478976$

divide them!

(a) $r^2 = \frac{36}{25}$

$r = \pm \frac{6}{5}$ $r = +1.2$

(b) 5th term

$ar^4 = 3.1104$
$a(1.2)^4 = 3.1104$
$a = 1.5$

(c) a 1.5
r +1.2
S
n 12

$\therefore 1.5 \times (+1.2)^{11}$
$= +11.14512556$

(d) $1.5(+1.2)^{n-1} > 200$

$1.2^{n-1} > \frac{400}{3}$

$n - 1 > \log_{1.2}\left(\frac{400}{3}\right)$

$n > \log_{1.2}\left(\frac{400}{3}\right) + 1$

$n > 27.8$

$\therefore n = 28$

(2) $\frac{4p}{p+1} = \frac{12p}{4p}$

(a) $\frac{4p}{p+1} = 3$

$4p = 3(p+1)$
$4p = 3p + 3$
$p = 3$

(b) $4, 12, 36 \ldots$

(c) $4 \times 3^{n-1}$

(d) $4 \times 3^{n-1} < 500$

$3^{n-1} < 125$

$n - 1 < \log_3(125)$

$n < \log_3(125) + 1$

$n < 5.39$

$\therefore n = 5$

(3) $a = 10$ $10 \times 0.5^{n-1} = 0.01$
$r = \frac{1}{2}$
$S =$
$n =$

$0.5^{n-1} = 0.001$

$n - 1 = \log_{0.5}(0.001)$

$n = \log_{0.5}(0.001) + 1$

$n = 10.96 \ldots$

$\therefore n = 11$

$10 \times 0.5^{10} = \frac{5}{512}$
$= 0.00976 \ldots$

(1) $\frac{2K-11}{K} = \frac{3K+1}{K(2K-11)}$

$(2K-11)^2 = 3K+1$ as $K \neq 0$

$4K^2 - 44K + 121 = 3K + 1$
$4K^2 - 47K + 120 = 0$
$K = 8$ or $K = 3.75$

when $K = 8$ $a_1 = 8$
$a_2 = 5$, $a_3 = \frac{25}{8}$

$\therefore K = 3.75$ or $\frac{15}{4}$ ✓

(2) (a) $a \times r^3 = 100$

If r is negative r^3 is negative $\therefore$ a must be negative to give +100
If r is positive r^3 is positive so a is + to give 100.

(b) log is $\log_{10}$ $\therefore$

$ar^3 = 100$
$\log ar^3 = \log 100$
$\log a + \log r^3 = 2$
$\log a + 3\log r = 2$
$3\log r = 2 - \log a$
$\log r = \frac{2 - \log a}{3}$

(c) If $0 < r < 1$ then $\log r < 0$ $\therefore$ $2 - \log a < 0$
$2 < \log a$
$a > 10^2$, $a > 100$

(3) $\frac{a+1}{a} = \frac{a+2}{a+1}$ if geometric $\therefore$

$(a+1)^2 = a(a+2)$

$a^2 + 2a + 1 = a^2 + 2a$ ... $1 \neq 0$ not