

Integration

(1) Integrate the following expressions with respect to t :

- (a) $3t$
- (b) $4t^3$
- (c) t^{-2}
- (d) $\frac{1}{2}t^3$
- (e) 1

(2) Find the following indefinite integrals:

- (a) $\int (2x+1) dx$
- (b) $\int (3x^3 - 2x) dx$
- (c) $\int (4x^{-3} - 2x^{-5}) dx$
- (d) $\int (-x^{-5} + \sqrt{x}) dx$
- (e) $\int (7 - x - x^{-5}) dx$
- (f) $\int \left(4x - \frac{c}{2}\right) dx$
(c is a constant)
- (g) $\int (ax^3 + bx^2) dx$
(a and b are constants)

(3) Find an expression for $f(x)$ for each of the following:

- (a) $f'(x) = 3x - 4$
- (b) $f'(x) = x^3 - 4x^{-5}$
- (c) $f'(x) = x^{\frac{1}{2}} + \sqrt{2}x^{-2}$
- (d) $f''(x) = \frac{1}{2}x^{\frac{1}{3}} + 8x^{-5}$

(4) Find an expression for y given the following:

- (a) $\frac{dy}{dx} = 4x - x^2$
- (b) $\frac{dy}{dx} = 3 + 4x - 2x^3$
- (c) $\frac{dy}{dx} = 9x + c$
(c is a constant)
- (d) $\frac{d^2y}{dx^2} = 9 - x$
- (e) $\frac{dy}{dx} = (ax + b)^2$
(a and b are constants)

(5) Find the following indefinite integrals in the form $y = f(x)$:

- (a) $\int (2 + 3x)(1 - x) dx$
- (b) $\int x(\sqrt{x} - 3) dx$
- (c) $\int \left(\frac{3x-1}{x^{0.5}}\right) dx$
- (d) $\int \left(\frac{2\sqrt[3]{x}-3}{x^{0.25}}\right) dx$
- (e) $\int (1 - 4\sqrt{x^5}) dx$

(6) Given $y = x\sqrt{x} - 3$, $x > 0$ find $\int y dx$.

(7) Find an equation for each of curves with the derivatives given below:

- (a) $\frac{dy}{dx} = 2x + 3$
Point (3, 20)
- (b) $\frac{dy}{dx} = 6x^2 + 10x - 3$
Point (1, 11)
- (c) $\frac{dy}{dx} = (1 - x)(2 + x)$
Point $\left(2, -\frac{8}{3}\right)$
- (d) $\frac{dy}{dx} = \frac{3x^2 - 1}{\sqrt{x}}$
Point (1, 3)
- (e) $\frac{dy}{dx} = \frac{1}{\sqrt{x}} + 0.25x^{-0.75}$
Point (16, 11)

- (f) $\frac{dm}{dt} = t(1 - t)^2$
Point (0, 1)

- (g) $\frac{dp}{dt} = \sqrt[4]{t} \left(\frac{t-1}{2}\right)$
Point (1, 1)

(8) The curve $y = f(x)$ passes through the point (2, 6).
Given $f'(x) = \frac{1}{2}x^3 - 2x + 3$ find an equation for y in terms of x .

(9) The curve C passes through the point (4, 3). Find an equation for y in terms of x given $\frac{dy}{dx} = 3x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} + 2x + 1$

(10) The curve $y = f(x)$ passes through the origin.
Given $f'(x) = 3x^2 - 2x - 6$ find the solutions to the equation $f(x) = 0$.

(11) The gradient of a particular curve is given by $\frac{dy}{dx} = 2x + c$ where c is a constant.
Given the curve passes through the points (1, 2) and (3, 14) find an equation for y in terms of x .

(12) The diagram below shows the line with equation $\frac{d^2y}{dx^2} = 6(x + 1)$ and the curves of $f(x)$ and $g(x)$. Find an equation for $y = f(x)$ given $g(0) = -2$ and $f(0) = 2$.

