

To print higher-resolution math symbols, click the  
**Hi-Res Fonts for Printing** button on the jsMath control panel.

# MathsNet : A-Level<sup>+</sup>



**steve blades**

Your account expires on: 27 Mar, 2013

## s2 full exam paper



Syllabus: **EdExcel** Questions: **76** Time: **11 hours 41 minutes** Total Marks: **585**

This paper contains a set of questions followed by the corresponding mark schemes. The time you should spend on each question together with its worth in marks is also given. The content of this paper is based on material from a wide selection of national and international examination boards and organisations.

**You are advised to have:**

a set of geometrical equipment, pen, HB pencil, eraser. Check if you are allowed a calculator. Some examinations, but not all, allow calculators, including graphical models.

**NOTES:** The following browsers have been tested with this facility: Mozilla Firefox 3.x, 4.x; Microsoft Internet Explorer versions 6, 7, 8 and 9 RC (see the website for the small font problem with IE7 and IE8 was tested in IE7 compatibility mode), Apple Safari and Google Chrome. Best results are when the background printing of images and colours is enabled (not available in Chrome on Windows/Mac or Safari on Windows). There are known printing format issues with the Opera web browser and we do not recommend using this browser.

Many of the questions use the jsMath applet. This requires special fonts to be installed for successful printing. These fonts can be downloaded from: <http://www.math.union.edu/~dpvc/jsMath/>. Use the **Download the TeX** fonts option. Full instructions for their use can be found at: <http://pubpages.unh.edu/~jsh3/jsMath/>

**www.mathsnetalevel.com** is available on annual subscription, where you will find 8024 pages of content, including 2906 interactive questions that can be printed and 955 on-line assessments. Visit **www.chartwellyorke.com** to find out further details about subscribing.

## s2 full exam paper

Questions: 76

Time: 11 hours 41 minutes

Total Marks: 585

**Q1** - ID: 4378

[4 marks, 5 minutes]

- (a) How many different teams of 9 people can be chosen, without regard to order, from a squad of 22?
- (b) The squad consists of 9 forwards and 13 defenders. How many different teams containing 3 forwards and 6 defenders can be chosen?

**Q2** - ID: 3141

[5 marks, 6 minutes]

Codes of 5 letters are made up using only the letters X, W, Q, N, M, Z, A.

Find how many different codes are possible

- (a) if all 5 letters used must be different,
- (b) if letters may be repeated.

**Q3** - ID: 6091

[7 marks, 8 minutes]

The letters A, B, C, D, E are arranged in a straight line.

- (a) How many different arrangements are possible?
- (b) In how many of these arrangements are the letters A and B next to each other?
- (c) From the letters A, B, C, D, E two different letters are selected at random. Find the probability that these two letters are A and B.

**Q4** - ID: 6101

[5 marks, 6 minutes]

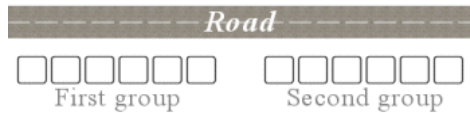
A class consists of 7 students from Acle and 8 students from Beccles.

A committee of 5 students is chosen at random from the class.

- (a) Find the probability that 2 students from Acle and 3 from Beccles are chosen.
- (b) In fact 2 students from Acle and 3 from Beccles are chosen. In order to watch a video, all 5 committee members must sit in a row. In how many different orders can they sit if no two students from Beccles sit next to each other?

Q5 - ID: 6194

[7 marks, 8 minutes]



A builder is planning to build 12 houses along one side of a road. He will build 4 houses in style A, 2 houses in style B, 2 houses in style C, 2 houses in style D and 2 house in style E.

(a) Find the number of possible arrangements of these 12 houses.

(b) The 12 houses will be in two groups of 6 (see diagram). Find the number of possible arrangements if all the houses in styles A and D are in the first group and all the houses in styles B, C and E are in the second group.

4 of the 12 houses will be selected for a survey. Exactly one house must be in style B and exactly one house in style C. Find the number of ways in which these 4 houses can be selected.

Q6 - ID: 4572

[7 marks, 8 minutes]

(a) The random variable  $X$  has the distribution  $B(10, 0.8)$ . Using the tables of cumulative binomial probabilities, or otherwise, find  $P(X \geq 6)$ .

(b) The random variable  $Y$  has the distribution  $B(11, 0.22)$ . Find  $P(Y = 4)$ .

(c) The random variable  $Z$  has the distribution  $B(n, 0.22)$ .

Find the smallest value of  $n$  such that  $P(Z \geq 1) > 0.93$ .

Q7 - ID: 5369

[5 marks, 6 minutes]

The random variable  $X$  has the distribution  $B(n, 0.1)$ . Given that the mean and standard deviation of  $X$  are equal, find the value of  $n$ .

Q8 - ID: 4371

[8 marks, 10 minutes]

Bob plays 17 squash games. In each game he either wins or loses.

(a) State, in this context, two conditions needed for a binomial distribution to arise.

(b) Assuming these conditions are satisfied, define a variable in this context which has a binomial distribution.

(c) The random variable  $X$  has the distribution  $B(17, p)$  where  $0 < p < 1$

Given that  $P(X = 6) = P(X = 5)$  find the value of  $p$ .

---

**Q9** - ID: 5471

[9 marks, 11 minutes]

The random variable  $X$  has the distribution  $B(25, 0.3)$ .

Given that  $Y = 3X + 3$  find

(a)  $E(Y)$ , (b)  $Var(Y)$ , (c)  $P(Y = 15)$ .

---

**Q10** - ID: 6094

[9 marks, 11 minutes]

(a) The random variable  $X$  has the distribution  $B(11, 0.54)$ . Find

(a)  $P(X < 9)$ .

(b)  $P(X = 4)$ .

(c)  $P(3 \leq X < 7)$ .

A random variable  $Y$  has the distribution  $B(14, \frac{4}{13})$ . Find

(d)  $P(Y = 3)$ .

(e)  $Var(Y)$ .

---

**Q11** - ID: 910

[4 marks, 5 minutes]

Letters are sent by second class post unless they are marked first class.

Over a long period of time it has been established that 20% of letters to be posted are marked first class.

In a random selection of 8 letters to be posted, find the probability that the number marked first class is

(a) at least 2.

(b) fewer than 1.

---

**Q12** - ID: 539

[8 marks, 10 minutes]

A farmer notices that some of the chickens in his pens have damaged limbs.

He estimated the probability of this happening to be 0.1.

Chickens live in pens of 10.

Find the probability that in a pen, the number of chickens with damaged limbs will be

(a) exactly 1, (b) more than 5.

Another farmer took over 3 of these pens.

(c) Find the probability that only 2 of them contained exactly 1 chicken with a damaged limb.

**Q13** - ID: 749

[7 marks, 8 minutes]

In the production of long life electric light bulbs it is found that 15 percent are defective.

The light bulb is produced in batches of 12.

(a) Write down a suitable model for the distribution of defective light bulbs in a batch

Find the probability that a batch contains

(b) no defective light bulbs.

(c) more than 1 defective light bulbs.

(d) Find the mean and variance of the defective light bulbs in a batch.

**Q14** - ID: 2044

[7 marks, 8 minutes]

A fair coin is tossed 4 times.

Find the probability that

(a) an equal number of head and tails occur

(b) all the outcomes are the same,

(c) the first tail occurs on the third throw.

**Q15** - ID: 3146

[8 marks, 10 minutes]

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of the caller being connected to the wrong agent is 0.11.

(a) Find the probability that 4 consecutive calls will be connected to the wrong agent.

(b) Find the probability that more than 3 call in 15 consecutive calls are connected to the wrong agent.

The call centre receives 1000 calls each day.

(c) Find the mean and variance of the number of wrongly connected calls.

**Q16** - ID: 4376

[7 marks, 8 minutes]

On average, 30% of the packets of a certain kind of soup contain a voucher. Kim buys one packet of soup each week for 12 weeks. The number of vouchers she obtains is denoted by  $X$ .  $X$  is modelled by the distribution  $B(12, 0.3)$ .

(a) Find  $P(X \leq 4)$

In order to claim a free gift, 5 vouchers are needed.

(b) Find the probability that Kim will be able to claim a free gift at some time during the 12 weeks.

(c) Find the probability that Kim will be able to claim a free gift in the 12th week but not before.

**Q17** - ID: 3137

[7 marks, 8 minutes]

The probability of a bolt being faulty is 0.5. Find the probability that in a random sample of 12 bolts there are

- (a) exactly 12 faulty bolts,
- (b) more than 3 faulty bolts.

These bolts are sold in bags of 12. John buys 9 bags.

- (c) Find the probability that exactly 5 of these bags contain more than 3 faulty bolts.

**Q18** - ID: 3145

[10 marks, 12 minutes]

A particular product is made from human blood given by donors.

The product is stored in bags. The production process is such that, on average, 5% of bags are faulty. Each bag is carefully tested before use.

- (a) 15 bags are selected at random.

- (i) Find the probability that exactly 2 bag is faulty.

- (ii) Find the probability that at least 2 bags are faulty.

- (iii) Find the expected number of faulty bags in the sample.

- (b) A random sample of  $n$  bags is selected. The production manager wishes there to be a probability of 0.327 or less of finding any faulty bags in the sample.

Find the maximum possible value of  $n$ , showing your working clearly.

**Q19** - ID: 4369

[8 marks, 10 minutes]

20% of people in the town of Repps support the Broads Party.

10 people from Repps are selected at random. Out of these

10 people, the number who support the Broads Party is denoted by  $B$ . Find

- (a)  $P(B \leq 2)$

- (b)  $P(B \geq 4)$ .

28% of people from Repps support the Fenn Party.

19 people from Repps are selected at random. Out of these

19 people, the number who support the Fenn Party is denoted by  $F$ . Find

- (c)  $P(F = 4)$ .

**Q20** - ID: 4529

[8 marks, 10 minutes]

A travel agency in Tunisia offers customers a 3-day tour into the Sahara desert by either coach or minibus.

The agency accepts bookings from 15 customers for seats on the coach.

The probability that a customer, who has booked a seat on the coach, will not turn up to claim the seat is 0.09, and may be assumed to be independent of the behaviour of other customers. Determine the probability that, of the customers who have booked a seat on the coach:

- (a) 3 or more will not turn up;
- (b) 4 or more will not turn up;

The agency accepts bookings from 17 customers for seats on the minibus.

The probability that a customer, who has booked a seat on the minibus, will not turn up to claim the seat is 0.02, and may be assumed to be independent of the behaviour of other customers. Calculate the probability that, of the customers who have booked a seat on the minibus:

- (c) all will turn up;
- (d) one or more will not turn up.

**Q21** - ID: 5377

[14 marks, 17 minutes]

A salesman makes 25 house calls during a particular week. You may assume that, independently for each house visited, the probability of a sale is 0.3.

Find the probability that, during this week, he makes

- (a) exactly 10 sales,
  - (b) between 8 and 13 (both inclusive) sales.
  - (c) his first sale on the third house visited.
  - (d) At the end of the week, he is paid £100 plus a commission of £100 for every sale.
- Find the mean and standard deviation of his total pay for this week.

**Q22** - ID: 5456

[7 marks, 8 minutes]

A small business has 6 workers. On a given day the probability that any particular worker is off sick is 0.03, independently of the other workers.

A day is selected at random., Find the probability that

- (a) no workers are off sick,
- (b) more than 1 worker is off sick.
- (c) There are 295 working days in a year. Find the expected number of days in the year when more than 1 worker is off sick.

---

**Q23** - ID: 6193

[4 marks, 5 minutes]

A fair die has one face numbered 1, one face numbered 3, two faces numbered 5 and two faces numbered 6.  
Find the probability of obtaining at least 2 odd numbers in 3 throws of the die.

---

**Q24** - ID: 2046

[4 marks, 5 minutes]

The random variable  $X \sim B(110, 0.05)$ .  
Use a suitable approximation to estimate  $P(X > 8)$ .

---

**Q25** - ID: 607

[4 marks, 5 minutes]

The probability of an expert hitting the target in a computer game in under 0.2 seconds is 0.085  
A group of 100 experts each played the game once.  
Using a suitable approximation, find the probability that more than 8 of them hit the target in under 0.2 seconds.

---

**Q26** - ID: 3064

[10 marks, 12 minutes]

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of the caller being connected to the wrong agent is 0.01.  
(a) Find the probability that 4 consecutive calls will be connected to the wrong agent.  
(b) Find the probability that more than 1 call in 7 consecutive calls are connected to the wrong agent.  
The call centre receives 910 calls each day.  
(c) Find the mean and variance of the number of wrongly connected calls.  
(d) Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent.

---

**Q27** - ID: 3147

[5 marks, 6 minutes]

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of the caller being connected to the wrong agent is 0.04.  
The call centre receives 150 calls each day.  
(a) Find the mean and variance of the number of wrongly connected calls.  
(b) Use a Poisson approximation to find, to 3 decimal places, the probability that more than 9 calls each day are connected to the wrong agent.



**Q28** - ID: 5503

[9 marks, 11 minutes]

A factory produces components of which 4% are defective.

The components are packed in boxes of 6. A box is selected at random.

- Find the probability that the box contains exactly one defective component.
- Find the probability that there are at least 2 defective components in the box.
- Using a suitable approximation, find the probability that a batch of 230 components contains between 1 and 5 (inclusive) defective components.

**Q29** - ID: 2328

[6 marks, 7 minutes]

The random variable  $X$  has a Poisson distribution with mean 10. Calculate

- $P(7 \leq X \leq 14)$
- $P(X > 7)$
- $P(7 \leq X \leq 14 | X > 7)$ .

**Q30** - ID: 5500

[11 marks, 13 minutes]

A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be 4. The daisies are distributed randomly throughout the field.

Find the probability that, in a randomly chosen square there will be

- more than 3 daisies,
- either 4 or 5 daisies.

The botanist decides to count the number of daisies,  $x$ , in each of 80 randomly selected squares within the field. The results are summarised below

$$\sum x = 299, \sum x^2 = 1387$$

- Calculate the mean and the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places.
- Explain how the answers from part (c) support the choice of a Poisson distribution as a model.
- Using your mean from part (c), estimate the probability that exactly 6 daisies will be found in a randomly selected square.

**Q31 - ID: 5375***[8 marks, 10 minutes]*

The number of accidents occurring per week on a certain stretch of motorway has a Poisson distribution with mean 0.5.

(a) Using tables, find the probability that, in a randomly chosen week, there are between 1 and 4 (both inclusive) accidents on this stretch of motorway.

The number of accidents occurring per week on another stretch of road has a Poisson distribution with mean 3.44. Without the use of tables, find the probability that the number of accidents occurring on this stretch of road during a randomly chosen week is

- (b) exactly 4.
- (c) less than 3

**Q32 - ID: 3138***[11 marks, 13 minutes]*

(a) State two conditions under which a Poisson distribution is a suitable model to use in statistical work.

The number of cars passing an observation point in a 10 minute interval is modelled by a Poisson distribution with mean 1.

(b) Find the probability that in a randomly chosen 60 minute period there will be

- (i) exactly 5 cars passing the observation point,
- (ii) at least 7 cars passing the observation point.

The number of other vehicles, other than cars, passing the observation point in a 60 minute interval is modelled by a Poisson distribution with mean 18.

(c) Find the probability that exactly 1 vehicle, of any type, passes the observation point in a 10 minute period.

**Q33 - ID: 3063***[8 marks, 10 minutes]*

An engineering company manufactures an electronic component. At the end of the manufacturing process, each component is checked to see if it is faulty. Faulty components are detected at a rate of 2.4 per hour.

(a) Suggest a suitable model for the number of faulty components detected per hour.

(b) Describe, in the context of this question, two assumptions you have made in part (a) for this model to be suitable.

(c) Find the probability of 3 faulty components being detected in a 1 hour period.

(d) Find the probability of at least 1 faulty component being detected in a 3 hour period.

---

**Q34** - ID: 2045

[9 marks, 11 minutes]

Accidents on a particular stretch of motorway occur at an average rate of 3.1 per week.

- (a) Write down a suitable model to represent the number of accidents per week on this stretch of motorway. Find the probability that
- (b) there will be 5 accidents in the same week,
- (c) there is at least one accident per week for 5 consecutive weeks,
- (d) there are more than 3 accidents in a 6 week period.

---

**Q35** - ID: 713

[6 marks, 7 minutes]

The number of breakdowns per day in a large fleet of hire cars has a Poisson distribution with mean  $\frac{3}{7}$

- (a) Find the probability that on a particular day there are fewer than 2 breakdowns.
- (b) Find the probability that during a 28-day period there are at most 4 breakdowns.

---

**Q36** - ID: 868

[6 marks, 7 minutes]

A local tyre-fitting company replaces worn tyres at an average rate of 6 per hour. Find the probability that in a randomly chosen hour the number of jobs is

- (a) exactly 5.
- (b) more than 11.

---

**Q37** - ID: 386

[6 marks, 7 minutes]

Before being sent to supermarkets, pre-cooked meat has to be checked for weight. The produce pass along a conveyor belt, and are checked automatically by a weighing machine. Underweight packs arrive at random times, but at a constant average rate of 1.2 per minute.

- (a) Find the probability of at least one underweight pack arriving in a one minute period.
- (b) In a period of  $t$  minutes, the probability of at least one underweight pack is 0.683. Find the value of  $t$ .

**Q38** - ID: 930

[7 marks, 8 minutes]

In a big city false 999 calls are made at a rate of 0.6 per hour.

(a) Show that the probability of no false alarms in the next hour is 0.5488.

Find the probability of

(b) exactly 7 false alarms in the next 9 hour period.

(c) no false alarms in exactly 4 of the next 7 hours.

**Q39** - ID: 5378

[12 marks, 14 minutes]

The continuous random variable  $X$  has cumulative distribution function  $F$  given by

$$F(x) = \begin{cases} 0 & x < 0 \\ 7x^6 - 6x^7 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

(a) Evaluate  $P(0.23 \leq X \leq 0.75)$ .

(b) By evaluating  $F(0.5)$ , determine whether the median of  $X$  is greater or less than 0.5.

(c) Obtain an expression for  $f(x)$ , valid for  $0 \leq x \leq 1$ , where  $f$  denotes the probability density function of  $X$ .

(d) Evaluate  $E(X)$ .

**Q40** - ID: 3139

[14 marks, 17 minutes]

The continuous random variable  $Y$  has cumulative distribution function  $F(y)$  given by

$$F(y) = \begin{cases} 0 & y < 3 \\ k(y^4 + y^2 - 90) & 3 \leq y \leq 6 \\ 1 & y > 6 \end{cases}$$

(a) Show that  $k = \frac{1}{1242}$ .

(b) Find  $P(Y > 3.7)$

(c) Specify fully the probability density function  $f(y)$ .

**Q41** - ID: 3026

[8 marks, 10 minutes]

The time (in milliseconds) taken by my computer to perform a particular task is modelled by the random variable  $T$ . The probability that it takes more than  $t$  milliseconds to perform this task is given by the expression

$$P(T > t) = \frac{k}{t^2} \text{ for } t \geq 2 \text{ where } k \text{ is a constant.}$$

(a) Write down the cumulative distribution function of  $T$  and hence show that  $k = 4$ .

(b) Find the probability density function of  $T$ .

(c) Find the mean time for the task.

**Q42** - ID: 2048

[13 marks, 16 minutes]

A continuous random variable  $X$  has probability density function  $f(x)$  where:

$$f(x) = \begin{cases} kx(x-4) & 4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that  $k = 0.4286$ ,  
 (b) Find  $E(X)$ ,  
 (c) Find the cumulative distribution function  $F(x)$

**Q43** - ID: 875

[14 marks, 17 minutes]

The continuous random variable  $X$  has probability density function

$$f(x) = \begin{cases} \frac{x}{40} & 0 \leq x \leq 4 \\ \frac{4}{40} & 4 < x < 10 \\ \frac{7}{20} - \frac{4x}{160} & 10 \leq x \leq 14 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find expressions for the cumulative distribution function,  $F(x)$ , for  $0 \leq x \leq 4$  and for  $10 \leq x \leq 14$   
 (b) Show that for  $4 < x < 10$ ,  $F(x) = \frac{4x}{40} - \frac{8}{40}$ .  
 (c) Specify  $F(x)$  for  $x < 0$  and  $x > 14$ .  
 (d) Find  $P(X \leq 10.5)$   
 (e) Find, to 3 significant figures,  $E(X)$ .

**Q44** - ID: 774

[14 marks, 17 minutes]

The continuous random variable  $X$  has cumulative distribution function  $F(x)$  given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{6}x^2(7-x^2) & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

- (a) Find  $P(X > 0.7)$   
 (b) Find the probability density function  $f(x)$  of  $X$ .  
 (c) Calculate  $E(X)$  and show that, to 3 decimal places,  $\text{Var}(X) = 0.057$ .

One measure of skewness is

$$\frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$$

- (d) Calculate the skewness of the distribution of  $X$ .

**Q45** - ID: 831

[4 marks, 5 minutes]

The continuous random variable  $X$  has probability density function  $f(x)$  given by

$$f(x) = \begin{cases} \frac{1}{63.75}x^3 & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Specify fully the cumulative distribution function of  $X$ .

**Q46** - ID: 640

[11 marks, 13 minutes]

The continuous random variable  $X$  has cumulative distribution function  $F(x)$  given by

$$F(x) = \begin{cases} 0 & x < 0 \\ kx^2 + 2kx & 0 \leq x \leq 2 \\ 8k & x > 2 \end{cases}$$

- Show that  $k = \frac{1}{8}$ .
- Find the median of  $X$ .
- Find the probability density function  $f(x)$ .
- Write down the mode of  $X$ .
- Find  $E(X)$ .

**Q47** - ID: 461

[8 marks, 10 minutes]

The continuous random variable  $X$  has cumulative distribution function  $F(x)$  given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{17}(-x^3 + 8x^2 - 7) & 1 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$$

- Find the probability density function  $f(x)$ .
- Find the mode of  $X$ .
- Find the mean  $\mu$  of  $X$ .

**Q48** - ID: 5502

[11 marks, 13 minutes]

The length of a telephone call made to a company is denoted by the continuous random variable  $T$ . It is modelled by the probability density function

$$f(x) = \begin{cases} kt & 0 \leq t \leq 16 \\ 0 & \text{otherwise} \end{cases}$$

- Show that the value of  $k$  is  $\frac{1}{128}$ .
- Find  $P(T > 4)$ .
- Calculate an exact value for  $E(T)$  and for  $Var(T)$ .
- Write down the mode of the distribution of  $T$ .

**Q49** - ID: 843

[7 marks, 8 minutes]

The continuous random variable  $X$  has probability density function  $f(x)$  given by

$$f(x) = \begin{cases} kx^2(8-x) & 0 \leq x \leq 8 \\ 0 & \text{otherwise} \end{cases}$$

- Show that  $k = \frac{12}{4096}$ .
- Find  $E(X)$ .
- Find  $P(X < 5)$ .

**Q50** - ID: 5973

[7 marks, 8 minutes]

The random variable  $T$  has the probability density function

$$f(t) = \frac{\pi}{4} \cos\left(\frac{\pi t}{2}\right), -1 \leq t \leq 1$$

- (a) Find  $P(T = 0)$ .
- (b) Find the interquartile range.

**Q51** - ID: 5501

[5 marks, 6 minutes]

The continuous random variable  $X$  is uniformly distributed over the interval  $[-2, 6]$ .

- (a) Write down fully the probability density function  $f(x)$  of  $X$ .
- (b) Find  $E(X^2)$
- (c) Find  $P(-1.9 < X < 2)$

**Q52** - ID: 3061

[6 marks, 7 minutes]

A string AB of length 16 cm is cut, in a random place C, into two pieces. The random variable  $X$  is the length of AC.

- (a) Write down the name of the probability distribution of  $X$ .
- (b) Find the values of  $E(X)$  and  $Var(X)$ .
- (c) Find  $P(X > 6)$ ,
- (d) Write down the probability that AC is 6 cm long.

**Q53** - ID: 2054

[7 marks, 8 minutes]

The continuous random variable  $L$  represents the error, in mm, made when a machine cuts rods to a target length.

The distribution of  $L$  is continuous uniform over the interval  $[-8, 8]$ .

- (a) Find  $P(L < -3)$ ,
- (b) Find  $P(L < -7 \text{ or } L > 7)$ .

A random sample of 12 rods cut by the machine was checked.

- (c) Find the probability that more than half of them were within 7 mm of the target length.

**Q54** - ID: 2047

[5 marks, 6 minutes]

The random variable  $X$  is uniformly distributed over the interval  $[-1, 5]$ .

- (a) Find  $E(X)$
- (b) Find  $Var(X)$
- (c) Find  $P(-0.5 < X < 1)$

---

**Q55** - ID: 800

[5 marks, 6 minutes]

The continuous random variable  $X$  is uniformly distributed over the interval  $[-2, 1]$ .

- (a) Find  $P(X < 0.8)$
- (b) Find  $E(X)$
- (c) Find  $Var(X)$

---

**Q56** - ID: 951

[6 marks, 7 minutes]

An engineer measures, to the nearest cm, the lengths of metal rods.

- (a) Suggest a suitable model to represent the differences between the true lengths and the measured lengths.
- (b) Find the probability that for a randomly chosen rod the measured length will be within 0.45 cm of the true length. Two rods are chosen at random.
- (c) Find the probability that for both rods the measured lengths will be within 0.45 cm of their true lengths.

---

**Q57** - ID: 953

[7 marks, 8 minutes]

A piece of string AB has length 15 cm. The string is cut at a randomly chosen point P, into two pieces. The random variable  $X$  represents the length, in cm, of the piece AP.

- (a) Describe the distribution of  $X$ .
- (b) Find the cumulative distribution function of  $X$ .
- (c) Write down  $P(X < 11)$

---

**Q58** - ID: 667

[8 marks, 10 minutes]

$T$  is a continuous uniform random variable defined over the interval  $[0, 6]$ .

- (a) Find  $P(T < 0.8)$
- (b) Find  $E(T)$
- (c) Find  $Var(T)$



---

**Q59** - ID: 538

[18 marks, 22 minutes]

On a typical weekday morning customers arrive at a village shop independently at a rate of 6 per 10 minute period.

Find the probability that

(a) at least 6 customers arrive in the next 10 minutes.

(b) no more than 7 customers arrive between 11.00am and 11.30am.

The period from 11.00am to 11.30 next Tuesday will be divided into 6 periods of 5 minutes each.

(c) Find the probability that no customers arrive in at most one of those periods.

The village shop is open for  $3\frac{1}{2}$  hours on Wednesday mornings.

(d) Using a suitable approximation, estimate the probability that more than 137 customers arrive at the village shop next Wednesday morning.

---

**Q60** - ID: 610

[6 marks, 7 minutes]

The probability of a centre forward scoring in a football game within 1 minute of kick-off is 0.13

A group of 52 centre forwards each played a game once.

Using a suitable approximation, find the probability that more than 10 of them scored a goal within 1 minute of kick-off.

---

**Q61** - ID: 863

[5 marks, 6 minutes]

At a particular road junction, most vehicles passing are cars, the rest are lorries.

Over a long period of time it has been established that

15% of vehicles passing are lorries.

Given that one day there are 70 passing vehicles, use a suitable approximation to find the probability that the number of lorries is no more than 17

**Q62** - ID: 889

[11 marks, 13 minutes]

An internet service provider has a large number of users regularly connecting to its computers. On average only 5 users every hour fail to connect at their first attempt. Find the probability that in a randomly chosen hour

- (a) all internet users connect at their first attempt.
- (b) more than 4 users fail to connect at their first attempt.
- (c) Write down the distribution of numbers of users failing to connect at the first attempt in an 7-hour period.
- (d) Using a suitable approximation, estimate the probability that 39 or more users fail to connect at the first attempt in a randomly chosen 7-hour period.

**Q63** - ID: 809

[12 marks, 14 minutes]

A botanist suggests that the number of a particular variety of weed growing in a meadow can be modelled by a Poisson distribution.

- (a) Write down two conditions that must apply for this model to be applicable.
- Assuming this model and a mean of 0.8 weeds per  $m^2$ , find
- (b) the probability that in a randomly chosen plot of size  $6m^2$  there will be fewer than 3 of these weeds.
  - (c) Using a suitable approximation, find the probability that in a plot of  $70m^2$  there will be more than 47 of these weeds.

**Q64** - ID: 3065

[8 marks, 10 minutes]

A factory manufactures 1700 DVDs every day. It is known that 3% of DVDs are faulty.

- (a) Using a normal approximation, estimate the probability that at least 38 faulty DVDs are produced in one day
- The quality control system in the factory identifies and destroys every faulty DVD at the end of the manufacturing process. It costs 87p to manufacture a DVD and the factory sells non-faulty DVDs for £9.
- (b) Find the expected profit made by the factory per day.

---

**Q65** - ID: 3140

[12 marks, 14 minutes]

The probability that a sunflower plant grows over 1.5 metres high is 0.32. A random sample of 60 sunflower plants is taken and each sunflower plant is measured and its height recorded.

(a) Find the probability that the number of sunflower plants over 1.5 m high is between 19 and 23 (inclusive) using

(i) a Poisson approximation,

(ii) a Normal approximation.

(b) Write down which of the approximations used in part (a) is the most accurate estimate of the probability. You must give a reason for your answer.

---

**Q66** - ID: 6189

[5 marks, 6 minutes]

On a production line making toys, the probability of any toy being faulty is 0.08.

A random sample of 170 toys is checked.

Use a suitable approximation to find the probability that there are at least 17 faulty toys.

---

**Q67** - ID: 628

[6 marks, 7 minutes]

In a box containing many sheets of paper,  $\frac{1}{4}$  of them are red and the remainder are of different colours. A group of children remove some sheets and the teacher wants to know if the proportion of red sheets has changed. From a random sample of 15 sheets he finds that 2 are red.

Test at the 5 percent level whether or not there is evidence that the proportion has changed.

---

**Q68** - ID: 884

[6 marks, 7 minutes]

In a coffee bar, the probability of a customer asking for an espresso coffee is 0.5. During one particular day, in a random sample of 15 customers, 2 asked for espresso coffee.

Test at the 15 percent level whether the proportion of espresso coffees ordered that day is unusually low.

---

**Q69** - ID: 733

[5 marks, 6 minutes]

In a pilot study, the editor of a journal took a random sample of 40 subscribers to see if they agreed to a name change of the journal. He believed that 35 percent would. In fact only 5 subscribers agreed to the name change.

Stating your hypotheses clearly, test at the 5 percent level whether the percent agreeing to the change is less than the editor believes.

---

**Q70** - ID: 2049

[5 marks, 6 minutes]

A teacher thinks that 20% of the pupils in a school read the Deano comic regularly. He chooses 30 pupils at random and finds 11 of them read the Deano. Test, at the 5 % level of significance, whether or not there is evidence that the percentage of pupils that read the Deano is different from 20%. State your hypotheses clearly.

---

**Q71** - ID: 375

[7 marks, 8 minutes]

Brad planted 25 seeds in his greenhouse. He has read in a gardening manual that the probability of one of these seeds germinating is 0.25. 9 of the seeds germinate. He claimed that the gardening book had underestimated this probability. Stating your hypotheses clearly, test at the 10 percent level of significance, his claim.

---

**Q72** - ID: 5457

[8 marks, 10 minutes]

A psychology student is investigating memory. In an experiment volunteers are given 30 seconds to memorise a set of objects. The items are removed and the volunteers asked to name them all. It is found that the probability a volunteer names them all is 0.33. The student believes this probability may be increased if the volunteer listens to ambient music at the same time. Of 30 volunteers asked to do the experiment to music, 15 of them recall all the items.

- (a) Write down suitable hypotheses for a test to determine if there is evidence to support the student's belief, giving a reason for your choice of alternate hypothesis.  
(b) Carry out the test at the 10 percent level.

**Q73** - ID: 2053

[3 marks, 4 minutes]

Before introducing a new rule, the secretary of a golf club decided to find out how members might react to this rule.

- (a) Explain why the secretary decided to take a random sample of club members rather than ask all the members.
- (b) Suggest a suitable sampling frame.
- (c) Identify the sampling units.

**Q74** - ID: 925

[7 marks, 8 minutes]

The number of cars sold per week at a car showroom follows a Poisson distribution with mean 2. The firm appoints a new salesman and wants to find out whether or not car sales increase as a result. After the appointment of the salesman, the number of sales in a randomly chosen 4-week period is 13. Stating your hypotheses clearly, test at the 5 percent level of significance, whether or not the new salesman has increased car sales.

**Q75** - ID: 882

[8 marks, 10 minutes]

A single observation  $x$  is to be taken from a Poisson distribution with parameter  $\lambda$ .

This observation is to be used to test  $H_0 : \lambda = 7$  against

$$H_1 : \lambda \neq 7$$

- (a) Using a 5% level of significance, find the critical region for this test assuming that the probability of rejecting in either tail is as close as possible to 2.5 percent.
  - (b) Write down the significance level of this test.
- The actual value of  $x$  obtained was 11.
- (c) State a conclusion that can be drawn on this value.

**Q76** - ID: 356

[6 marks, 7 minutes]

A bus inspector believes that the number of late buses per day driven by the new driver is 1.75. Later the inspector notices that a 4-day report contains only 5 lates and wonders if the number of late buses per day driven by this driver is now less than 1.75.

Assuming a Poisson distribution, and stating your hypotheses clearly, carry out this test at the 10 percent level of significance.

## s2 full exam paper - Mark Scheme

**A1** - ID: 4378

[4 marks, 5 minutes]

- (a) number of teams =  ${}^{22}C_9 = \frac{22!}{9!13!} = 497420$  |M1A1
- (b) number of teams =  ${}^9C_3 \times {}^{13}C_6 = 144144$  |M1A1

**A2** - ID: 3141

[5 marks, 6 minutes]

- (a) number of codes =  ${}^nP_r = \frac{n!}{(n-r)!} = \frac{7!}{2!} = 2520$  |M2A1
- (b) number of codes =  $7^5 = 16807$  |M1A1

**A3** - ID: 6091

[7 marks, 8 minutes]

- (a) number of arrangements =  ${}^nP_r = \frac{n!}{(n-r)!} = \frac{5!}{0!} = 120$  |M1A1
- (b) number of arrangements =  $2 \times 4! = 48$  |M2A1
- (c) probability =  $\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$  |M1A1

**A4** - ID: 6101

[5 marks, 6 minutes]

- (a) Probability =  $\frac{{}^7C_2 \times {}^8C_3}{{}^{15}C_5}$  |M2
- = 0.392 |A1
- (b) number =  $3! \times 2! = 12$  |M1A1

**A5** - ID: 6194

[7 marks, 8 minutes]

- (a) Number =  $\frac{12!}{4!2!2!2!} = 1247400$  |M1A1
- (b) Number =  $\frac{6!}{4!2!} \times \frac{6!}{2!2!2!}$  |B1
- = 1350 |M1A1
- (c) Number =  $2 \times 2 \times {}^8C_2 = 112$  |M1A1

**A6** - ID: 4572

[7 marks, 8 minutes]

- (a)  $P(X \geq 6) = 1 - P(X \leq 5)$  |B1  
 $= 1 - 0.0328 = 0.9672$  |B1
- (b)  $P(Y = 4) = \binom{11}{4} (1 - 0.22)^7 0.22^4 = 0.136$  |M1A1
- (c)  $P(Z \geq 1) > 0.93 \Rightarrow P(Z = 0) < 0.07$   
 $\Rightarrow (1 - 0.22)^n < 0.07$   
 $\Rightarrow 0.78^n < 0.07$  |M1  
 $\Rightarrow n \log 0.78 < \log 0.07$   
 $\Rightarrow n > 10.7$   
 $\Rightarrow n = 11$  |M1A1

**A7** - ID: 5369

[5 marks, 6 minutes]

- mean =  $np = 0.1n$  |B1  
 standard deviation =  $\sqrt{np(1-p)} = \sqrt{0.09n}$  |B1  
 $\Rightarrow 0.1n = \sqrt{0.09n}$   
 $\Rightarrow 0.01n^2 = 0.09n$  |M1A1  
 $\Rightarrow 0.01n = 0.09$   
 $\Rightarrow n = 9$  |A1

**A8** - ID: 4371

[8 marks, 10 minutes]

- (a) two conditions  $\Rightarrow$  Results are independent |B1  
 Probability of winning is constant |B1
- (b) variable : Number of wins |B1
- (c)  $P(X = 6) = P(X = 5) \Rightarrow {}^{17}C_6 p^6 (1-p)^{11} = {}^{17}C_5 p^5 (1-p)^{12}$  |B1  
 $\Rightarrow \frac{17!}{6! \times 11!} p = \frac{17!}{5! \times 12!} (1-p)$  |M1  
 $\Rightarrow \frac{1}{6} p = \frac{1}{12} (1-p)$  |M1  
 $\Rightarrow 12p = 6(1-p)$   
 $\Rightarrow 18p = 6$   
 $\Rightarrow p = \frac{1}{3}$  |M1A1

**A9** - ID: 5471

[9 marks, 11 minutes]

- (a)  $E(X) = np = 7.5$  |M1A1  
 $\Rightarrow E(Y) = 3E(X) + 3 = 25.5$  |M1A1
- (b)  $Var(X) = np(1-p) = 5.25$   
 $\Rightarrow VAR(Y) = 9Var(X) = 47.25$  |M1A1
- (c)  $P(Y = 15) = P(X = 4)$  |B1  
 $= 0.0573$  |M1A1

**A10** - ID: 6094

[9 marks, 11 minutes]

- (a)  $P(X < 9) = P(X \leq 8) = 0.9428$  |B1
- (b)  $P(X = 4) = \binom{11}{4} (1 - 0.54)^7 0.54^4 = 0.1223$  |M1A1
- (c)  $P(3 \leq X < 7) = P(X \leq 6) - P(X \leq 2)$  |M2  
 $= 0.6288 - 0.0175 = 0.6113$  |A1
- (d)  $P(Y = 3) = \binom{14}{3} \left(1 - \frac{4}{13}\right)^{11} \left(\frac{4}{13}\right)^3 = 0.1857$  |M1A1
- (e)  $Var(Y) = 14 \times \frac{4}{13} \times \left(1 - \frac{4}{13}\right) = 2.98$  |A1

**A11** - ID: 910

[4 marks, 5 minutes]

- B(n,p) :  $n = 8, p = 0.2$
- (a)  $P(X \geq 2) = 1 - P(X \leq 1)$  |M1  
 $= 1 - 0.5033$  (Binomial tables)  
 $= 0.4967$  |A1
- (b)  $P(X < 1) = P(X \leq 0)$  |M1  
 $= 0.1678$  (Binomial tables) |A1

**A12** - ID: 539

[8 marks, 10 minutes]

- B(n,p),  $n = 10, p = 0.1$  |B1
- (a)  $P(X = 1) = 0.3874$  (Binomial tables) |M1A1
- (b)  $P(X > 5) = 1 - P(X \leq 5)$   
 $= 1 - 0.9999$  (Binomial tables) |M1  
 $= 0.0001$  |A1
- (c) B(3,0.3874)  
 $P(X = 2) = 3(0.3874)^2(1 - 0.3874)$  |M1A1  
 $= 0.276$  |A1

**A13** - ID: 749

[7 marks, 8 minutes]

- (a) model = Bin(12, 0.15) |B1
- (b)  $P(X = 0) = 0.1422$  (Binomial tables) |M1A1
- (c)  $P(X > 1) = 1 - P(X \leq 1)$   
 $= 1 - 0.4435$  (Binomial tables) |M1  
 $= 0.5565$  |A1
- (d) mean =  $np = 1.8$  |A1  
variance =  $np(1 - p) = 1.53$  |A1



**A14** - ID: 2044

[7 marks, 8 minutes]

- (a) model =  $B(4, 0.5)$   
 $P(X = 2) = \binom{4}{2} 0.5^2 0.5^2 = 0.375$  |M1A1
- (b)  $P(\text{same}) = P(X = 4) + P(X = 0)$  |B1  
 $= 2 \times 0.5^4 = 0.125$  |M1A1
- (c) Probability =  $P(HHT)$  |M1  
 $= 0.5^3 = 0.125$  |A1

**A15** - ID: 3146

[8 marks, 10 minutes]

- (a) Probability =  $0.11^4 = 0.00015$  |M1A1
- (b)  $X \sim B(15, 0.11)$  |B1  
 $P(X \geq 3) = 1 - P(X \leq 3) = 1 - 0.9258$  (Binomial tables)  
 $= 0.0742$  |M1A1
- (c)  $X \sim B(1000, 0.11)$  |B1  
mean =  $np = 110$  |B1  
variance =  $np(1 - p) = 97.9$  |B1

**A16** - ID: 4376

[7 marks, 8 minutes]

- (a)  $P(X \leq 4) = 0.7237$  (Binomial tables) |B2
- (b)  $P(B \geq 5) = 1 - P(X \leq 4)$   
 $= 1 - 0.7237 = 0.276$  (Binomial tables) |B1
- (c)  $V \sim B(n, p)$  :  $n = 11, p = 0.3$   
 $P(V = 4) = {}^{11}C_4 \times 0.3^4 \times 0.7^7$  |B1M2  
 $= 0.220133$   
 $P(5 \text{ in } 12 \text{th}) = 0.220133 \times 0.3 = 0.06604$  |A1

**A17** - ID: 3137

[7 marks, 8 minutes]

- (a)  $B(n, p), n = 12, p = 0.5$   
 $P(X = 12) = P(X \leq 12) - P(X \leq 11) = 1 - 0.9998$  (Binomial tables)  
 $= 0.0002$  |M1A1
- (b)  $P(X > 3) = 1 - P(X \leq 3)$   
 $= 1 - 0.073$  (Binomial tables) |M1  
 $= 0.927$  |A1
- (c)  $B(9, 0.927)$   
 $P(X = 5) = \frac{9!}{5!4!} (0.927)^5 (1 - 0.927)^4$  |M1A1  
 $= 0.0024$  |A1

**A18** - ID: 3145

[10 marks, 12 minutes]

- (a)  $B(n,p), \quad n = 15, p = 0.05$  | **M1**
- (i)  $P(X = 2) = P(X \leq 2) - P(X \leq 1)$   
 $= 0.9638 - 0.829$  (Binomial tables)  
 $= 0.1348$  | **M1A1**
- (ii)  $P(X \geq 2) = 1 - P(X \leq 1)$   
 $= 1 - 0.829$  (Binomial tables)  
 $= 0.171$  | **M1A1**
- (iii)  $E(X) = np = 0.75$  | **M1A1**
- (b)  $P(X > 0) = 0.327 \Rightarrow P(X = 0) = 0.673$   
 $\Rightarrow 0.95^n = 0.673$  | **M1**  
 $\Rightarrow n = \log 0.673 \div \log 0.95 = 7.721$  | **M1**  
 $\Rightarrow n = 7$  | **A1**

**A19** - ID: 4369

[8 marks, 10 minutes]

- (a)  $B(n,p) : n = 10, p = 0.2$  | **B1**  
 $P(B \leq 2) = 0.6778$  (Binomial tables) | **A1**
- (b)  $P(B \geq 4) = 1 - P(X \leq 3)$  | **M1A1**  
 $= 1 - 0.8791 = 0.121$  (Binomial tables) | **A1**
- (c)  $B(n,p) : n = 19, p = 0.28$   
 $P(V = 4) = {}^{19}C_4 \times 0.28^4 \times 0.72^{15}$  | **M2**  
 $= 0.173$  | **A1**

**A20** - ID: 4529

[8 marks, 10 minutes]

- (a)  $B(n,p) : n = 15, p = 0.09$  | **B1**  
 $P(B \geq 3) = 1 - P(X \leq 2)$   
 $= 1 - 0.8531 = 0.147$  (Binomial tables) | **A1**
- (b)  $P(B \geq 4) = 1 - P(X \leq 3)$   
 $= 1 - 0.9601 = 0.04$  (Binomial tables) | **M1A1**
- (c)  $B(n,p) : n = 17, p = 0.02$   
 $P(all) = (1 - 0.02)^{17} = 0.71$  | **M1A1**
- (d)  $P = 1 - P(all) = 0.29$  | **M1A1**

**A21** - ID: 5377

[14 marks, 17 minutes]

- (a)  $S \sim B(n, p)$  :  $n = 25, p = 0.3$  |B1  
 $P(S = 10) = P(S \leq 10) - P(S \leq 9)$  |M1A1  
 $= 0.9022 - 0.8106 = 0.0916$  (Binomial tables)
- (b)  $P(8 \leq S \leq 13) = P(S \leq 13) - P(S \leq 7)$  |B3  
 $= 0.994 - 0.5118 = 0.4822$  (Binomial tables) |M1A1
- (c)  $P(\text{third house}) = (1 - 0.3)^2 \times 0.3$  |A1  
 $= 0.147$  |A1
- (d)  $P = 100 + 100S$  |B1  
 $E(P) = E(100 + 100S) = 100 + 100 \cdot 25 \cdot 0.3 = \text{£}850$  |M1A1  
 $\text{Var}(P) = \text{Var}(100 + 100S) = 100^2 \cdot 25 \cdot 0.3 \cdot (1 - 0.3)$   
 $= 52500$   
standard deviation =  $\sqrt{52500} = 229.1$  |M1A1

**A22** - ID: 5456

[7 marks, 8 minutes]

- (a)  $W \sim B(n, p)$  :  $n = 6, p = 0.03$   
 $P(W = 0) = (1 - 0.03)^6 = 0.833$  |M1A1
- (b)  $P(W = 1) = 6 \times 0.03 \times (1 - 0.03)^5 = 0.1546$  |M1  
 $P(W > 1) = 1 - P(W = 0) - P(W = 1)$   
 $= 1 - 0.833 - 0.1546 = 0.0125$  |M1A1
- (c) expected number =  $295 \times 0.0125$  |M1A1  
 $= 3.7$

**A23** - ID: 6193

[4 marks, 5 minutes]

$$P(\text{odd}) = p = \frac{2}{3} \quad |B1$$

$$P(2 \text{ odds}) = {}^3C_2 p^2 (1 - p) = 0.444 \quad |M1$$

$$P(3 \text{ odds}) = p^3 = 0.296 \quad |M1$$

$$P(2 \text{ or } 3 \text{ odds}) = 0.741 \quad |A1$$

**A24** - ID: 2046

[4 marks, 5 minutes]

$$B(n, p), \quad : n = 110, p = 0.05$$

$$\Rightarrow P_o(\lambda) \quad : \lambda = 5.5 \quad |B2$$

$$P(X > 8) = 1 - P(X \leq 8) \quad |M1$$

$$= 1 - 0.8944 \quad (\text{using Poisson tables})$$

$$= 0.1056 \quad |A1$$

**A25** - ID: 607

[4 marks, 5 minutes]

$$\begin{aligned}
 & B(n,p), \quad n = 100, p = 0.085 \\
 & \Rightarrow P(\lambda) \quad \lambda = 8.5 & |B1 \\
 & P(X > 8) = 1 - P(X \leq 8) \\
 & = 1 - 0.5231 \quad (\text{using Poisson tables}) & |M1A1 \\
 & = 0.4769 & |A1
 \end{aligned}$$

**A26** - ID: 3064

[10 marks, 12 minutes]

$$\begin{aligned}
 (a) \text{ Probability} &= 0.01^4 = 1e-8 & |M1A1 \\
 (b) \quad X &\sim B(7, 0.01) & |B1 \\
 P(X > 1) &= 1 - P(X \leq 1) & |M1 \\
 &= 1 - (7(0.01)(0.99)^6 + (0.99)^7) \quad (\text{using Binomial tables}) \\
 &= 0.00203 & |A1 \\
 (c) \quad X &\sim B(910, 0.01) & |B1 \\
 \mu &= np = 9.1 \\
 \sigma^2 &= np(1-p) = 9.009 & |B2 \\
 (d) \quad X &\sim P_o(9.1) \\
 &\Rightarrow P(X > 6) = 1 - P(X \leq 6) & |M1 \\
 &\Rightarrow P(X > 6) = 1 - 0.1978 = 0.8022 & |A1
 \end{aligned}$$

**A27** - ID: 3147

[5 marks, 6 minutes]

$$\begin{aligned}
 (a) \quad X &\sim B(150, 0.04) & |B1 \\
 \text{mean} &= np = 6 & |B1 \\
 \text{variance} &= np(1-p) = 5.76 & |B1 \\
 (b) \quad X &\sim P_o(6) \\
 P(X > 9) &= 1 - P(X \leq 9) \\
 &= 1 - 0.9161 = 0.0839 & |M1A1
 \end{aligned}$$

**A28** - ID: 5503

[9 marks, 11 minutes]

$$\begin{aligned}
 (a) \quad X &\sim B(6, 0.04) \\
 P(X = 1) &= 6 \times 0.96^5 \times 0.04 = 0.1957 & |M1A1 \\
 (b) \quad P(X \geq 2) &= 1 - P(X = 0) - P(X = 1) & |M1 \\
 &= 1 - 0.96^6 - 0.1957 \\
 &= 0.0216 & |M1A1 \\
 (c) \quad X &\sim P_o(9.2000000000000001) & |B1 \\
 P(1 \leq X \leq 5) &= P(X \leq 5) - P(X = 0) & |M1 \\
 &= 0.1041 - 0.0001 \\
 &= 0.104 & |M1A1
 \end{aligned}$$

**A29** - ID: 2328

[6 marks, 7 minutes]

- (a)  $P(7 \leq X \leq 14) = P(X \leq 14) - P(X \leq 6)$   
 $= 0.9165 - 0.1301$  (using Poisson tables)  
 $= 0.7864$  |M1A1
- (b)  $P(X \geq 7) = 1 - P(X \leq 6)$   
 $= 1 - 0.1301$  (using Poisson tables)  
 $= 0.8699$  |M1A1
- (c)  $P(7 \leq X \leq 14 | X \geq 7) = \frac{P(7 \leq X \leq 14)}{P(X \geq 7)} = \frac{0.7864}{0.8699} = 0.904$  |M1A1

**A30** - ID: 5500

[11 marks, 13 minutes]

- (a) distribution :  $P_o(4)$  |B1  
 $P(X > 3) = 1 - P(X \leq 3)$   
 $= 1 - 0.4335 = 0.5665$  (using Poisson tables) |M1A1
- (b)  $P(4 \text{ or } 5) = P(X = 5) - P(X = 3)$   
 $= 0.7851 - 0.4335 = 0.3516$  |M1A1
- (c)  $\mu = \frac{299}{80} = 3.74$  |B1  
 $\sigma^2 = \frac{1387}{80} - \left(\frac{299}{80}\right)^2 = 3.37$  |M1A1
- (d)  $\mu \approx \sigma^2 \Rightarrow$  Poisson dist applicable |B1
- (e)  $P(X = 6) = \frac{e^{-3.74} 3.74^6}{6!} = 0.09$  |M1A1

**A31** - ID: 5375

[8 marks, 10 minutes]

- (a)  $P(1 \leq X \leq 4) = P(X \leq 4) - P(X \leq 0)$   
 $= 0.9998 - 0.6065$  (using Poisson tables) |B2  
 $= 0.3933$  |A1
- (b)  $P(X = 4) = \frac{e^{-3.44} 3.44^4}{4!}$   
 $= 0.187$  |M1A1
- (c)  $P(X < 3) = e^{-3.44} \left(1 + 3.44 + \frac{3.44^2}{2}\right)$   
 $= 0.332$  |M1A1  
|A1

**A32** - ID: 3138

[11 marks, 13 minutes]

- (a) conditions : Events occur - at a constant rate  
independently or randomly  
singly - any 2 comments
- (b)  $Po(6)$
- (i)  $P(X = 5) = P(X \leq 5) - P(X \leq 4) = 0.4457 - 0.2851 = 0.1606$
- (ii)  $P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.6063 = 0.3937$
- (c) Probability =  $P(1 \text{ car } 0 \text{ other} + 0 \text{ car } 1 \text{ other})$   
 $= (e^{-1} \times e^{-3}) + (e^{-1} \times 3e^{-3})$   
 $= 0.0733$

|B2

|A1

|M1A1

|M1A1

|B1

|M1A1

|A1

**A33** - ID: 3063

[8 marks, 10 minutes]

- (a) model =  $P_o(2.4)$
- (b) assumptions : faulty components occur at constant rate, independently, singly
- (c)  $P(X = 3) = \frac{e^{-2.4} 2.4^3}{3!}$   
 $= 0.209$
- (d)  $\lambda = 7.2$   
 $P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-7.2} = 0.9993$

|B1

|B2

|M1

|A1

|B1

|M1A1

**A34** - ID: 2045

[9 marks, 11 minutes]

- (a) model =  $P_o(3.1)$
- (b)  $P(X = 5) = \frac{e^{-3.1} 3.1^5}{5!}$   
 $= 0.1075$
- (c)  $P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-3.1} = 0.955$   
 $\Rightarrow P((c)) = 0.955^5 = 0.7944$
- (d)  $Po(\lambda) : \lambda = 18.6$   
 $P(X > 3) = 1 - P(X \leq 3)$   
 $= 1 - 0 = 1$

|B1

|A1

|A1

|B1

|M1A1

|B1

|M1A1

**A35** - ID: 713

[6 marks, 7 minutes]

- (a)  $P(X < 2) = P(X \leq 1)$   
 $= e^{-\frac{3}{7}} + \frac{3}{7} e^{-\frac{3}{7}}$   
 $= 0.651 + 0.279$   
 $= 0.931$
- (b) Poisson dist  $\lambda = 12$   
 $\Rightarrow P(X \leq 4) = 0.0076$

|M1

|M1A1

|B1

|M1A1

**A36** - ID: 868

[6 marks, 7 minutes]

- (a)  $P(\lambda), \lambda = 6$   
 $P(X = 5) = P(X \leq 5) - P(X \leq 4)$   
 $= 0.4457 - 0.2851$  (using Poisson tables) |M1A1  
 $= 0.1606$  |A1
- (b)  $P(X > 11) = 1 - P(X \leq 11)$  |M1A1  
 $= 1 - 0.9799$  (using Poisson tables) |A1  
 $= 0.0201$

**A37** - ID: 386

[6 marks, 7 minutes]

- (a)  $Po(1.2)$  1 minute period |B1  
 $P(X \geq 1) = 1 - e^{-1.2} = 0.699$  |M1A1
- (b)  $Po(1.2t)$   $t$  minute period  
 $P(X \geq 1) = 0.683$   
 $\Rightarrow P(X = 0) = 1 - 0.683 = 0.317$   
 $\Rightarrow e^{-1.2t} = 0.317$  |B1  
 $\Rightarrow \ln(e^{-1.2t}) = \ln(0.317)$   
 $\Rightarrow -1.2t = \ln(0.317)$  |M1  
 $\Rightarrow 1.2t = 1.148853505$   
 $\Rightarrow t = 0.96$  minutes |A1

**A38** - ID: 930

[7 marks, 8 minutes]

- (a)  $Po(0.6)$   
 $P(X = 0) = e^{-0.6} = 0.5488$  |B1
- (b)  $Po(\lambda) : \lambda = 0.6 \times 9 = 5.4$  |B1  
 $P(X = 7) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-5.4} 5.4^7}{7!}$  |M1  
 $= 0.12$  |A1
- (c)  $Po(\lambda) : \lambda = 0.6$   
 $P(X = 0) = e^{-0.6} = 0.5488$   
 $B(n, p) : n = 7, p = 0.5488$  |B1  
 $P(X = 4) = \binom{n}{x} p^x (1 - p)^{n-x} = \binom{7}{4} 0.5488^4 (0.4512)^3$  |M1  
 $= 0.2916$  |A1

**A39** - ID: 5378

[12 marks, 14 minutes]

- (a)  $P(0.23 \leq X \leq 0.75) = F(0.75) - F(0.23)$   
 $= 0.4449 - 0.0008 = 0.4441$  |M1A2
- (b)  $F(0.5) = 0.0625$  |B1  
 $\Rightarrow F(0.5) < 0.5$   
 $\Rightarrow \text{median} > 0.5$  |M1A1
- (c)  $f(x) = F'(x) = 42x^5 - 42x^6$  |M1A1
- (d)  $E(X) = \int_0^1 x \cdot (42x^5 - 42x^6) dx$  |M1A1  
 $= \int_0^1 (42x^6 - 42x^7) dx$   
 $= \left[ \frac{42}{7}x^7 - \frac{42}{8}x^8 \right]_0^1 = 0.75$  |A2

**A40** - ID: 3139

[14 marks, 17 minutes]

- (a)  $F(6) = 1 \Rightarrow k((6)^4 + (6)^2 - 90) = 1$   
 $\Rightarrow k = \frac{1}{1242}$  |M1A1
- (b)  $P(Y > 3.7) = 1 - P(Y < 3.7) = 1 - F(3.7)$   
 $= 1 - k((3.7)^4 + (3.7)^2 - 90) = 0.911$  |M1A1
- (c)  $f(y) = \frac{dF(y)}{dy} = k(4y^3 + 2y)$  |M1A1  
 $\Rightarrow f(y) = \begin{cases} \frac{1}{1242}(4y^3 + 2y) & 3 \leq y \leq 6 \\ 0 & \text{otherwise} \end{cases}$  |B1

**A41** - ID: 3026

[8 marks, 10 minutes]

- (a)  $F(t) = P(T \leq t) = 1 - P(T > t) = 1 - \frac{k}{t^2}$  |M1
- $F(2) = 0 \Rightarrow 1 - \frac{k}{2^2} = 0$   
 $\Rightarrow k = 4$  |M1A1
- (b)  $f(t) = \frac{d}{dt}F(t) = \frac{d}{dt}\left(1 - \frac{4}{t^2}\right) = \frac{8}{t^3}$  |M1A1
- (c)  $\mu = \int_2^{\infty} t\left(\frac{8}{t^3}\right) dt = \int_2^{\infty} \frac{8}{t^2} dt = \left[-\frac{8}{t}\right]_2^{\infty}$  |M1A1  
 $= (0) - (-4) = 4$  |A1



**A42** - ID: 2048

[13 marks, 16 minutes]

(a)  $\int_4^5 f(x) dx = 1 \Rightarrow \int_4^5 kx(x-4) dx = 1 \Rightarrow k \int_4^5 x^2 - 4x dx = 1$  |M1  
 $\Rightarrow k \left[ \frac{1}{3}x^3 - \frac{4}{2}x^2 \right]_4^5 = 1$  |A1  
 $\Rightarrow k \left( \frac{125}{3} - \frac{100}{2} - \frac{64}{3} + \frac{64}{2} \right) = 1$   
 $\Rightarrow \frac{14}{6}k = 1 \Rightarrow k = 0.4286$  |M1A1

(b)  $E(X) = \int_4^5 x \cdot kx(x-4) dx = k \int_4^5 x^3 - 4x^2 dx$  |M1  
 $= k \left[ \frac{1}{4}x^4 - \frac{4}{3}x^3 \right]_4^5 = 4.6786$  |M1A1

(c)  $F(x) = \int_4^x kt(t-4) dt = k \left[ \frac{1}{3}t^3 - \frac{4}{2}t^2 \right]_4^x$  |M1A2  
 $= k \left( \frac{1}{3}x^3 - \frac{4}{2}x^2 + 10.667 \right)$  |A1  
 $\Rightarrow F(x) = \begin{cases} 0 & x < 4 \\ k \left( \frac{1}{3}x^3 - \frac{4}{2}x^2 + 10.667 \right) & 4 < x < 5 \\ 0 & x \geq 5 \end{cases}$  |B2

**A43** - ID: 875

[14 marks, 17 minutes]

(a)  $F(x) = \int_0^x \frac{x}{40} dx = \frac{x^2}{80}$  |B1  
 $F(x) = \frac{32}{40} + \int_1^x 0x \cdot \frac{7}{20} - \frac{4x}{160} dx = \frac{32}{40} + \left[ \frac{7x}{20} - \frac{2x^2}{160} \right]_1^x$  |B1M1  
 $= \frac{32}{40} + \frac{7x}{20} - \frac{2x^2}{160} - \frac{70}{20} + \frac{200}{160} = \frac{7x}{20} - \frac{2x^2}{160} - \frac{29}{20}$  |A1

(b)  $F(x) = \frac{8}{40} + \int_4^x \frac{4}{40} dx = \frac{8}{40} + \left[ \frac{4x}{40} \right]_4^x$  |B1M1  
 $= \frac{8}{40} + \frac{4x}{40} - \frac{16}{40} = \frac{4x}{40} - \frac{8}{40}$  |A1

(c)  $F(x) = 0$  for  $x < 0$  and  $F(x) = 1$  for  $x > 14$  |B1

(d)  $P(X \leq 10.5) = F(10.5) = 0.847$  |M1A1

(e)  $E(X) = \int_0^4 \frac{x^2}{40} dx + \int_4^1 0 \frac{4x}{40} dx + \int_{10}^{14} \frac{7x}{20} - \frac{4x^2}{160} dx$  |M1A1  
 $= \left[ \frac{x^3}{120} \right]_0^4 + \left[ \frac{2x^2}{40} \right]_4^1 0 + \left[ \frac{3.5x^2}{20} - \frac{4x^3}{480} \right]_{10}^{14}$   
 $= \frac{64}{120} + \frac{200}{40} - \frac{32x^2}{40} + \frac{686}{20} - \frac{10976}{480} - \frac{350}{20} + \frac{4000}{480}$   
 $= 7$  |A2

**A44** - ID: 774

[14 marks, 17 minutes]

- (a)  $P(X > 0.7) = 1 - P(X \leq 0.7)$   
 $= 1 - \frac{1}{6}(0.7)^2(7 - (0.7)^2) = 0.468$  |M1A1
- (b)  $f(x) = \frac{dF(x)}{dx} = \frac{14}{6}x - \frac{4}{6}x^3$  |M1A1
- (c)  $E(X) = \int_0^1 x \left( \frac{14}{6}x - \frac{4}{6}x^3 \right) dx = \int_0^1 \frac{14}{6}x^2 - \frac{4}{6}x^4 dx$  |M1A1  
 $= \left[ \frac{14}{18}x^3 - \frac{4}{30}x^5 \right]_0^1 = 0.644$  |A1  
 $E(X^2) = \int_0^1 x^2 \left( \frac{14}{6}x - \frac{4}{6}x^3 \right) dx = \int_0^1 \frac{14}{6}x^3 - \frac{4}{6}x^5 dx$  |M1  
 $= \left[ \frac{14}{24}x^4 - \frac{4}{36}x^6 \right]_0^1 = 0.472$  |A1  
 $Var(X) = E(X^2) - E^2(X) = 0.472 - 0.644^2 = 0.057$  |A1
- (d)  $\frac{df(x)}{dx} = 0 \Rightarrow \frac{14}{6} - \frac{12}{6}x^2 = 0 \Rightarrow x^2 = \frac{14}{12} \Rightarrow \text{mode} = 1.08$  |M1A1  
 $\Rightarrow \text{skewness} = \frac{0.644 - 1.08}{\sqrt{0.057}} = -1.823$  |M1A1

**A45** - ID: 831

[4 marks, 5 minutes]

$$P(X \leq x) = \int_1^x \frac{1}{63.75} t^3 dt = \left[ \frac{t^4}{255} \right]_1^x = \frac{x^4}{255} - \frac{1}{255}$$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{x^4}{255} - \frac{1}{255} & 1 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

|M1A2

|A1

**A46** - ID: 640

[11 marks, 13 minutes]

- (a)  $8k = 1 \Rightarrow k = \frac{1}{8}$  |A1
- (b) median  $\Rightarrow F(m) = 0.5$  |M1  
 $\Rightarrow \frac{1}{8}m^2 + \frac{2}{8}m = 0.5 \Rightarrow m^2 + 2m - 4 = 0$  |A1  
 $\Rightarrow m = 1.236$  |A1
- (c)  $f(x) = \frac{dF(x)}{dx} = \frac{2}{8}(x+1)$  |M1A1  
 $= \begin{cases} 0 & x < 0 \\ \frac{2}{8}(x+1) & 0 \leq x \leq 2 \\ 0 & x > 2 \end{cases}$  |A1
- (d) mode = 2 |A1
- (e)  $E(X) = \int_0^2 x \frac{2}{8}(x+1) dx$  |M1  
 $= \left[ \frac{2}{24}x^3 + \frac{2}{8}x^2 \right]_0^2$  |A1  
 $= \frac{2}{24}2^3 + \frac{2}{8}2^2 = 1.667$  |A1

**A47** - ID: 461

[8 marks, 10 minutes]

$$(a) \quad f(x) = \frac{d}{dx}(F(x)) = \frac{d}{dx}\left(\frac{1}{17}(-x^3 + 8x^2 - 7)\right) = \frac{1}{17}(-3x^2 + 16x) \quad |M1$$

$$f(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{17}(-3x^2 + 16x) & 1 \leq x \leq 2 \\ 0 & x > 2 \end{cases} \quad |A2$$

$$(b) \quad \text{mode} \Rightarrow \frac{d}{dx}f(x) = 0 \Rightarrow \frac{d}{dx}\left(\frac{1}{17}(-3x^2 + 16x)\right) = 0 \Rightarrow \frac{1}{17}(-6x + 16) = 0 \Rightarrow x = 2.67 \Rightarrow \text{mode} = 2.67 \quad |M1$$

$$(c) \quad \mu = \int_1^2 x \frac{1}{17}(-3x^2 + 16x) dx = \int_1^2 \frac{1}{17}(-3x^3 + 16x^2) dx = \left[\frac{1}{17}\left(-\frac{3}{4}x^4 + \frac{16}{3}x^3\right)\right]_1^2 = \frac{1}{17}\left(-\frac{3}{4}2^4 + \frac{16}{3}2^3\right) - \frac{1}{17}\left(-\frac{3}{4} + \frac{16}{3}\right) = 1.534 \quad |M1$$

**A48** - ID: 5502

[11 marks, 13 minutes]

$$(a) \quad \int ktdx = 1 \Rightarrow \int_0^{16} kt dx = 1 \quad |B1$$

$$\Rightarrow \left[\frac{1}{2}kt^2\right]_0^{16} = 1 \Rightarrow 128k = 1$$

$$\Rightarrow k = \frac{1}{128} \quad |M1A1$$

$$(b) \quad P(T > 4) = \int_4^{16} kt dx = \left[\frac{1}{2}kt^2\right]_4^{16} = \frac{15}{16} \quad |M1A1$$

$$(c) \quad E(T) = \int_0^{16} kt^2 dx = \left[\frac{1}{3}kt^3\right]_0^{16} = \frac{32}{3} \quad |M1A1$$

$$E(T^2) = \int_0^{16} kt^3 dx = \left[\frac{1}{4}kt^4\right]_0^{16} = \frac{128}{1} \quad |M1A1$$

$$\text{Var}(T) = \frac{128}{1} - \left(\frac{32}{3}\right)^2 = \frac{128}{9} \quad |A1$$

$$(d) \quad \text{mode} = 16 \quad |B1$$

**A49** - ID: 843

[7 marks, 8 minutes]

$$(a) \quad \int_0^8 f(x) dx = 1 \Rightarrow \int_0^8 kx^2(8-x) dx = 1 \Rightarrow k \int_0^8 8x^2 - x^3 dx = 1 \quad |M1$$

$$\Rightarrow k \left[\frac{8}{3}x^3 - \frac{1}{4}x^4\right]_0^8 = 1 \quad |A1$$

$$\Rightarrow k \left(\frac{4096}{3} - \frac{4096}{4}\right) = 1 \Rightarrow \frac{4096}{12}k = 1$$

$$\Rightarrow k = \frac{12}{4096} \quad |A1$$

$$(b) \quad E(X) = \int_0^8 x \cdot kx^2(8-x) dx = k \int_0^8 8x^3 - x^4 dx \quad |M1$$

$$= k \left[\frac{8}{4}x^4 - \frac{1}{5}x^5\right]_0^8 = \frac{24}{5} \quad |A1$$

$$(c) \quad P(X < 5) = \frac{12}{4096} \left[\frac{8}{3}x^3 - \frac{1}{4}x^4\right]_0^5 = 0.519 \quad |M1$$

$$|A1$$

**A50** - ID: 5973

[7 marks, 8 minutes]

$$\begin{aligned}
 \text{(a)} \quad P(T=0) &= \int_0^0 f(t) \, dt = 0 && |M1A1 \\
 \text{(b) let UQ and LQ} &= a \Rightarrow \int_{-a}^a f(t) \, dt = \frac{1}{2} \\
 &\Rightarrow \int_{-a}^a \frac{\pi}{4} \cos\left(\frac{\pi t}{2}\right) = \frac{1}{2} \\
 &\Rightarrow \int_0^a \frac{\pi}{2} \cos\left(\frac{\pi t}{2}\right) = \frac{1}{2} && |M1A1 \\
 &\Rightarrow \left[ \sin\left(\frac{\pi t}{2}\right) \right]_0^a = \frac{1}{2} && |A1 \\
 &\Rightarrow \sin\left(\frac{\pi a}{2}\right) = \frac{1}{2} \\
 &\Rightarrow \frac{\pi a}{2} = \frac{\pi}{6} \Rightarrow a = \frac{1}{3} && |A1 \\
 &\Rightarrow \text{interquartile range} = \frac{2}{3} && |A1
 \end{aligned}$$

**A51** - ID: 5501

[5 marks, 6 minutes]

$$\begin{aligned}
 \text{(a)} \quad X \text{ has pdf } f(x) &= \begin{cases} \frac{1}{8} & -2 \leq x \leq 6 \\ 0 & \text{otherwise} \end{cases} && |B2 \\
 \text{(b)} \quad E(X^2) &= \int_{-2}^6 x^2 f(x) \, dx && |B1 \\
 &= \int_{-2}^6 \frac{x^2}{8} \, dx = \left[ \frac{x^3}{24} \right]_{-2}^6 = 9.333 && |M1A1 \\
 \text{(c)} \quad P(-1.9 < X < 2) &= \frac{1}{8}(2 - -1.9) \\
 &= 0.488 && |M1A1
 \end{aligned}$$

**A52** - ID: 3061

[6 marks, 7 minutes]

$$\begin{aligned}
 \text{(a)} \quad \text{name} &= \text{Continuous uniform (or rectangular) distribution} && |B1 \\
 \text{(b)} \quad E(X) &= \frac{\alpha + \beta}{2} = 8 && |B1 \\
 \text{Var}(X) &= \frac{(\beta - \alpha)^2}{12} = \frac{256}{12} && |M1A1 \\
 \text{(c)} \quad P(X > 6) &= \frac{10}{16} && |B1 \\
 \text{(d)} \quad P(X = 6) &= 0 && |B1
 \end{aligned}$$

**A53** - ID: 2054

[7 marks, 8 minutes]

$$\begin{aligned}
 L \quad \text{has pdf } f(x) &= \frac{1}{16}, \text{ for } -8 \leq x \leq 8 \\
 \text{(a)} \quad P(L < -3) &= \frac{1}{16}(-3 - -8) = 0.313 && |B1 \\
 \text{(b)} \quad P(L < -7 \text{ or } L > 7) &= 2 \times \frac{1}{16}(-7 - -8) = 0.125 && |M1A1 \\
 \text{(c)} \quad P(-7 < L < 7) &= 0.875 && |B1 \\
 &\Rightarrow X \sim B(12, 0.875) && |M1 \\
 &\Rightarrow P(X > 6) = 1 - P(X \leq 6) = 0.998 && |M1A1
 \end{aligned}$$

**A54** - ID: 2047

[5 marks, 6 minutes]

- $X$  has pdf  $f(x) = \frac{1}{6}$ , for  $-1 \leq x \leq 5$
- (a)  $E(X) = \frac{\alpha+\beta}{2} = \frac{-1+5}{2} = 2$  |B1
- (b)  $Var(X) = \frac{(\beta-\alpha)^2}{12} = \frac{(5-(-1))^2}{12}$   
 $= 3$  |M1A1
- (c)  $P(-0.5 < X < 1) = \frac{1}{6}(1 - -0.5)$   
 $= 0.25$  |M1A1

**A55** - ID: 800

[5 marks, 6 minutes]

- $X$  has pdf  $f(x) = \frac{1}{3}$ , for  $-2 \leq x \leq 1$
- (a)  $P(X < 0.8) = \frac{1}{3}(0.8 - -2)$   
 $= 0.933$  |B1
- (b)  $E(X) = \frac{\alpha+\beta}{2} = \frac{-2+1}{2} = -0.5$  |M1A1
- (c)  $Var(X) = \frac{(\beta-\alpha)^2}{12} = \frac{(1-(-2))^2}{12}$   
 $= \frac{9}{12}$  |M1A1

**A56** - ID: 951

[6 marks, 7 minutes]

- (a)  $X \sim$  continuous Uniform dist,  $U(-0.5, 0.5)$  |B1  
with pdf  $f(x) = 1$ , for  $-0.5 \leq x \leq 0.5$  |B1
- (b)  $\int_{-0.45}^{0.45} 1 \, dx = [x]_{-0.45}^{0.45} = 0.9$  |M1A1
- (c) Prob =  $0.9 \times 0.9$   
 $= 0.81$  |M1A1

**A57** - ID: 953

[7 marks, 8 minutes]

- (a)  $X \sim$  continuous Uniform distribution |B1  
with pdf  $f(x) = \frac{1}{15}$ , for  $0 \leq x \leq 15$  |B1
- (b)  $\int_0^x \frac{1}{15} \, dt = \left[ \frac{1}{15}t \right]_0^x = \frac{1}{15}$  |M1A1
- $F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{15}x, & 0 \leq x \leq 15 \\ 1, & x > 15 \end{cases}$  |B2
- (c)  $P(X < 11) = \frac{1}{15}11 = 0.7333$  |B1

**A58** - ID: 667

[8 marks, 10 minutes]

- T has pdf  $f(x) = \frac{1}{6}$ , for  $0 \leq x \leq 6$  |B1
- (a)  $P(T < 0.8) = \int_0^{0.8} \frac{1}{6} dx = \left[ \frac{1}{6}x \right]_0^{0.8} = 0.13$  |M1A1
- (b)  $E(T) = \int_0^6 x \cdot \frac{1}{6} dx = \left[ \frac{1}{12}x^2 \right]_0^6 = 3$  |M1A1
- (c)  $Var(T) = E(T^2) - E(T)^2$   
 $= \int_0^6 x^2 \cdot \frac{1}{6} dx - E(T)^2 = \left[ \frac{1}{18}x^3 \right]_0^6 - E(T)^2$   
 $= \frac{216}{18} - \frac{36}{4} = \frac{36}{12}$  |M1A2

**A59** - ID: 538

[18 marks, 22 minutes]

- (a)  $X \sim Po(6)$  : 10 minute period  
 $P(X \geq 6) = 1 - P(X \leq 5)$  |M1  
 $= 1 - 0.4457 = 0.5543$  |A1
- (b)  $Y \sim Po(18)$  : 30 minute period  
 $P(X \leq 7) = 0.0029$  |B1  
|M1A1
- (c)  $Z \sim Po(3)$  : 5 minute period  
 $P(Z = 0) = e^{-3} = p$  |B1  
 $\Rightarrow C \sim B(6, e^{-3})$  |M1  
 $\Rightarrow P(C \leq 1) = (1 - p)^6 + 6(1 - p)^5 p = 0.9675$  |M2A2
- (d)  $W \sim Po(126)$  : 210 minute period  
 $\Rightarrow W \approx N(126, 126)$  |B1  
 $\Rightarrow P(W > 137) \approx P(W \geq 137.5)$  |M1A1  
 $= 1 - P(W \leq 137.5)$  |M1  
 $= 1 - \Phi\left(\frac{137.5 - 126}{\sqrt{126}}\right) = 1 - \Phi(1.024)$  |M1A1  
 $= 0.1528$  |A1

**A60** - ID: 610

[6 marks, 7 minutes]

- $B(n, p)$  :  $n = 52, p = 0.13$   
 $\Rightarrow N(\mu, \sigma^2)$  :  $\mu = 6.76, \sigma^2 = 5.88$  |B2
- $P(X > 10) = 1 - P(X < 10.5)$ , with continuity correction  
 $= 1 - \Phi\left(\frac{10.5 - 6.76}{\sqrt{5.88}}\right) = 1 - \Phi(1.5)$  |M1A1  
 $= 0.0618$  using Normal tables |M1A1

**A61** - ID: 863

[5 marks, 6 minutes]

$$\begin{aligned}
 & B(n,p), \quad : n = 70, p = 0.15 \\
 \Rightarrow & N(\mu, \sigma^2) \quad : \mu = 10.5, \sigma^2 = 8.93 & |B2 \\
 & P(X \leq 17) = P(X < 17.5) \quad \text{using continuity correction} \\
 & = \Phi\left(\frac{17.5 - 10.5}{\sqrt{8.93}}\right) & |M1A1 \\
 & = \Phi(2.3425) = 1 - \Phi(-2.3425) \\
 & = 0.9904 \text{ using Normal tables} & |A1
 \end{aligned}$$

**A62** - ID: 889

[11 marks, 13 minutes]

$$\begin{aligned}
 (a) \quad & X \sim P(5) \\
 & P(X = 0) = 0.0067 & |M1A1 \\
 (b) \quad & P(X > 4) = 1 - P(X \leq 4) \\
 & = 1 - 0.4405 \\
 & = 0.5595 & |M1A1 \\
 (c) \quad & Y \sim P(35) & |B1 \\
 (d) \quad & Z \sim N(35, 35) & |B2 \\
 & \Rightarrow P(Z \geq 39) \approx P(Z \geq 38.5) & |M1 \\
 & = 1 - P(Z \leq 38.5) \\
 & = 1 - \Phi\left(\frac{38.5 - 35}{\sqrt{35}}\right) & |M1 \\
 & = 1 - \Phi(0.5916) & |A1 \\
 & = 0.2771 & |A1
 \end{aligned}$$

**A63** - ID: 809

[12 marks, 14 minutes]

$$\begin{aligned}
 (a) \quad & \text{conditions} = \text{independent growth; at random} & |B2 \\
 (b) \quad & X \sim Po(4.8) \quad 6m^2 & |B1 \\
 & P(X < 3) = P(X \leq 2) & |B1 \\
 & = 0.1425 & |M1A1 \\
 (c) \quad & Y \sim Po(56) \quad 70m^2 & |B1 \\
 & \Rightarrow Z \approx N(56, 56) & |B1 \\
 & \Rightarrow P(Z > 47) \approx P(Z \geq 47.5) & |M1 \\
 & = 1 - P(Z \leq 47.5) \\
 & = 1 - \Phi\left(\frac{47.5 - 56}{\sqrt{56}}\right) & |M1 \\
 & = 1 - \Phi(-1.1359) & |A1 \\
 & = 0.872 & |A1
 \end{aligned}$$

**A64** - ID: 3065

[8 marks, 10 minutes]

- (a)  $B(n,p)$ , :  $n = 1700, p = 0.03$   
 $\Rightarrow N(\mu, \sigma^2)$  :  $\mu = 51, \sigma^2 = 49.47$  |B2  
 $P(X > 38) = 1 - P(X < 37.5)$ , with continuity correction |M1  
 $= 1 - \Phi\left(\frac{37.5-51}{\sqrt{49.47}}\right) = 1 - \Phi(-1.92)$   
 $= 0.9726$  using Normal tables |M1A1
- (b)  $E(X) = 51$  |B1  
 expected profit =  $(1700 - 51) \times 9 - 1700 \times 0.87 = £13362$  |M1A1

**A65** - ID: 3140

[12 marks, 14 minutes]

- (a) Distribution :  $X$  is number of plants growing over 1.5 m |B1  
 $\Rightarrow X \sim B(60, 0.32)$
- (i) Poisson  $\Rightarrow X \sim Po(19.2)$  |B1  
 $P(19 \leq X \leq 23) = P(X \leq 23) - P(X \leq 18)$   
 $= 0.8376 - 0.4514 = 0.3862$  |M1A1
- (ii) Normal  $\Rightarrow X \sim N(19.2, 13.056)$   
 $P(19 \leq X \leq 23) = P(X \leq 23.5) - P(X \leq 18.5)$  |M2  
 $= \Phi\left(\frac{23.5-19.2}{\sqrt{13.056}}\right) - \Phi\left(\frac{18.5-19.2}{\sqrt{13.056}}\right)$   
 $= \Phi(1.19) - \Phi(-0.193)$  |A2  
 $= 0.883 - 0.4232 = 0.4598$  |M1A1
- (b) Normal : large  $n$ ,  $p$  close to 0.5 |B2

**A66** - ID: 6189

[5 marks, 6 minutes]

- $B(n,p)$ , :  $n = 170, p = 0.08$   
 $\Rightarrow N(\mu, \sigma^2)$  :  $\mu = 13.6, \sigma^2 = 12.51$  |B1  
 $P(X > 17) = 1 - P(X < 16.5)$ , with continuity correction |M1  
 $= 1 - \Phi\left(\frac{16.5-13.6}{\sqrt{12.51}}\right)$  |M1  
 $= 1 - \Phi(0.82)$  |M1  
 $= 0.2061$  using Normal tables |A1

**A67** - ID: 628

[6 marks, 7 minutes]

- $H_0 : p = \frac{1}{4}, H_1 : p \neq \frac{1}{4}$  |B2  
 $B(15, \frac{1}{4})$   
 $P(X \leq 2) = 0.2361$  (using Binomial tables) |M1A1  
 $0.2361 > 0.025 \Rightarrow$  Non-significant result |B1  
 $\Rightarrow$  Accept  $H_0$   
 $\Rightarrow$  The proportion has not changed |B1



**A68** - ID: 884

[6 marks, 7 minutes]

$$H_0 : p = 0.5, H_1 : p < 0.5 \quad |B2$$

$$B(15, 0.5)$$

$$P(X \leq 2) = 0.0037 \text{ (using Binomial tables)} \quad |M1A1$$

$$0.0037 < 0.05 \Rightarrow \text{Significant result}$$

$$\Rightarrow \text{Reject } H_0 \text{ accept } H_1$$

$$\Rightarrow \text{Proportion unusually low} \quad |M1A1$$

**A69** - ID: 733

[5 marks, 6 minutes]

$$H_0 : p = 0.35, H_1 : p < 0.35 \quad |B1$$

$$B(40, 0.35) \quad |B1$$

$$P(X \leq 5) = 0.0013 \text{ (using Binomial tables)} \quad |M1A1$$

$$0.0013 < 0.05 \Rightarrow \text{Significant result}$$

$$\Rightarrow \text{Reject } H_0 \text{ accept } H_1$$

$$\Rightarrow \text{Percent is lower than editor believes} \quad |M1$$

**A70** - ID: 2049

[5 marks, 6 minutes]

$$H_0 : p = 0.2, H_1 : p \neq 0.2 \quad |B2$$

$$B(30, 0.2)$$

$$P(X \geq 11) = 1 - P(X \leq 10) \quad |M1$$

$$= 0.0256 \text{ (using Binomial tables)}$$

$$0.0256 > 0.025 \Rightarrow \text{Non-significant result} \quad |A1$$

$$\Rightarrow \text{Accept } H_0$$

$$\Rightarrow \text{The percentage has not changed} \quad |A1$$

**A71** - ID: 375

[7 marks, 8 minutes]

$$H_0 : p = 0.25, H_1 : p > 0.25 \quad |B1$$

$$B(25, 0.25) \quad |B1$$

$$P(X \geq 9) = 1 - P(X \leq 8) \quad |M1$$

$$= 1 - 0.8506 = 0.1494 \text{ (using Binomial tables)} \quad |M1A1$$

$$0.1494 > 0.1 \Rightarrow \text{Non-significant result}$$

$$\Rightarrow \text{Accept } H_0 \quad |A1$$

$$\Rightarrow \text{Probability not underestimated} \quad |A1$$

**A72** - ID: 5457

[8 marks, 10 minutes]

- (a)  $H_0 : p = 0.33, H_1 : p > 0.33$  |B2  
 where  $p$  is the probability a volunteer names them all |B1  
 $H_1$  because student believes probability will increase |B1
- (b)  $B(30, 0.33)$   
 $P(X \geq 15) = 1 - P(X \leq 14)$  |M1  
 $= 1 - 0.9601 = 0.0399$  (using Binomial tables) |M1  
 $0.0399 < 0.1 \Rightarrow$  Significant result |M1  
 $\Rightarrow$  Reject  $H_0$  accept  $H_1$  |A1  
 $\Rightarrow$  Probability has increased |A1

**A73** - ID: 2053

[3 marks, 4 minutes]

- (a) reasons : saves time/cheaper/easier |B1  
 (b) sampling frame : List/register/database of all club members/golfers |B1  
 (c) sampling units : Club member(s) |B1

**A74** - ID: 925

[7 marks, 8 minutes]

- $H_0 : \lambda = 2, H_1 : \lambda > 2$  |B2  
 $X \sim P(8)$  |M1  
 $P(X \geq 13) = 1 - P(X \leq 12)$   
 $= 0.0638$  (Poisson tables) |M1A1  
 $0.0638 > 0.05 \Rightarrow$  Non-significant result |M1  
 $\Rightarrow$  Accept  $H_0$   
 $\Rightarrow$  salesman has not increased sales |A1

**A75** - ID: 882

[8 marks, 10 minutes]

- (a)  $P(X \leq 3) = 0.0818$  (Poisson tables) |B1  
 $P(X \leq 2) = 0.0296$  (Poisson tables) |A1  
 $P(X \geq 12) = 0.0533$  (Poisson tables)  
 $P(X \geq 13) = 0.027$  (Poisson tables) |A1  
 Critical region:  $\leq 2, \geq 13$  |M1A1
- (b) level =  $0.0296 + 0.027 = 0.0566\%$  |A1
- (c) conclusion = insufficient evidence that  $\lambda \neq 7$  |A2

**A76** - ID: 356

[6 marks, 7 minutes]

$$H_0 : \lambda = 1.75, H_1 : \lambda < 1.75$$

$$X \sim P(7)$$

$$P(X \leq 5) = 0.3007 \text{ (Poisson tables)}$$

$$0.3007 > 0.1 \Rightarrow \text{Non-significant result}$$

$$\Rightarrow \text{Accept } H_0$$

$$\Rightarrow \text{The number of lates is not less}$$

|B2

|M1

|M1A1

|A1