To print higher-resolution math symbols, click the
Hi-Res Fonts for Printing button on the jsMath control panel.

- - -

MathsNet : A-Level+

Steve Blades

Your account expires on: **7 May, 2012**

my fp3 paper

Syllabus: **EdExcel** Questions: **56** Time: **8 hours 11 minutes** Total Marks: **411**

This paper contains a set of questions followed by the corresponding mark schemes. The time you should spend on each question together with its worth in marks is also given. The content of this paper is based on material from a wide selection of national and international examination boards and organisations.

You are advised to have:
a set of geometrical equipment, pen, HB pencil, eraser. Check if you are allowed a calculator. Some examinations, but not all, allow calculators, including graphical models.

NOTES: The following browsers have been tested with this facility: Mozilla Firefox 3.x, 4.x; Microsoft Internet Explorer versions 6, 7, 8 and 9 RC (see the website for the small font problem with IE7 and IE8 was tested in IE7 compatibility mode), Apple Safari and Google Chrome. Best results are when the background printing of images and colours is enabled (not available in Chrome on Windows/Mac or Safari on Windows). There are known printing format issues with the Opera web browser and we do not recommend using this browser.

Many of the questions use the jsMath applet. This requires special fonts to be installed for successful printing. These fonts can be downloaded from: **http://www.math.union.edu/~dpvc/jsMath/**. Use the **Download the TeX** fonts option. Full instructions for their use can be found at: **http://pubpages.unh.edu/~jsh3/jsMath/**

my fp3 paper

Questions: **56** Time: **8 hours 11 minutes** Total Marks: **411**

Q1 - ID: 439 *[7 marks, 8 minutes]*

Solve the equation

$$18\cosh x + 6\sinh x = 17$$

Give each answer in the form $\ln a$, where a is a rational number.

Q2 - ID: 598 *[5 marks, 6 minutes]*

The displacement of a particle from a fixed point O at time t is given by $x = \sinh t$

At time T the displacement $x = \frac{4}{5}$

(a) Find $\cosh T$.

(b) Hence find e^T and T.

Q3 - ID: 2066 *[6 marks, 7 minutes]*

Find the values of x for which

$$25\cosh x - 19\sinh x = 35$$

giving your answers as natural logarithms.

Q4 - ID: 2069 *[7 marks, 8 minutes]*

(a) Using the definition of $\cosh x$ in terms of exponentials, prove that

$$4\cosh^3 x - 3\cosh x = \cosh 3x.$$

(b) Hence, or otherwise, solve the equation

$$\cosh 3x = 17\cosh x$$

giving your answers as natural logarithms.

Q5 - ID: 4398 *[10 marks, 12 minutes]*

(a) Using the definition of $\sinh x$ in terms of exponentials, prove that

$$4\sinh^3 x = \sinh 3x - 3\sinh x.$$

(b) Find the range of values of the constant k for which the equation

$$\sinh 3x = k\sinh x$$

has real roots other than $x = 0$.

(c) Given that $k = 5$ solve the equation in part (b), giving the non-zero answers in logarithmic form.

Q6 - ID: 415 *[4 marks, 5 minutes]*

An ellipse has equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(a) Find the value of of the eccentricity e.
(b) State the coordinates of the foci of the ellipse

Q7 - ID: 520 *[5 marks, 6 minutes]*

The point S, which lies on the positive x-axis, is a focus of the ellipse with equation $\frac{x^2}{9} + \frac{y^2}{1} = 1$
Given that S is also the focus of a parabola P, with vertex at the origin, find
(a) a cartesian equation for P,
(b) an equation for the directrix of P.

Q8 - ID: 2067 *[4 marks, 5 minutes]*

The hyperbola H has equation $\frac{x^2}{36} - \frac{y^2}{16} = 1$
Find
(a) the value of the eccentricity of H,
(b) the distance between the foci of H.

Q9 - ID: 885 *[6 marks, 7 minutes]*

Find the exact value of the radius of curvature of the curve with equation $y = \arcsin x$ at the point where $x = \frac{\sqrt{2}}{2}$

Q10 - ID: 750 *[6 marks, 7 minutes]*

The curve C has equation $y = \tan^{-1} x^2, 0 \leq y < \frac{\pi}{2}$.
Find, in exact form, the value of the radius of curvature of C at the point where $x = 1$

Q11 - ID: 657 *[7 marks, 8 minutes]*

(a) Show that, for $x = \ln k$, where k is a positive constant,

$$\cosh 5x = \frac{k^{10} + 1}{2k^5}.$$

Given that $f(x) = px - \tanh 5x$, where p is a constant,
(b) find the value of p for which $f(x)$ has a stationary value at $x = \ln 2$, giving your answer as an exact fraction.

Q12 - ID: 438 *[7 marks, 8 minutes]*

The curve with equation

$$y = -x + \tanh 64x, x \geq 0,$$

has a maximum turning point A.

(a) Find, in exact logarithmic form, the x-coordinate of A.

(b) Show that the y-coordinate of A is $\frac{\sqrt{63}}{8} - \frac{1}{64}\ln(8 + \sqrt{63})$.

Q13 - ID: 516 *[4 marks, 5 minutes]*

Given that $y = \arctan 5x$, and assuming the derivative of $\tan x$, prove that

$$\frac{dy}{dx} = \frac{5}{1 + 25x^2}$$

Q14 - ID: 2605 *[6 marks, 7 minutes]*

Given that $f(x) = \arccos 2x$,

(a) Find $f'(x)$

(b) Use a standard series to expand $f'(x)$ and hence find the series for $f(x)$ in ascending powers of x^2, up to the term in x^5.

Q15 - ID: 5305 *[4 marks, 5 minutes]*

Show that $f(x) = \sin^{-1} x - 6x^{\frac{3}{2}} + 1$ has a stationary value when x satisfies

$$81x^3 - 81x + 1 = 0$$

Q16 - ID: 5987 *[6 marks, 7 minutes]*

A normal to the graph of $y = \arctan(x - 6)$, for $x > 0$, has equation $y = -2x + c$, where $c \in \mathbb{R}$. Find the value of c.

Q17 - ID: 473 *[7 marks, 8 minutes]*

A curve has parametric equations

$$x = 5\cos^3 t, y = 5\sin^3 t, 0 \leq t \leq \frac{\pi}{2},$$

The curve is rotated through 2π radians about the x-axis. Find the exact value of the area of the curved surface generated.

Q18 - ID: 794 *[7 marks, 8 minutes]*

(a) Find the integral of $\int \frac{3 + x}{\sqrt{1 - 16x^2}}\,dx$,

(b) Find, to 3 decimal places, the value of

$$\int_0^{0.2} \frac{3 + x}{\sqrt{1 - 16x^2}}\,dx$$

Q19 - ID: 2068 *[5 marks, 6 minutes]*

Evaluate $\int_2^6 \frac{1}{\sqrt{x^2 - 4x + 20}}\,dx$,
giving your answer as an exact logarithm.

Q20 - ID: 1747 *[5 marks, 6 minutes]*

Evaluate $\int_1^3 \frac{1}{\sqrt{x^2 + 4x + 8}}\,dx$,
giving your answer as an exact logarithm.

Q21 - ID: 2607 *[5 marks, 6 minutes]*

Find $\int_0^2 \frac{1}{\sqrt{16x^2 + 25}}\,dx$,
giving your answer in an exact logarithmic form.

Q22 - ID: 2608 *[4 marks, 5 minutes]*

Show that $\int_{-\ln 2}^{\ln 2} (22\cosh x - 4\cosh 2x)\,dx = 25.5$

Q23 - ID: 4234 *[7 marks, 8 minutes]*

Show that

$$\int_5^6 \frac{7 + x}{\sqrt{x^2 - 9}}\,dx = 7\ln\left(\frac{6 + \sqrt{27}}{5 + \sqrt{16}}\right) + \sqrt{27} - \sqrt{16}$$

Q24 - ID: 4700 *[5 marks, 6 minutes]*

(a) Prove that $\frac{d}{dx}(\cosh^{-1} 8x) = \frac{8}{\sqrt{64x^2 - 1}}$.

(b) Hence, or otherwise, find $\int \frac{1}{\sqrt{64x^2 - 1}}\,dx$

Q25 - ID: 545 *[10 marks, 12 minutes]*

(a) Given that $y = arctan3x$, and assuming the derivative of $\tan x$, prove that

$$\frac{dy}{dx} = \frac{3}{1 + 9x^2}$$

(b) Show that

$$\int_0^{\frac{\sqrt{3}}{3}} 6x \arctan 3x\,dx = \frac{1}{9}(4\pi - 3\sqrt{3})$$

Q26 - ID: 2604 *[4 marks, 5 minutes]*

Use a trigonometrical substitution to show that

$$\int_0^3 \frac{1}{(36 - x^2)^{\frac{3}{2}}}\,dx = \frac{1}{36\sqrt{3}}$$

Q27 - ID: 856 *[8 marks, 10 minutes]*

$$I_n = \int x^n e^{5x}\, dx, n \geq 0$$

(a) Prove that for $n \geq 1$

$$I_n = \frac{1}{5}(x^n e^{5x} - nI_{n-1})$$

(b) Find, in terms of e, the exact value of

$$\int_0^1 x^2 e^{5x}\, dx$$

Q28 - ID: 5106 *[6 marks, 7 minutes]*

The line l has equation $r = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 3 \\ 4 \end{pmatrix}$.

(a) Write down a vector equation for l in the form $(r - a) \times b = 0$.
(b) Write down cartesian equations for l.
(c) Find the direction cosines of l and explain, geometrically, what these represent.

Q29 - ID: 5988 *[6 marks, 7 minutes]*

Given any two non-zero vectors a and b , show that

$$|a \times b|^2 = |a|^2|b|^2 - (a \cdot b)^2$$

Q30 - ID: 854 *[10 marks, 12 minutes]*

The line l_1 has equation

$$r = 1i + 3j + 3k + \lambda(4i + 4j + 2k)$$

and the line l_2 has equation

$$r = 3i + pj + 3k + \mu(3i + 3j + 1k)$$

The plane Π_1 contains l_1 and l_2.
(a) Find a vector which is normal to Π_1.
(b) Show that an equation for Π_1 is $-2x + 2y - 0z = 4$.
(c) Find the value of p.
The plane Π_2 has equation $r.(2i + 5j + 1k) = 2$.
d) Find an equation for the line of intersection of Π_1 and Π_2
giving your answer in the form

$$(r - a) \times b = 0.$$

Q31 - ID: 2554 *[4 marks, 5 minutes]*

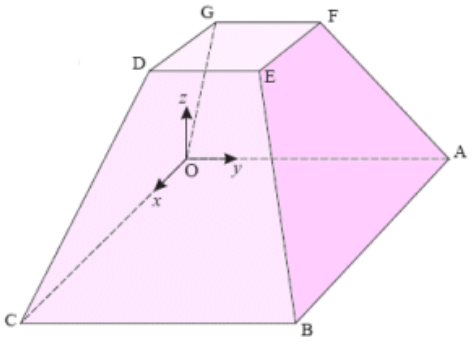
Write down normal vectors to the planes $2x + 6y + 10z = 2$
and $4x - 8y + 4z = 7$.
Hence show that these planes are perpendicular to each other

Q32 - ID: 5107 *[5 marks, 6 minutes]*

The plane Π has equation $r = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 10 \\ 4 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 8 \\ 6 \\ 6 \end{pmatrix}$.

(a) Find an equation for Π in the form $r \cdot n = d$.
(b) State the geometrical significance of the value of d in this case.

Q33 - ID: 5747 *[18 marks, 22 minutes]*



A glass ornament OABCDEFG is a truncated pyramid on a rectangular base. All dimensions are in centimetres.
The vertices are at $A(0, 30, 0), B(30, 30, 0), C(30, 0, 0), D(18, 9, 18)$, $E(18, 21, 18), F(6, 21, 18), G(6, 9, 18)$.
(a) Write down the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$.
(b) Find the length of the edge CD.
(c) Show that the vector $1.5i + k$ is perpendicular to the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$.
(d) Hence find the cartesian equation of the plane BCDE.
(e) Write down vector equations for the lines OG and AF.
(f) Show OG and AF meet at the point P with coordinates $(10, 15, 30)$.
You may assume that the lines CD and BE also meet at the point P.
The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ height.
(g) Find the volumes of the pyramids POABC and PDEFG.
Hence find the volume of the ornament.

Q34 - ID: 5975 *[14 marks, 17 minutes]*

The points A, B, C have position vectors $i + j + 4k, i + 3j + 5k, 4i + k$ respectively and lie in the plane Π. Find
(a) the area of the triangle ABC
(b) the shortest distance from C to the line AB
(c) the cartesian equation of the plane Π.

Q35 - ID: 5976 *[6 marks, 7 minutes]*

The line L passes through the origin and is normal to the plane $\Pi : 7x - 4y + 6z = -5$
It intersects Π at the point D. Find
(a) the coordinates of the point D
(b) the distance of Π from the origin.

Q36 - ID: 5994 *[6 marks, 7 minutes]*

The line L has equation $r = 2i + 4j + 7k + t(2i + j + 3k)$, where $t \in \Re$.
Find the Cartesian equation of the plane which contains both the line L and the point $A(2, -2, 5)$.

Q37 - ID: 7446 *[6 marks, 7 minutes]*

A ray of light coming from the point $(-2, -1, -1)$ is travelling in the direction of vector $\begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix}$ and meets the plane $\Pi : x + 4y + 3z - 20 = 0$.

Find the angle that the ray of light makes with the plane.A.

Q38 - ID: 7448 *[6 marks, 7 minutes]*

Find the vector equation of the line of intersection of the three planes represented by the following system of equations.

$5x - 8y + 2z = 4$
$10x + 4y - 6z = 1$
$-15x - 36y - -24z = 9$

Q39 - ID: 5849 *[8 marks, 10 minutes]*

(a) Write down normal vectors to the planes $7x - y + z = 3$ and $x - z = 4$.
(b) Hence find the acute angle between the planes.
(c) Write down a vector equation of the line through $(3, 1, 3)$ perpendicular to the plane $7x - y + z = 3$.
(d) Find the point of intersection of this line with the plane.

Q40 - ID: 413 *[12 marks, 14 minutes]*

The points A, B, C and D have position vectors a = $2i + 2j + 2k$
b = $4i + 0j + 4k$, c = $5i + 5j + 1k$ and d = $6i + 2j + 1k$ respectively.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$ and hence find the area of triangle ABC.
(b) Find the volume of the tetrahedron ABCD.
(c) Find the perpendicular distance of D to the plane containing A, B and C.

Q41 - ID: 379 *[18 marks, 22 minutes]*

The plane P passes through the points A(-1, 2, 2), B(3, 3, 3) and C(1, 2, 1)
(a) Find a vector equation of the line perpendicular to P which passes through the point D(3, 5, 3)
(b) Find the volume of the tetrahedron ABCD.
(c) Find the equation of P in the form r.n = p.
The perpendicular from D to the plane P meets P at the point E.
(d) Find the coordinates of E.
(e) Show that DE = $\frac{12}{41}\sqrt{41}$
The point F is the reflection of D in P.
(f) Find the coordinates of F.

Q42 - ID: 587 *[5 marks, 6 minutes]*

Find the inverse of the matrix

$$\begin{pmatrix} 2 & 2 & 3 \\ 1 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix}$$

Q43 - ID: 2076 *[9 marks, 11 minutes]*

$$A = \begin{pmatrix} k & 2 & -5 \\ 0 & 0 & k \\ 7 & 4 & 2 \end{pmatrix}$$

(a) Find values of k for which A is singular.
(b) Given that A is non-singular, find, in terms of k, A^{-1}.

Q44 - ID: 5450 *[5 marks, 6 minutes]*

You are given that $A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 3 & 2 \\ 6 & -9 & -1 \end{pmatrix}$.
(a) Calculate AB
(b) Write down A^{-1}

Q45 - ID: 5406 *[11 marks, 13 minutes]*

The matrix A is given by $A = \begin{pmatrix} a & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$
The matrix B is such that $AB = \begin{pmatrix} a & 2 & 3 \\ 1 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix}$
(a) Show that AB is non-singular.
(b) Find $(AB)^{-1}$
(c) Find B^{-1}

Q46 - ID: 5966 *[6 marks, 7 minutes]*

Let M be the matrix $\begin{pmatrix} a & 3a & 0 \\ 0 & a & 1 \\ -3 & -3 & a \end{pmatrix}$

Find all the values of a for which M is singular.

Q47 - ID: 5866 *[5 marks, 6 minutes]*

By using the determinant of an appropriate matrix, or otherwise, find the value of k for which the simultaneous equations

$6x - y + z = 6$

$5y + z = 5$

$x + ky + kz = 6$

do not have a unique solution for x, y and z.

Q48 - ID: 357 *[10 marks, 12 minutes]*

$$A = \begin{pmatrix} 7 & 4 \\ 10 & 1 \end{pmatrix}$$

(a) Find the eigenvalues of A.
(b) Obtain the corresponding normalized eigenvectors.

Q49 - ID: 858 *[13 marks, 16 minutes]*

$$A = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 5 & 4 \\ 4 & 4 & 3 \end{pmatrix}$$

(a) Verify that $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector of A and find the corresponding eigenvalue.
(b) Show that 9 is another eigenvalue of A and find the corresponding eigenvector.
(c) Give that the third eigenvector of A is $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, write down a matrix P and a diagonal matrix D such that $P^T AP = D$

Q50 - ID: 555 *[10 marks, 12 minutes]*

$$A = \begin{pmatrix} 6 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & k \end{pmatrix}$$

(a) Show that $det A = 8 - 4k$
(b) Find A^{-1}.

Given that $k = 2$ and that $E = \begin{pmatrix} 0 \\ 4 \\ -4 \end{pmatrix}$ is an eigenvector of A,

(c) find the corresponding eigenvalue.

Q51 - ID: 2073 *[8 marks, 10 minutes]*

The eigenvalues of the matrix M, where $M = \begin{pmatrix} 3 & -4 \\ 0 & 5 \end{pmatrix}$ are λ_1 and λ_2.
(a) Find the value of λ_1 and the value of λ_2.
(b) Find M^{-1}.
(c) Verify that the eigenvalues of M^{-1} are λ_1^{-1} and λ_2^{-1}.

Q52 - ID: 2075 *[7 marks, 8 minutes]*

A transformation $T : R^2 \to R^2$ is represented by the matrix

$A = \begin{pmatrix} k & 3 \\ 16 & -2 \end{pmatrix}$ where k is a constant.

For the case $k = 6$ find

(a) the two eigenvalues of A,

(b) a cartesian equation for each of the two lines passing through the origin which are invariant under T

Q53 - ID: 2077 *[6 marks, 7 minutes]*

Given that $\begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$ is an eigenvector of the matric A, where

$A = \begin{pmatrix} 2 & 3 & p \\ -3 & q & -3 \\ 4 & 4 & 4 \end{pmatrix}$ where p and q are constants.

(a) find the eigenvalue of A corresponding to $\begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$

(b) find the value of p and the value of q.

Q54 - ID: 4399 *[10 marks, 12 minutes]*

You are given the matrix $M = \begin{pmatrix} 8 & 6 \\ -4 & -2 \end{pmatrix}$

(a) Find the eigenvalues and the corresponding eigenvectors of M.

(b) Write down a matrix P and a diagonal matrix D such that $P^{-1}MP = D$.

Q55 - ID: 5104 *[8 marks, 10 minutes]*

The matrix T has eigenvalues 2 and -2, with corresponding eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ respectively.

(a) Given that $T = UDU^{-1}$, where D is a diagonal matrix, write down suitable matrices U, D and U^{-1}.

(b) Hence prove that, for all even positive integers n, $T^n = f(n)I$ where $f(n)$ is a function of n, and I is the 2×2 identity matrix.

Q56 - ID: 5105 *[12 marks, 14 minutes]*

A system of equations is given by

$$2x + 2y + 2z = -4$$
$$1x - 3y + 8z = 5$$
$$ax + 6y + 2z = b$$

where a and b are constants.

(a) Find the unique solution of the system in the case when $a = 4$ and $b = 2$.

(b) Determine the value of a for which the system does not have a unique solution.

(c) For this value of a, find the value of b such that the system of equations is consistent.

my fp3 paper - Mark Scheme

A1 - ID: 439 *[7 marks, 8 minutes]*

equation $\Rightarrow 18\left(\frac{e^x + e^{-x}}{2}\right) + 6\left(\frac{e^x - e^{-x}}{2}\right) = 17$	$\vert M1$
$\Rightarrow 18(e^x + e^{-x}) + 6(e^x - e^{-x}) = 34$	
$\Rightarrow 18(e^{2x} + 1) + 6(e^{2x} - 1) = 34e^x$	
$\Rightarrow 24e^{2x} - 34e^x + 12 = 0$	
$\Rightarrow 12e^{2x} - 17e^x + 6 = 0$	$\vert M1A1$
$\Rightarrow (4e^x - 3)(3e^x - 2) = 0$	$\vert M1$
$\Rightarrow e^x = \frac{3}{4}$ or $\frac{2}{3}$	$\vert A1$
$\Rightarrow x = \ln\frac{3}{4}$ or $\ln\frac{2}{3}$	$\vert M1A1$

A2 - ID: 598 *[5 marks, 6 minutes]*

(a) $\cosh^2 T = 1 + \sinh^2 T = 1 + \frac{16}{25} = \frac{41}{25}$	$\vert M1$
$\Rightarrow \cosh T = \frac{\sqrt{41}}{5}$	$\vert A1$
(b) $e^T = \cosh T + \sinh T$	
$= \frac{4}{5} + \frac{\sqrt{41}}{5}$	
$= \frac{4+\sqrt{41}}{5}$	$\vert M1A1$
$T = \ln\frac{4+\sqrt{41}}{5}$	$\vert A1$

A3 - ID: 2066 *[6 marks, 7 minutes]*

equation $\Rightarrow 25\left(\frac{e^x + e^{-x}}{2}\right) - 19\left(\frac{e^x - e^{-x}}{2}\right) = 35$	$\vert B1$
$\Rightarrow 25(e^x + e^{-x}) - 19(e^x - e^{-x}) = 70$	
$\Rightarrow 25(e^{2x} + 1) - 19(e^{2x} - 1) = 70e^x$	
$\Rightarrow 6e^{2x} - 70e^x + 44 = 0$	$\vert M1A1$
$\Rightarrow (6e^x - 4)(e^x - 11) = 0$	$\vert M1$
$\Rightarrow e^x = \frac{4}{6}$ or 11	$\vert A1$
$\Rightarrow x = \ln\frac{4}{6}$ or $\ln 11$	$\vert A1$

A4 - ID: 2069 *[7 marks, 8 minutes]*

(a) $\text{LHS} = 4\left(\frac{e^x + e^{-x}}{2}\right)^3 - 3\left(\frac{e^x + e^{-x}}{2}\right)$ |M1

$= \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{2} - \frac{3e^x + 3e^{-x}}{2}$ |A1

$= \frac{e^{3x} + e^{-3x}}{2} = RHS$ |A1

(b) equation $\Rightarrow 4\cosh^3 x - 3\cosh x = 17\cosh x$

$\Rightarrow \cosh x(4\cosh^2 x - 20) = 0 \Rightarrow \cosh x = \sqrt{5}$ |M2

$\Rightarrow x = \pm\cosh^{-1}\sqrt{5}$

$\Rightarrow x = \ln(\sqrt{5} \pm 2)$ |A2

A5 - ID: 4398 *[10 marks, 12 minutes]*

(a) $\text{LHS} = 4\left(\frac{e^x - e^{-x}}{2}\right)^3$

$= \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{2}$ |B1M1

$\text{RHS} = \frac{e^{3x} - e^{-3x}}{2} - 3\frac{e^x - e^{-x}}{2}$

$= \frac{e^{3x} - e^{-3x}}{2} - 3\frac{e^x - e^{-x}}{2} = \text{LHS}$ |B1A1

(b) equation $\Rightarrow 4\sinh^3 x + 3\sinh x = k\sinh x$ |M1A1

$\Rightarrow 4\sinh^3 x - (k-3)\sinh x = 0$

$\Rightarrow 4\sinh^2 x - (k-3) = 0 \Rightarrow 4\sinh^2 x = k - 3$

$\Rightarrow k > 3$ |A1

(c) $k = 5 \Rightarrow 4\sinh^2 x = 2$ |M1

$\Rightarrow \sinh x = \pm\frac{\sqrt{2}}{2}$

$\Rightarrow x = \pm\sinh^{-1}\frac{\sqrt{2}}{2} = \pm\ln\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}\right)$ |A2

A6 - ID: 415 *[4 marks, 5 minutes]*

(a) $b^2 = a^2(1 - e^2) \Rightarrow 9 = 16(1 - e^2)$ |M1

$\Rightarrow 0.5625 = 1 - e^2$

$\Rightarrow e^2 = 1 - 0.5625$

$\Rightarrow e = 0.661$ |A1

(b) foci $= (\pm ae, 0)$ |M1

$= (\pm 2.65, 0)$ |A1

A7 - ID: 520 *[5 marks, 6 minutes]*

(a) $b^2 = a^2(1 - e^2) \Rightarrow e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{\sqrt{8}}{3}$ |M1A1

$\Rightarrow$ coords of $S = (\sqrt{8}, 0)$

$\Rightarrow$ equation of $P : y^2 = 4\sqrt{8}x$ |M1A1

(b) directrix : $x = -\sqrt{8}$ |B1

A8 - ID: 2067 *[4 marks, 5 minutes]*

(a) $b^2 = a^2(e^2 - 1) \Rightarrow 16 = 36(e^2 - 1)$

$\Rightarrow e = \sqrt{\frac{16}{36} + 1} = \frac{\sqrt{52}}{6}$ |M1A1

(b) distance $= 2ae = 2 \times 6 \times \frac{\sqrt{52}}{6} = 2\sqrt{52}$ |M1A1

A9 - ID: 885 *[6 marks, 7 minutes]*

$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}} = \frac{1}{\sqrt{1 - \frac{2}{4}}}$ at $x = \frac{\sqrt{2}}{2}$ |B1

$\frac{d^2y}{dx^2} = -\frac{1}{2}(1 - x^2)^{-3/2}.2x = x(1 - x^2)^{-3/2}$ |B1

$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \frac{1}{1-x^2}\right]^{3/2}}{x(1 - x^2)^{-3/2}}$ |M1A1

$= \frac{[1 - x^2 + 1]^{3/2}}{x} = \frac{[2 - x^2]^{3/2}}{x}$

$= \frac{[2 - (\frac{\sqrt{2}}{2})^2]^{3/2}}{\frac{\sqrt{2}}{2}} = \frac{[2 - \frac{2}{4}]^{3/2}}{\frac{\sqrt{2}}{2}} = \frac{2\left[\frac{3}{2}\right]^{3/2}}{\sqrt{2}}$

$= \frac{2\frac{3\sqrt{3}}{2\sqrt{2}}}{\sqrt{2}} = \frac{3\sqrt{3}}{2}$ |M1A1

A10 - ID: 750 *[6 marks, 7 minutes]*

$\frac{dy}{dx} = \frac{2x}{1 + (x^2)^2} = \frac{2}{2}$ at $x = 1$ |M1A1

$\frac{d^2y}{dx^2} = \frac{(1 + x^4).2 - 2x.4x^3}{(1 + x^4)^2} = \frac{2 - 6x^4}{(1 + x^4)^2}$ |M1A1

$= \frac{-4}{4}$ at $x = 1$

$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + (\frac{2}{2})^2\right]^{3/2}}{\frac{-4}{4}}$ |M1

$= \frac{4}{-4}\left[1 + \frac{4}{4}\right]^{3/2} = -\frac{4}{4}\left[\frac{8}{4}\right]^{3/2}$ |A1

A11 - ID: 657 *[7 marks, 8 minutes]*

(a) $x = \ln k \Rightarrow e^x = k$

$\cosh 5x = \dfrac{e^{5x} + e^{-5x}}{2} = \dfrac{k^5 + k^{-5}}{2}$ | $M1$

$= \dfrac{k^{10} + 1}{2k^5}$ | $M1A1$

(b) $f'(x) = p - 5sech^2 5x = 0$ | $M1A1$

$\Rightarrow p = \dfrac{5}{\cosh^2 5x}$

$x = \ln 2 \Rightarrow \cosh 5x = \dfrac{1025}{64}$ | $B1$

$\Rightarrow p = 5 \times \dfrac{4096}{1050625} = \dfrac{20480}{1050625}$ | $A1$

A12 - ID: 438 *[7 marks, 8 minutes]*

(a) $\frac{dy}{dx} = -1 + 64sech^2 64x$ |B1

$\frac{dy}{dx} = 0 \Rightarrow -1 + 64sech^2 64x = 0 \Rightarrow \cosh^2 64x = 64$

$\Rightarrow \cosh 64x = 8$

$\Rightarrow 64x = \cosh^{-1} 8 = \ln(8 \pm \sqrt{63})$ |M1A1

$\Rightarrow x = \frac{1}{64}\ln(8 + \sqrt{63})$ |A1

(b) $\tanh^2 64x = 1 - sech^2 64x = 1 - \frac{1}{64} = \frac{63}{64}$

$\Rightarrow \tanh 64x = \frac{\sqrt{63}}{8}$ |M1

$\Rightarrow y = -\frac{1}{64}\ln(8 + \sqrt{63}) + \tanh 64x$ |M1

$\Rightarrow y = \frac{\sqrt{63}}{8} - \frac{1}{64}\ln(8 + \sqrt{63})$ |A1

A13 - ID: 516 *[4 marks, 5 minutes]*

$y = arctan\, 5x \Rightarrow \tan y = 5x$ |M1

$\Rightarrow \sec^2 y \frac{dy}{dx} = 5$ |A1

$\Rightarrow \frac{dy}{dx} = \frac{5}{\sec^2 y} = \frac{5}{1 + \tan^2 y} = \frac{5}{1 + 25x^2}$ |M1A1

A14 - ID: 2605 *[6 marks, 7 minutes]*

(a) $y = arccos\, 2x \Rightarrow \cos y = 2x$ |M1

$\Rightarrow -\sin y \frac{dy}{dx} = 2$

$\Rightarrow \frac{dy}{dx} = -\frac{2}{\sin y} = -\frac{2}{\sqrt{1 - \cos^2 y}} = -\frac{2}{\sqrt{1 - 4x^2}}$ |A1

(b) $f'(x) = -2(1 - 4x^2)^{-\frac{1}{2}}$ |M1

$= -2(1 + 2x^2 + 6x^4 + ...)$ |A1

$f(x) = C - 2x - \frac{4}{3}x^3 - \frac{12}{5}x^5 + ...$ |M1

$x = 0 \Rightarrow C = \frac{\pi}{2}$

$\Rightarrow f(x) = \frac{\pi}{2} - 2x - \frac{4}{3}x^3 - \frac{12}{5}x^5 + ...$ |A1

A15 - ID: 5305 *[4 marks, 5 minutes]*

$f'(x) = \frac{1}{\sqrt{1 - x^2}} - 9x^{\frac{1}{2}}$ |M1A1

$f'(x) = 0 \Rightarrow \frac{1}{\sqrt{1 - x^2}} - 9x^{\frac{1}{2}} = 0$

$\Rightarrow 1 = 9x^{\frac{1}{2}}\sqrt{1 - x^2}$

$\Rightarrow 1 = 81x(1 - x^2)$

$\Rightarrow 1 = 81x - 81x^3$

$\Rightarrow 81x^3 - 81x + 1 = 0$ |M1A1

A16 - ID: 5987 *[6 marks, 7 minutes]*

$$\frac{dy}{dx} = \frac{1}{1+(x-6)^2} \qquad |A1$$

$$\text{gradient of normal} - 2 \Rightarrow \text{gradient of tangent} = \frac{1}{2} \qquad |A1$$

$$\Rightarrow \frac{1}{1+(x-6)^2} = \frac{1}{2}$$

$$\Rightarrow 1+(x-6)^2 = 2$$

$$\Rightarrow (x-6)^2 = 1 \Rightarrow x = 7 \qquad |M1A1$$

$$\Rightarrow y = \frac{\pi}{4}$$

$$\Rightarrow \frac{\pi}{4} = -14 + c$$

$$\Rightarrow c = \frac{\pi}{4} - -14 \qquad |M1A1$$

A17 - ID: 473 *[7 marks, 8 minutes]*

$$\frac{dx}{dt} = -15\cos^2 t \sin t, \quad \frac{dy}{dt} = 15\sin^2 t \cos t \qquad |M1A1$$

$$\text{area} = 2\pi \int y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dx$$

$$= 2\pi \int 5\sin^3 t\sqrt{225\cos^4 t \sin^2 t + 225\sin^4 t\cos^2 t}\,dx \qquad |M1A1$$

$$= 150\pi \int \sin^4 t \cos t\sqrt{\cos^2 t + \sin^2}\,dx = \left[150\pi \frac{1}{5}\sin^5 t\right]_0^{\frac{\pi}{2}} \qquad |M1A1$$

$$= \frac{150\pi}{5} \qquad |A1$$

A18 - ID: 794 *[7 marks, 8 minutes]*

(a)

$$\int \frac{3+x}{\sqrt{1-16x^2}}\,dx = \int \frac{3}{\sqrt{1-16x^2}}\,dx + \int \frac{x}{\sqrt{1-16x^2}}\,dx \qquad |M1$$

$$= \frac{3}{4}\sinh^{-1} 4x + \int \frac{x}{\sqrt{1-16x^2}}\,dx \qquad |M1A1$$

$$= \frac{3}{4}\sinh^{-1} 4x - \frac{2}{32}\sqrt{1-16x^2} + c \qquad |M1A1$$

(b)

$$\int_0^{0.2} \frac{3+x}{\sqrt{1-16x^2}}\,dx = \left(\frac{3}{4}\sinh^{-1} 0.8 - \frac{2}{32}\sqrt{0.36}\right) - \left(-\frac{2}{32}\right)$$

$$= 0.575 \qquad |M1A1$$

A19 - ID: 2068 *[5 marks, 6 minutes]*

$$\int_2^6 \frac{1}{\sqrt{x^2 - 4x + 20}}\,dx = \int_2^6 \frac{1}{\sqrt{(x-2)^2 + 16}}\,dx \qquad |B1$$

$$= \left[\sinh^{-1}\frac{x-2}{4}\right]_2^6$$

$$= \sinh^{-1}\frac{4}{4} \qquad |M1A1$$

$$= \ln\left(\frac{4}{4} + \sqrt{\frac{16}{16} + 1}\right)$$

$$= \ln\left(\frac{4 + \sqrt{32}}{4}\right) \qquad |M1A1$$

A20 - ID: 1747 *[5 marks, 6 minutes]*

$$\int_1^3 \frac{1}{\sqrt{x^2+4x+8}}\,dx = \int_1^3 \frac{1}{\sqrt{(x+2)^2+4}}\,dx \qquad |M1A1$$

$$= \left[\sinh^{-1}\frac{x+2}{2}\right]_1^3 \qquad |A1$$

$$= \sinh^{-1}2.5 - \sinh^{-1}1.5$$

$$= \ln(2.5+\sqrt{7.25}) - \ln(1.5+\sqrt{3.25}) \qquad |A1$$

$$= \ln\left(\frac{2.5+\sqrt{7.25}}{1.5+\sqrt{3.25}}\right) \qquad |A1$$

A21 - ID: 2607 *[5 marks, 6 minutes]*

$$\int_0^2 \frac{1}{\sqrt{16x^2+25}}\,dx = \frac{1}{4}\int_0^2 \frac{1}{\sqrt{x^2+\frac{25}{16}}}\,dx \qquad |M1$$

$$= \left[\frac{1}{4}\sinh^{-1}\frac{4x}{5}\right]_0^2 \qquad |A1$$

$$= \frac{1}{4}\sinh^{-1}\frac{8}{5} \qquad |A1$$

$$= \frac{1}{4}\ln\left(\frac{8}{5}+\sqrt{\frac{64}{25}+1}\right) \qquad |M1$$

$$= \frac{1}{4}\ln\left(\frac{8}{5}+\sqrt{\frac{89}{25}}\right) \qquad |A1$$

A22 - ID: 2608 *[4 marks, 5 minutes]*

$$\int_{-\ln 2}^{\ln 2} (22\cosh x - 4\cosh 2x)\,dx = \left[22\sinh x - \frac{4}{2}\sinh 2x\right]_{-\ln 2}^{\ln 2} \qquad |B2$$

$$= 2\left[\frac{22}{2}(2-\frac{1}{2}) - \frac{4}{4}(4-\frac{1}{4})\right] \qquad |M1$$

$$= 2\left[\frac{66}{4} - \frac{60}{16}\right] = 25.5 \qquad |A1$$

A23 - ID: 4234 *[7 marks, 8 minutes]*

$$\int_5^6 \frac{7+x}{\sqrt{x^2-9}}\,dx = \int_5^6 \frac{7}{\sqrt{x^2-9}}\,dx + \int_5^6 \frac{x}{\sqrt{x^2-9}}\,dx \qquad |B1$$

$$= \left[7\cosh^{-1}\frac{x}{3} + \sqrt{x^2-9}\right]_5^6 \qquad |M1A2$$

$$= \left[7\ln\left(\frac{x+\sqrt{x^2-9}}{3}\right) + \sqrt{x^2-9}\right]_5^6$$

$$= \left(7\ln\left(\frac{6+\sqrt{6^2-9}}{3}\right) + \sqrt{6^2-9}\right) - \left(7\ln\left(\frac{5+\sqrt{5^2-9}}{3}\right) + \sqrt{5^2-9}\right)$$

$$= 7\ln\left(\frac{6+\sqrt{27}}{3}\right) + \sqrt{27} - 7\ln\left(\frac{5+\sqrt{16}}{3}\right) - \sqrt{16}$$

$$= 7\ln\left(\frac{6+\sqrt{27}}{5+\sqrt{16}}\right) + \sqrt{27} - \sqrt{16} \qquad |M1A2$$

A24 - ID: 4700 *[5 marks, 6 minutes]*

(a) $y = \cosh^{-1} 8x \Rightarrow \cosh y = 8x$

$\Rightarrow \sinh y \dfrac{dy}{dx} = 8$ |M1

$\Rightarrow \dfrac{dy}{dx} = \dfrac{8}{\sinh y}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{8}{\sqrt{\cosh^2 y - 1}}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{8}{\sqrt{64x^2 - 1}}$ |M1A1

(b) $\displaystyle\int \frac{1}{\sqrt{64x^2 - 1}}\, dx = \frac{1}{8} \cosh^{-1} 8x$ |M1A1

A25 - ID: 545 *[10 marks, 12 minutes]*

(a) $y = arctan3x \Rightarrow \tan y = 3x$ |M1

$\Rightarrow \sec^2 y \dfrac{dy}{dx} = 3$ |A1

$\Rightarrow \dfrac{dy}{dx} = \dfrac{3}{\sec^2 y} = \dfrac{3}{1 + \tan^2 y} = \dfrac{3}{1 + 9x^2}$ |M1A1

(b) $\text{integral} = 3x^2 . \arctan 3x - \displaystyle\int \frac{9x^2}{1 + 9x^2}\, dx$ |M1A1

$= 3x^2 . \arctan 3x - \displaystyle\int \frac{1 + 9x^2 - 1}{1 + 9x^2}\, dx$ |M1

$= \left[3x^2 \arctan 3x - x + \dfrac{1}{3} \arctan 3x\right]_0^{\frac{\sqrt{3}}{3}}$ |A1

$= \dfrac{\pi}{3} - \dfrac{\sqrt{3}}{3} + \dfrac{\pi}{9}$ |M1

$= \dfrac{1}{9}(4\pi - 3\sqrt{3})$ |A1

A26 - ID: 2604 *[4 marks, 5 minutes]*

$x = 6 \sin\theta \Rightarrow I = \displaystyle\int_0^{\frac{\pi}{6}} \frac{1}{(36 - 36 \sin^2\theta)^{\frac{3}{2}}} (6 \cos\theta)\ d\theta$ |M1A1

$= \displaystyle\int_0^{\frac{\pi}{6}} \frac{6 \cos\theta}{216 \cos^3\theta}\, d\theta$

$= \displaystyle\int_0^{\frac{\pi}{6}} \frac{1}{36} \sec^2\theta\, d\theta$

$= \left[\dfrac{1}{36} \tan\theta\right]_0^{\frac{\pi}{6}}$ |M1

$= \dfrac{1}{36} \times \dfrac{1}{\sqrt{3}} = \dfrac{1}{36\sqrt{3}}$ |A1

A27 - ID: 856 *[8 marks, 10 minutes]*

(a) $I_n = \frac{1}{5}x^n e^{5x} - \frac{n}{5}\int x^{n-1}e^{5x}\,dx = \frac{1}{5}(x^n e^{5x} - nI_{n-1})$ |*M1A2*

(b) $I_2 = \left[\frac{1}{5}x^2e^{5x}\right]_0^1 - \frac{2}{5}I_1 = \frac{1}{5}e^5 - \frac{2}{5}I_1$ |*M1*

$I_1 = \left[\frac{1}{5}xe^{5x}\right]_0^1 - \frac{1}{5}I_0 = \frac{1}{5}e^5 - \frac{1}{5}I_0$

$I_0 = \int e^{5x}\,dx = \left[\frac{1}{5}e^{5x}\right]_0^1 = \frac{1}{5}e^5 - \frac{1}{5}$

$\Rightarrow I_2 = \frac{1}{5}e^5 - \frac{2}{5}(\frac{1}{5}e^5 - \frac{1}{5}(\frac{1}{5}e^5 - \frac{1}{5}))$ |*M1A2*

$= \frac{1}{5}e^5 - \frac{2}{25}e^5 + \frac{2}{125}e^5 - \frac{2}{125} = \frac{17}{125}e^5 - \frac{2}{125}$ |*A1*

A28 - ID: 5106 *[6 marks, 7 minutes]*

(a) vector equation : $\left(r - \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}\right) \times \begin{pmatrix} -3 \\ 3 \\ 4 \end{pmatrix} = 0$ |B1

(b) cartesian equation: $\dfrac{x - -3}{-3} = \dfrac{y - 0}{3} = \dfrac{z - 0}{4}$ |M1A1

(c) $\sqrt{-3^2 + 3^2 + 4^2} = \sqrt{34}$ |B1

direction cosines: $\dfrac{-3}{\sqrt{34}}, \dfrac{3}{\sqrt{34}}, \dfrac{3}{\sqrt{34}}$ |B1

: cosines of the angles between the line and the x, y and z-axes respectively |B1

A29 - ID: 5988 *[6 marks, 7 minutes]*

$$\begin{aligned}
|a \times b| &= |a||b| \sin\theta \\
|a \times b|^2 &= |a|^2|b|^2 \sin^2\theta && |M1A1 \\
&= |a|^2|b|^2(1 - \cos^2\theta) && |A1 \\
&= |a|^2|b|^2 - |a|^2|b|^2 \cos^2\theta && |A1 \\
&= |a|^2|b|^2 - (|a||b| \cos\theta)^2 && |A1 \\
&= |a|^2|b|^2 - (a \cdot b)^2 && |A1
\end{aligned}$$

A30 - ID: 854 *[10 marks, 12 minutes]*

(a) $\text{vector} = (4i + 4j + 2k) \times (3i + 3j + 1k)$

$= (4 - 6)i - (4 - 6)j + (12 - 12)k = -2i + 2j - 0k$ |M1A1

(b) $d = -2x + 2y - 0z$

$= -2(1) + 2(3) - 0(3) = 4$ |M1A1

(c) $4 = -2(3) + 2p - 0(3)$

$\Rightarrow p = 5$ |B1

(d) direction, $b = (-2i + 2j - 0k) \times (2i + 5j + 1k)$

$= 2i + 2j + -14k$ |M1

point, $x = 1 \Rightarrow -2 + 2y - 0z = 4 \Rightarrow 2y - 0z = 6$

$\Rightarrow 2 + 5y + 1z = 2 \Rightarrow 5y + 1z = 0$

$\Rightarrow y = 3, z = -15$ |M1A1

equation : $\left[r - (1i + 3j + -15k)\right] \times (2i + 2j + -14k) = ($ |M1A1

A31 - ID: 2554 *[4 marks, 5 minutes]*

normal vectors = $\begin{pmatrix} 2 \\ 6 \\ 10 \end{pmatrix}, \begin{pmatrix} 4 \\ -8 \\ 4 \end{pmatrix}$ |B2

$\begin{pmatrix} 2 \\ 6 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 8 \\ 4 \end{pmatrix} = 8 - 48 + 40 = 0$ |M1

$\Rightarrow$ planes are perpendicular |A1

A32 - ID: 5107 *[5 marks, 6 minutes]*

(a) $n = \det\begin{pmatrix} i & j & k \\ 10 & 4 & 6 \\ 8 & 6 & 6 \end{pmatrix} = -12i - 12j + 28k$ |M1A1

$d = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -12 \\ -12 \\ 28 \end{pmatrix} = -24 - 60 + 84 = 0$ |M1A1

(b) $d = 0 \Rightarrow$ plane passes through origin |B1

A33 - ID: 5747 *[18 marks, 22 minutes]*

(a) $\vec{CD} = \begin{pmatrix} -12 \\ 9 \\ 18 \end{pmatrix}, \quad \vec{CB} = \begin{pmatrix} 0 \\ 30 \\ 0 \end{pmatrix}$ |B2

(b) $CD = \sqrt{-12^2 + 9^2 + 18^2} = \sqrt{549}$ |M1A1

(c) $\begin{pmatrix} 1.5 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -12 \\ 9 \\ 18 \end{pmatrix} = 0, \quad \begin{pmatrix} 1.5 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 30 \\ 0 \end{pmatrix} = 0$ |M1A2

(d) equation : $1.5x + z = c$

$C(30, 0, 0) \Rightarrow 1.5x + z = 45$ |M1A1

(e) $OG : \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 9 \\ 18 \end{pmatrix}$ |B1

$AF : \begin{pmatrix} 0 \\ 30 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 6 \\ -9 \\ 18 \end{pmatrix}$ |B1

(f) P on OG $\Rightarrow \lambda = \frac{5}{3} \Rightarrow 9\lambda = 15 \Rightarrow 18\lambda = 30$

P on AF $\Rightarrow \mu = \frac{5}{3} \Rightarrow 30 + -9\mu = 15 \Rightarrow 18\mu =$ |M1A2

(g) $POABC = \frac{1}{3} \times 30 \times 30 \times 30 = 9000$ |M1A1

$PDEFG = \frac{1}{3} \times 12 \times 12 \times 12 = 576$ |A1

$volume = 8424\,cm^3$ |A1

A34 - ID: 5975 *[14 marks, 17 minutes]*

(a) $\vec{AB} = \begin{pmatrix} 1-1 \\ 3-1 \\ 5-4 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \quad \vec{AC} = \begin{pmatrix} 4-1 \\ 0-1 \\ 1-4 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ -3 \end{pmatrix}$ |A2

$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 3 & -1 & -3 \end{vmatrix} = i(-5) - j(-3) + k(-6)$

$= -5i - -3j - 6k$ |M1A2

$\text{area} = \frac{1}{2}|-5i - -3j - 6k| = \frac{\sqrt{70}}{2}$ |M1A1

(b) $AB = \sqrt{5}$ |A1

$\frac{\sqrt{70}}{2} = \frac{1}{2}\sqrt{5}h \Rightarrow h = \sqrt{14}$ |M1A1

(c) $\Pi : r \cdot \begin{pmatrix} -5 \\ 3 \\ -6 \end{pmatrix} = d$

$i + j + 4k$ in plane $\Rightarrow \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 3 \\ -6 \end{pmatrix} = -26 \Rightarrow -5x - -3y - 6z = -26$ |M1A1

A35 - ID: 5976 *[6 marks, 7 minutes]*

(a) equation of OD : $r = \lambda \begin{pmatrix} 7 \\ -4 \\ 6 \end{pmatrix}$ |$M1$

meets Π where : $7(7\lambda) + -4(-4\lambda) + 6(6\lambda) = -5$

$\Rightarrow \lambda = -\frac{5}{101}$ |$M1A1$

$\Rightarrow D(-\frac{35}{101}, \frac{20}{101}, -\frac{30}{101})$ |$A1$

(b) $OD = \sqrt{(-\frac{35}{101})^2 + (\frac{20}{101})^2 + (-\frac{30}{101})^2} = \sqrt{\frac{25}{101}}$ |$M1A1$

A36 - ID: 5994 *[6 marks, 7 minutes]*

let $t = -2 \Rightarrow (-2, 2, 1)$ is a point on L |A1

$$r = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 4 \\ -4 \end{pmatrix}$$ |A1

$\Rightarrow x = 2 + 2\lambda + -4\mu$

$\Rightarrow y = 4 + \lambda + 4\mu$

$\Rightarrow z = 7 + 3\lambda + -4\mu$ |M1A1

$\Rightarrow x + y = 6 + 3\lambda$

$\Rightarrow y + z = 11 + 4\lambda$ |A1

$\Rightarrow 4(x + y) = 24 + 12\lambda$

$\Rightarrow 3(y + z) = 33 + 12\lambda$

$\Rightarrow 4x + 1y - 3z = -9$ |A1

A37 - ID: 7446 *[6 marks, 7 minutes]*

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$= \frac{\begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}}{\sqrt{6^2 + 3^2 + -2^2}\sqrt{1 + 4^2 + 3^2}} = \frac{12}{\sqrt{49}\sqrt{26}}$$ |M1A2

$\Rightarrow \theta = 70.4$ |A1

$\Rightarrow$ angle $= 19.6°$ |A1

A38 - ID: 7448 *[6 marks, 7 minutes]*

$$n_1 = \begin{pmatrix} 5 \\ -8 \\ 2 \end{pmatrix}, \quad n_2 = \begin{pmatrix} 10 \\ 4 \\ -6 \end{pmatrix}$$

$$n_1 \times n_2 = \begin{pmatrix} 5 \\ -8 \\ 2 \end{pmatrix} \times \begin{pmatrix} 10 \\ 4 \\ -6 \end{pmatrix} = \begin{pmatrix} 40 \\ 50 \\ 100 \end{pmatrix}$$

$x = 1 \Rightarrow -8y + 2z = -1, \quad 4y + -6z = -9$

$\Rightarrow y = 0.6, z = 1.9$

$$l \ : \ r = \begin{pmatrix} 1 \\ 0.6 \\ 1.9 \end{pmatrix} + \lambda \begin{pmatrix} 40 \\ 50 \\ 100 \end{pmatrix}$$

A39 - ID: 5849 *[8 marks, 10 minutes]*

(a) normal to $7x - y + z = 3 = \begin{pmatrix} 7 \\ -1 \\ 1 \end{pmatrix}$ normal to $x - z = 4 = \begin{pmatrix} 1 \\ -0 \\ -1 \end{pmatrix}$ | *B*1

(b) $\cos\theta = \dfrac{a \cdot b}{|a||b|} = \dfrac{6}{\sqrt{51}\sqrt{2}}$ | *M*2

$\Rightarrow \theta = 53.6$ | *A*1

(c) equation : $r = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -1 \\ 1 \end{pmatrix}$ | *B*1

(d) equation : $r = \begin{pmatrix} 3 + 7\lambda \\ 1 - \lambda \\ 3 + \lambda \end{pmatrix}$

$\Rightarrow 7(3 + 7\lambda) - (1 - \lambda) + (3 + \lambda) = 3$

$\Rightarrow 23 + 51\lambda = 3 \Rightarrow 51\lambda = -20$

$\Rightarrow \lambda = -\dfrac{20}{51}$ | *M*1*A*1

$\Rightarrow$ intersection $= \left(\dfrac{13}{51}, \dfrac{71}{51}, \dfrac{133}{51}\right)$ | *A*1

A40 - ID: 413 *[12 marks, 14 minutes]*

(a) $\vec{AB} = 2i + -2j + 2k$

$\vec{AC} = 3i + 3j + -1k,\ \vec{AD} = 4i + 0j + -1k$ |M1A1

$\vec{AB} \times \vec{AC} = (2 - 6)i + (6 - -2)j + (6 - -6)k$

$= -4i + 8j + 12k$ |M1A2

Area $ABC = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{16 + 64 + 144} = \frac{1}{2}\sqrt{224}$ |M1A1

(b) Vol $= \frac{1}{6}|\vec{AD}.\vec{AB} \times \vec{AC}| = \frac{1}{6}|-16 + 0 + -12| = \frac{28}{6}$ |M1A1

(c) Unit vector, $n = \frac{1}{\sqrt{224}}(-4i + 8j + 12k)$ |M1

$p = |n.\vec{AD}| = \frac{1}{\sqrt{224}}|(-4i + 8j + 12k).(4i + 0j + -1k)|$

$= \frac{1}{\sqrt{224}}(-16 + 0 + -12) = \frac{28}{\sqrt{224}}$ |M1A1

A41 - ID: 379 *[18 marks, 22 minutes]*

(a) $\vec{AB} = 4i + 1j + 1k,\ \vec{AC} = 2i + 0j + -1k$

$\vec{AB} \times \vec{AC} = (-1 - 0)i + (2 - -4)j + (0 - 2)k$

$= -1i + 6j + -2k$ |M1A1

$r = 3i + 5j + 3k + \lambda(-1i + 6j + -2k)$ |B1

(b) $\vec{AD} = 4i + 3j + 1k$ |B1

Vol $= \frac{1}{6}|\vec{AD}.\vec{AB} \times \vec{AC}|$

$= \frac{1}{6}|-4 + 18 + -2| = \frac{12}{6}$ |M1A1

(c) equation : $r.(-1i + 6j + -2k) = (1i + 2j + 1k).(-1i + 6j + -2k)$

: $r.(-1i + 6j + -2k) = 9$ |M1A2

(d) : $[(3 + -1\lambda)i + (5 + 6\lambda)j + (3 + -2\lambda)k].(-1i + 6j + -2k) = 9$ |M1A1

$\Rightarrow -3 + 1\lambda + 30 + 36\lambda + -6 + 4\lambda = 9$

$\Rightarrow 21 + 41\lambda = 9 \quad \Rightarrow \lambda = \frac{-12}{41}$ |M1

$\Rightarrow$ E is at $(\frac{135}{41}, \frac{133}{41}, \frac{147}{41})$ |A1

(e) $DE = \frac{-12}{41}|-1i + 6j + -2k| = \frac{12}{41}\sqrt{41}$ |M1A1

(f) At F, $\lambda = 2 \times \frac{-12}{41} = \frac{-24}{41}$ |B1

$\Rightarrow$ F is at $(\frac{147}{41}, \frac{61}{41}, \frac{171}{41})$ |M1A1

A42 - ID: 587 *[5 marks, 6 minutes]*

$$\det = 2\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + 3\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}$$

$= 2(8) - 2(1) + 3(-5) = -1$ |B1

$$\text{cofactors} = \begin{pmatrix} 8 & -1 & -5 \\ -3 & 0 & 2 \\ -7 & 1 & 4 \end{pmatrix}$$ |M1A1

$$\text{transpose} = \begin{pmatrix} 8 & -3 & -7 \\ -1 & 0 & 1 \\ -5 & 2 & 4 \end{pmatrix}$$

$$\text{inverse} = \frac{1}{-1}\begin{pmatrix} 8 & -3 & -7 \\ -1 & 0 & 1 \\ -5 & 2 & 4 \end{pmatrix}$$ |M1A1

A43 - ID: 2076 *[9 marks, 11 minutes]*

(a) $det = k\begin{vmatrix}0 & k\\4 & 2\end{vmatrix} - 2\begin{vmatrix}0 & k\\7 & 2\end{vmatrix} + -5\begin{vmatrix}0 & 0\\7 & 4\end{vmatrix}$

$= k(0-4k) - 2(0-7k) + 0$

$= -4k^2 + 14k + 0 = -(4k-14)(k-0)$ |M1A1

$det = 0 \Rightarrow k = 0, 3.5$ |M1A1

(b) $\text{cofactors} = \begin{pmatrix}0-4k & 7k-0 & 0\\-24 & 2k--35 & 14-4k\\2k-0 & 0-k^2 & 0k-0\end{pmatrix}$ |B3

$A^{-1} = \frac{1}{det}\begin{pmatrix}0-4k & -24 & 2k-0\\7k-0 & 2k--35 & 0-k^2\\0 & 14-4k & 0k-0\end{pmatrix}$ |M1A1

A44 - ID: 5450 *[5 marks, 6 minutes]*

(a) $AB = \begin{pmatrix}3 & 2 & 1\\2 & 1 & 0\\0 & 3 & 1\end{pmatrix} \times \begin{pmatrix}1 & 1 & -1\\-2 & 3 & 2\\6 & -9 & -1\end{pmatrix}$

$= \begin{pmatrix}5 & 0 & 0\\0 & 5 & 0\\0 & 0 & 5\end{pmatrix}$ |B3

(b) $AB = 5I \Rightarrow \frac{1}{5}AB = I \Rightarrow A \times \frac{1}{5}B = I$

$\Rightarrow A^{-1} = \frac{1}{5}B = \frac{1}{5}\begin{pmatrix}1 & 1 & -1\\-2 & 3 & 2\\6 & -9 & -1\end{pmatrix}$ |M1A1

A45 - ID: 5406 *[11 marks, 13 minutes]*

(a) $\det = a\begin{vmatrix}1 & 0\\2 & 0\end{vmatrix} - 2\begin{vmatrix}1 & 0\\1 & 0\end{vmatrix} + 3\begin{vmatrix}1 & 1\\1 & 2\end{vmatrix}$

$= a(0) - 2(0) + 3(1) = 3$ |M1A1

(b) $\text{cofactors}= \begin{pmatrix}0 & 0 & 1\\6 & -3 & 2-2a\\-3 & 3 & 1a-2\end{pmatrix}$ |M1A1

$\text{transpose}= \begin{pmatrix}0 & 6 & -3\\0 & -3 & 3\\1 & 2-2a & 1a-2\end{pmatrix}$

$\text{inverse}= \frac{1}{3}\begin{pmatrix}0 & 6 & -3\\0 & -3 & 3\\1 & 2-2a & 1a-2\end{pmatrix}$ |M1A1

(c) $B^{-1} = (AB)^{-1} \times A = \frac{1}{3}\begin{pmatrix}0 & 6 & -3\\0 & -3 & 3\\1 & 2-2a & 1a-2\end{pmatrix} \times \begin{pmatrix}a & 2 & 3\\1 & 1 & 2\\1 & 2 & 4\end{pmatrix}$ |M1A1

$= \frac{1}{3}\begin{pmatrix}3 & 0 & 0\\0 & 3 & 6\\0 & 0 & -1\end{pmatrix}$ |M1A2

A46 - ID: 5966 *[6 marks, 7 minutes]*

$$\begin{aligned}
\det &= a\begin{vmatrix} a & 1 \\ -3 & a \end{vmatrix} - 3a\begin{vmatrix} 0 & 1 \\ -3 & a \end{vmatrix} + 0\begin{vmatrix} 0 & a \\ -3 & -3 \end{vmatrix} \\
&= a(a^2 - -3) - 3a(3) \\
&= a(a^2 - 6) && \mid M1A1 \\
\text{singular} &\Rightarrow a = 0, \pm\sqrt{6} && \mid M1A3
\end{aligned}$$

A47 - ID: 5866 *[5 marks, 6 minutes]*

$$\text{matrix equation: } \begin{pmatrix} 6 & -1 & 1 \\ 0 & 5 & 1 \\ 1 & k & k \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ 6 \end{pmatrix}$$

$$\begin{aligned}
\text{no unique solution} &\Rightarrow \det = 0 \\
&\Rightarrow 6\begin{vmatrix} 5 & 1 \\ k & k \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & k \end{vmatrix} + \begin{vmatrix} 0 & 5 \\ 1 & k \end{vmatrix} = 0 && \mid M1 \\
&\Rightarrow 24k - 1 - 5 = 0 && \mid M1A1 \\
&\Rightarrow 24k = 6 \\
&\Rightarrow k = \tfrac{1}{4} && \mid M1A1
\end{aligned}$$

A48 - ID: 357 *[10 marks, 12 minutes]*

(a) $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 7-\lambda & 4 \\ 10 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow (7-\lambda)(1-\lambda) - 40 = 0$ $|M1A1$

$\Rightarrow \lambda^2 - 8\lambda + 7 - 40 = 0 \Rightarrow \lambda^2 - 8\lambda - 33 = 0$

$\Rightarrow (\lambda - 11)(\lambda + 3) = 0 \Rightarrow \lambda = 11, -3$ $|M1A1$

(b) $\lambda = 11 \Rightarrow \begin{pmatrix} 7-11 & 4 \\ 10 & 1-11 \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = 0$

$\Rightarrow -4x_1 + 4y_1 = 0$ $|M1A1$

Normalized: $\frac{1}{\sqrt{32}}\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ $|M1A1$

$\lambda = -3 \Rightarrow \begin{pmatrix} 7--3 & 4 \\ 10 & 1--3 \end{pmatrix}\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = 0$

$\Rightarrow 10x_2 + 4y_2 = 0$ $|A1$

Normalized: $\frac{1}{\sqrt{116}}\begin{pmatrix} 4 \\ -10 \end{pmatrix}$ $|A1$

A49 - ID: 858 *[13 marks, 16 minutes]*

(a) $\begin{pmatrix} 1 & 0 & 4 \\ 0 & 5 & 4 \\ 4 & 4 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \\ 3 \end{pmatrix} = 3\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \Rightarrow \lambda = 3$ $|M1A2$

(b) $|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 4 \\ 0 & 5-\lambda & 4 \\ 4 & 4 & 3-\lambda \end{vmatrix}$

$= (1-\lambda)((5-\lambda)(3-\lambda) - 16) + 4(0 - 4(5-\lambda))$

$= (3-\lambda)(\lambda - 9)(3 + \lambda)$

$|A - \lambda I| = 0 \Rightarrow \lambda = 9$ $|M1A1$

eigenvector $\Rightarrow 1x + 0y + 4z = 9x$

$\Rightarrow 0x + 5y + 4z = 9y$

$4x + 4y + 3z = 9z$ $|M1$

$\Rightarrow y = 2x, \quad z = 2x$

$\Rightarrow$ eigenvector $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ $|A2$

(c) $P = \frac{1}{3}\begin{pmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{pmatrix}$ $|M1A1$

$D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & -3 \end{pmatrix}$ $|B1M1A1$

A50 - ID: 555 *[10 marks, 12 minutes]*

(a) $\det = 6\begin{vmatrix} 2 & 2 \\ 2 & k \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 2 & k \end{vmatrix} + 2\begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix}$

$= 6(0) - 2(2k - 4) + 2(0) = 8 - 4k$ $\quad$ $|M1A1$

(b) cofactors$= \begin{pmatrix} 0 & 4-2k & 0 \\ 4-2k & 6k-4 & -8 \\ 0 & -8 & 8 \end{pmatrix}$ $\quad$ $|M1A3$

$A^{-1} = \dfrac{1}{8-4k}\begin{pmatrix} 0 & 4-2k & 0 \\ 4-2k & 6k-4 & -8 \\ 0 & -8 & 8 \end{pmatrix}$ $\quad$ $|M1A1$

(c) $A \times E = \begin{pmatrix} 6 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 4 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \lambda = 0$ $\quad$ $|M1A1$

A51 - ID: 2073 *[8 marks, 10 minutes]*

(a) $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 3-\lambda & -4 \\ 0 & 5-\lambda \end{vmatrix} = 0$

$\Rightarrow (3-\lambda)(5-\lambda) - 0 = 0$ $|M1$

$\Rightarrow \lambda^2 - 8\lambda + 15 - 0 = 0 \Rightarrow \lambda^2 - 8\lambda + 15 = 0$ $|M1$

$\Rightarrow (\lambda - 5)(\lambda - 3) = 0 \Rightarrow \lambda_1 = 5, \lambda_2 = 3$ $|A1$

(b) $M^{-1} = \frac{1}{15}\begin{pmatrix} 5 & 4 \\ 0 & 3 \end{pmatrix}$ $|B2$

(c) $|M^{-1} - \lambda_1^{-1} I| = \begin{vmatrix} \frac{5}{15} - \frac{1}{5} & \frac{4}{15} \\ \frac{0}{15} & \frac{3}{15} - \frac{1}{5} \end{vmatrix} = 0$ $|M1A1$

$|M^{-1} - \lambda_2^{-1} I| = \begin{vmatrix} \frac{5}{15} - \frac{1}{3} & \frac{4}{15} \\ \frac{0}{15} & \frac{3}{15} - \frac{1}{3} \end{vmatrix} = 0$ $|A1$

A52 - ID: 2075 *[7 marks, 8 minutes]*

(a) $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 6-\lambda & 3 \\ 16 & -2-\lambda \end{vmatrix} = 0$ $|M1$

$\Rightarrow (6-\lambda)(-2-\lambda) - 48 = 0$ $|A1$

$\Rightarrow \lambda^2 - 4\lambda + -12 - 48 = 0 \Rightarrow \lambda^2 - 4\lambda - 60 = 0$

$\Rightarrow (\lambda + -10)(\lambda - -6) = 0 \Rightarrow \lambda = 10, -6$ $|M1A1$

(b) $y = mx$ invariant $\Rightarrow \begin{pmatrix} 6 & 3 \\ 16 & -2 \end{pmatrix} \times \begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} x' \\ mx' \end{pmatrix}$

$\Rightarrow 6x + 3mx = x', 16x + -2mx = mx'$

$\Rightarrow \frac{16 + -2m}{6 + 3m} = m$

$\Rightarrow 3m^2 + 8m - 16 = 0$ $|M1A1$

$\Rightarrow (3m - 4)(m + 4) = 0$

$\Rightarrow m = \frac{4}{3}, -4 \Rightarrow y = \frac{4}{3}x, y = -4x$ $|A1$

A53 - ID: 2077 *[6 marks, 7 minutes]*

(a) $\begin{pmatrix} 2 & 3 & p \\ -3 & q & -3 \\ 4 & 4 & 4 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 + -2p \\ 6 + 2q \\ 0 \end{pmatrix}$ $|M1$

$\Rightarrow \lambda = 0$ $|A1$

(b) $6 + -2p = 0 \Rightarrow p = 3$ $|M1A1$

$6 + 2q = 0 \Rightarrow q = -3$ $|M1A1$

A54 - ID: 4399 *[10 marks, 12 minutes]*

(a) $|M - \lambda I| = 0 \Rightarrow \begin{vmatrix} 8-\lambda & 6 \\ -4 & -2-\lambda \end{vmatrix} = 0$

$\Rightarrow (8-\lambda)(-2-\lambda) - -24 = 0$ $|M1$

$\Rightarrow \lambda^2 - 6\lambda + -16 - -24 = 0 \Rightarrow \lambda^2 - 6\lambda + 8 = 0$

$\Rightarrow (\lambda - 2)(\lambda - 4) = 0 \Rightarrow \lambda = 2, 4$ $|A2$

$\lambda = 2 \Rightarrow \begin{pmatrix} 8-2 & 6 \\ -4 & -2-2 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = 0$

$\Rightarrow 6x_1 + 6y_1 = 0$ $|M2$

eigenvector: $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $|A1$

$\lambda = 4 \Rightarrow \begin{pmatrix} 8-4 & 6 \\ -4 & -2-4 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = 0$

$\Rightarrow 4x_2 + 6y_2 = 0$ $|M1$

eigenvector: $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $|A1$

(b) $P = \begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}, \quad D = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$ $|B2$

A55 - ID: 5104 *[8 marks, 10 minutes]*

(a) $U = \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix}, \quad D = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$ $|B2$

$U^{-1} = \begin{pmatrix} 4 & -3 \\ -1 & 1 \end{pmatrix}$ $|B1$

(b) $T^n = UD^nU^{-1} = \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2^n & 0 \\ 0 & 2^n \end{pmatrix} \begin{pmatrix} 4 & -3 \\ -1 & 1 \end{pmatrix}$ $|B1M1$

$= \begin{pmatrix} 2^n & 3(2^n) \\ 2^n & 4(2^n) \end{pmatrix} \begin{pmatrix} 4 & -3 \\ -1 & 1 \end{pmatrix}$ $|B1M1$

$= \begin{pmatrix} 2^n & 0 \\ 0 & 2^n \end{pmatrix} = 2^n \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $|A1$

A56 - ID: 5105 *[12 marks, 14 minutes]*

(a) $M^{-1} = \begin{pmatrix} 2 & 2 & 2 \\ 1 & -3 & 8 \\ 4 & 6 & 2 \end{pmatrix}^{-1} = \frac{1}{-12} \begin{pmatrix} -54 & 8 & 22 \\ 30 & -4 & -14 \\ 18 & -4 & -8 \end{pmatrix}$ $|M1A2$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M^{-1} \begin{pmatrix} -4 \\ 5 \\ 2 \end{pmatrix} = \frac{1}{-12} \begin{pmatrix} 300 \\ -168 \\ -108 \end{pmatrix}$

$\Rightarrow x = -25, y = 14, z = 9$ $|M1A1$

(b) $\det \begin{pmatrix} 2 & 2 & 2 \\ 1 & -3 & 8 \\ a & 6 & 2 \end{pmatrix} = -108 - 4 + 16a + 12 - -6a$

$\det = 0 \Rightarrow -100 + 22a = 0 \Rightarrow a = 4.545$ $|M2A1$

(c) $x = -25, y = 14, z = 9 \Rightarrow b = 4.545(-25) + 6(14) + 2(9)$

$\Rightarrow b = -11.625$ $|M3A1$