steve blades

Your account expires on: **20 Feb, 2013**

My Full M2 Paper 1

Syllabus: **EdExcel** Questions: **54** Time: **10 hours 23 minutes** Total Marks: **520**

This paper contains a set of questions followed by the corresponding mark schemes. The time you should spend on each question together with its worth in marks is also given. The content of this paper is based on material from a wide selection of national and international examination boards and organisations.

You are advised to have:
a set of geometrical equipment, pen, HB pencil, eraser. Check if you are allowed a calculator. Some examinations, but not all, allow calculators, including graphical models.

My Full M2 Paper 1

Questions: **54** Time: **10 hours 23 minutes** Total Marks: **520**

Q1 - ID: 7686 *[17 marks, 20 minutes]*

A particle P is projected from a fixed point O on horizontal ground with velocity $u(i + cj)\, ms^{-1}$, where c and u are positive constants. The particle moves freely under gravity until it strikes the ground at A, where it immediately comes to rest. Relative to O, the position vector of a point on the path of P is $(xi + yj)\, m$.

(a) Show that $y = cx - \dfrac{4.9x^2}{u^2}$.

Given that $u = 10$, $OA = R\, m$ and the maximum vertical height of P above the ground is $H\, m$,

(b) using the result in part (a), or otherwise, find, in terms of c,

(i) R, (ii) H.

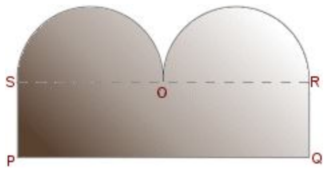
Given also that when P is at the point Q, the velocity of P is at right angles to its initial velocity,

(c) find, in terms of c, the value of x at Q.

Q2 - ID: 7680 *[8 marks, 10 minutes]*

A particle P moves along the x-axis. At time t seconds the velocity of P is $v\, ms^{-1}$ in the positive x-direction, where $v = 6t^2 - 4t + 4$. When $t = 0$, P is at the origin O. Find the distance of P from O when P is moving with minimum velocity.

Q3 - ID: 7687 *[11 marks, 13 minutes]*



A template T consists of a uniform plane lamina PQROS, as shown. The lamina is bounded by two semicircles, with diameters SO and OR, and by the sides SP, PQ and QR of the rectangle PQRS. The point O is the mid-point of SR, PQ = $12\, cm$ and QR = $2x\, cm$.

(a) Show that the centre of mass of T is a distance $\dfrac{8x^2 - 12}{8x + 3\pi}\, cm$ from SR.

The template T is freely suspended from the point P and hangs in equilibrium. Given that $x = 2$ and that θ is the angle that PQ makes with the horizontal,

(b) show that $\tan\theta = \dfrac{48 + 9\pi}{22 + 6\pi}$.

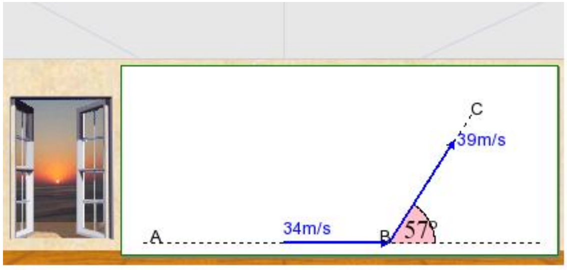
Q4 - ID: 7682 *[6 marks, 7 minutes]*

A particle of mass $0.9kg$ is projected vertically upwards from ground level with a speed of $22ms^{-1}$. It comes to instantaneous rest at a height of $6\, m$ above the ground. As the particle moves it is subject to air resistance of constant magnitude R newtons. Using the work-energy principle, or otherwise, find the value of R.

Q5 - ID: 7683 *[11 marks, 13 minutes]*

A cyclist and her bicycle have a total mass of 80 kg. She cycles along a straight horizontal road with constant speed $7ms^{-1}$. She is working at a constant rate of 480 W.
(a) Find the magnitude of the resistance to motion.
The cyclist now cycles down a straight road which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{14}$, at a constant speed $U\ ms^{-1}$. The magnitude of the non-gravitational resistance to motion is modelled as $45U$ newtons. She is now working at a constant rate of 45 W.
(b) Find the value of U.

Q6 - ID: 7685 *[8 marks, 10 minutes]*

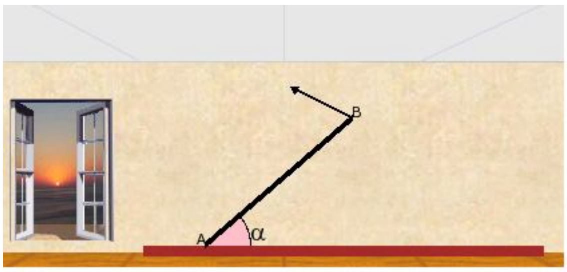


The points A, B and C lie in a horizontal plane. A batsman strikes a ball of mass 0.28 kg. Immediately before being struck, the ball is moving along the horizontal line AB with speed $34ms^{-1}$. Immediately after being struck, the ball moves along the horizontal line BC with speed 39 ms^{-1}. The line BC makes an angle of 57° with the original direction of motion AB, as shown.
Find, to 3 significant figures,
(a) the magnitude of the impulse given to the ball,
(b) the size of the angle that the direction of this impulse makes with the original direction of motion

Q7 - ID: 7681 *[7 marks, 8 minutes]*

Two particles, P, of mass $2m$, and Q, of mass m, are moving along the same straight line on a smooth horizontal plane. They are moving in opposite directions towards each other and collide. Immediately before the collision the speed of P is $4u$ and the speed of Q is u.
The coefficient of restitution between the particles is e, where $e < 1$. Find
(a) the speed of P immediately after the collision,
(b) the speed of Q immediately after the collision.

Q8 - ID: 7684 *[7 marks, 8 minutes]*



A uniform rod AB, of mass 24 kg and length 7 m, rests with one end A on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a force acting at B, as shown. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.4. Find the magnitude of the normal reaction of the ground on the rod at A.

Q9 - ID: 398 *[10 marks, 12 minutes]*

A particle P moves with constant acceleration $(3\,\underline{i} - 6\,\underline{j})\,m/s^2$

At time t its velocity is v m/s. When t = 0, v = $(-3\,\underline{i} + 6\,\underline{j})$.

(a) Find the value of t when P is moving parallel to the vector $\underline{i}$.

(b) Find the speed of P when t = 4.

(c) Find the angle between the vector $\underline{j}$ and the direction of motion of P when t = 4.

Q10 - ID: 483 *[9 marks, 11 minutes]*

A particle P moves in a horizontal plane. At time t seconds, the position vector of P is r metres relative to a fixed origin O, and r is given by

$$r = (18t - 7t^3)i + ct^2j.$$

where c is a positive constant. When $t = 1$, the speed of P is $14ms^{-1}$. Find

(a) the value of c,

(b) the acceleration of P when $t = 1$.

Q11 - ID: 477 *[8 marks, 10 minutes]*

A particle P of mass $0.5kg$ is moving so that its position vector r metres at time t seconds is given by

$$r = (t^2 + 4t)i + (3t - t^3)j.$$

(a) Calculate the speed of P when $t = 8$.

When $t = 8$, the particle P is given an impulse $(9i - 15j)Ns$.

(b) Find the velocity of P immediately after the impulse.

Q12 - ID: 3262 *[11 marks, 13 minutes]*

A particle has mass $880\,kg$. A single force of $(4400i - 5280tj)$ newtons acts on the particle at time t seconds. No other forces act on the particle.

(a) Find the acceleration of the particle at time t.

(b) At time $t = 0$, the velocity of the particle is $(4i + 31j)ms^{-1}$. The velocity of the particle at time t is $v\ ms^{-1}$. Show that

$$v = (4 + 5t)i + (31t - 3t^2)j$$

(c) Initially, the particle is at the point with position vector $(3i + 5j)m$

Find the position vector, r metres, of the particle at time t.

Q13 - ID: 3525 *[9 marks, 11 minutes]*

At time t seconds ($t \geq 0$), a particle P has position vector p metres, with respect to a fixed origin O, where

$$p = (6t^2 - 4t + 5)i + (2t^3 - 5t)j.$$

(a) Find the velocity of P at time t seconds.

(b) Find the value of t when P is moving parallel to the vector i.

When $t = 5$, the particle P receives an impulse of $(4i – 4j)\,N$. Given that the mass of P is $0.2\,kg$,

(c) find the velocity of P immediately after the impulse.

Q14 - ID: 569 *[8 marks, 10 minutes]*

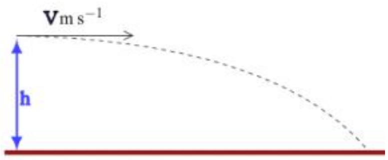
A vertical cliff is $67.5m$ high. Two stones A and B are projected simultaneously. Stone A is projected horizontally from the top of the cliff with speed $20ms^{-1}$. Stone B is projected from the bottom of the cliff with speed $25ms^{-1}$ at an angle α above the horizontal. The stones move freely under gravity in the same vertical plane and collide in mid-air.

By considering the horizontal motion of each stone,

(a) prove that $\cos\alpha = \frac{4}{5}$.

(b) Find the time which elapses between the instant when the stones are projected and the instant when they collide.

Q15 - ID: 2959 *[9 marks, 11 minutes]*



A tennis ball is hit from a height of $h = 2.46\,m$ above horizontal ground. Initially it travels horizontally at a speed of $V = 18\,ms^{-1}$ as shown.

(a) Show that the time taken for the tennis ball to reach the ground is $0.709\,s$.

(b) Find the horizontal distance travelled by the ball when it hits the ground.

(c) Find the angle between the velocity of the ball and the horizontal when the ball hits the ground.

Q16 - ID: 7771 *[12 marks, 14 minutes]*

A cricket ball is hit from the floor of a sports hall, which has a height of 5 metres. The initial velocity of the ball is $20\,ms^{-1}$ at an angle of 54° above the horizontal.

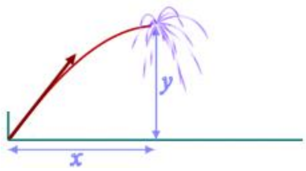
Assume that the cricket ball is a particle which moves only under the influence of gravity.

(a) Show that the ball hits the ceiling of the sports hall approximately 0.345 seconds after it was hit.

(b) Find the horizontal distance travelled by the ball before it hits the ceiling.

(c) Find the speed of the ball just before it hits the ceiling.

Q17 - ID: 2910 *[19 marks, 23 minutes]*



A firework is fired from a point O at ground level over horizontal ground. The highest point reached by the firework is a horizontal distance of $x = 60\ m$ from O and a vertical distance of $y = 40\ m$ from O, as shown. Air resistance is negligible. The initial horizontal component of the velocity of the firework is $21\ ms^{-1}$.

(a) Calculate the time for the firework to reach its highest point and show that the initial vertical component of its velocity is $28\ ms^{-1}$.

(b) Show that the firework is $28t - 4.9t^2\ m$ above the ground t seconds after its projection.

When the firework is at its highest point it explodes into several parts. Two of the parts initially continue to travel horizontally in the original direction, one with the original horizontal speed of $21\ ms^{-1}$ and the other with a quarter of this speed.

(c) State why the two parts are always at the same height as one another above the ground and hence find an expression in terms of t for the distance between the parts t seconds after the explosion.

(d) Find the distance between these parts of the firework when they reach the ground.

(e) Find the distance between these parts of the firework when they are $10\ m$ off the ground.

(f) Show that the cartesian equation of the trajectory of the firework before it explodes is $y = \frac{1}{441}(588x - 4.9x^2)$, referred to the coordinate axes shown.

Q18 - ID: 5463 *[6 marks, 7 minutes]*

In this question take g to be 10.

A golf ball is hit from ground level over horizontal ground. The initial velocity of the ball is $46\ ms^{-1}$ at an angle α to the horizontal, where $\cos\alpha = 0.6$ and $\sin\alpha = 0.8$. Air resistance may be neglected.

(a) Find an expression for the height of the ball above the ground t seconds after projection.

(b) Calculate the horizontal range of the ball.

Q19 - ID: 7306 *[14 marks, 17 minutes]*

A pellet is fired from a window at a height of 5 metres above horizontal ground.

Initially, the pellet travels at $69\ ms^{-1}$ at an angle of $13°$ above the horizontal.

(a) Show that the time for which the pellet travels before it hits the ground is 3.46 seconds, correct to three significant figures.

(b) Find the horizontal distance that the pellet travels before it hits the ground.

(c) Find the minimum speed of the pellet during its flight.

(d) Find the speed of the pellet when it hits the ground.

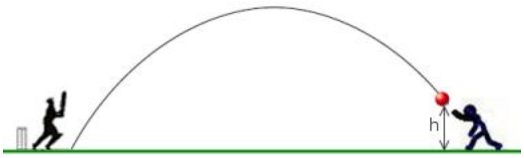
Q20 - ID: 7312

[8 marks, 10 minutes]

A ball is kicked so that it leaves a horizontal surface, at the point A, travelling at 13 ms^{-1}
and at an angle θ above the horizontal. The ball hits the surface again 2 seconds later, at the point B.
Assume that the ball is a particle that moves only under the influence of gravity.
(a) Show that $\theta = 48.9°$, correct to three significant figures.
(b) Find the time for which the ball is more than 3 metres above the surface.

Q21 - ID: 7328

[14 marks, 17 minutes]



A cricket ball is hit at ground level on a horizontal surface. It moves initially at 25 ms^{-1}
and at an angle 56° above the horizontal.
(a) Find the maximum height of the ball during its flight.
The ball is caught when it is at a height of $h = 2.1\ m$ above ground level, as shown.
(b) Show that the time it takes for the ball to travel from the point where it was hit to the point where it was caught is $4.13\ s$, correct to 3 significant figures.
(c) Find the speed of the ball when it is caught.

Q22 - ID: 7355

[6 marks, 7 minutes]

Sandy is throwing a stone at a plum tree. The stone is thrown from a point O at a
speed of 35 ms^{-1} at an angle of α to the horizontal, where $\cos\alpha = 0.93$. You are
given that, t seconds after being thrown, the stone is $(9.8t - 4.9t^2)\ m$ higher than O.
When descending, the stone hits a plum which is $4.704\ m$ higher than O.
Air resistance should be neglected. Calculate the horizontal distance of the plum from O.

Q23 - ID: 7768 *[10 marks, 12 minutes]*

An arrow is fired horizontally at a speed of 22 ms^{-1} from a point at a height of h metres above horizontal ground. The arrow hits the ground after it has been moving for 0.6 seconds. Model the arrow as a particle that moves only under the influence of gravity.

(a) Show that $h = 1.76$, correct to three significant figures.

(b) Find the horizontal distance travelled by the arrow during its flight.

(c) Find the speed of the arrow and the direction in which it is moving when it hits the ground.

Q24 - ID: 3902 *[11 marks, 13 minutes]*

In an automobile crash-test,a car is accelerated from rest at $2ms^{-2}$ for 7 seconds and then decelerated at $4ms^{-2}$ until it strikes a barrier.
The position function is:

$$s(t) = \begin{cases} t^2 & 0 \le t < 7 \\ -2t^2 + At + B & t > 7 \end{cases}$$

(a) Assuming that both $s(t)$ and $s'(t)$ are continuous at $t = 7$, determine A and B.

(b) The barrier is located at $s = 73$ meters. Determine the velocity of the car when it strikes the barrier.

Q25 - ID: 392 *[6 marks, 7 minutes]*

A particle P moves on the x-axis. At time t seconds, its acceleration is $(35 - 10t)ms^{-2}$, measured in the direction of x increasing. When $t = 0$, its velocity is $90ms^{-1}$ measured in the direction of x increasing. Find the time when P is instantaneously at rest in the subsequent motion.

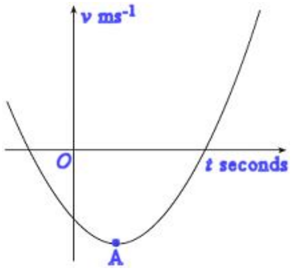
Q26 - ID: 3580 *[6 marks, 7 minutes]*

The displacement s metres of a moving body B from a fixed point O at time t is given by $s = 70t - 10t^2 + 1200$

(a) Find the velocity of B in ms^{-1}.

(b) Find its maximum displacement from O.

Q27 - ID: 2917 [14 marks, 17 minutes]



Its velocity, $v\ ms^{-1}$, at time t seconds for the time interval $-3 \leq t \leq 7$ is given by $v = t^2 - 4t - 12$

(a) Write down the velocity of the insect when $t = 0$.

(b) Show that the insect is instantaneously at rest when $t = -2$ and $t = 6$.

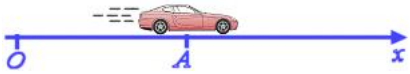
(c) Determine the velocity of the insect when its acceleration is zero.

(d) Write down the coordinates of the point A shown above

(e) Calculate the distance travelled by the insect from $t = 2$ to $t = 6$.

(f) Write down the distance travelled by the insect in the time interval $-2 \leq t \leq 6$.

Q28 - ID: 2908 [8 marks, 10 minutes]



A toy car is moving along the straight line Ox, where O is the origin.
The time t is in seconds. At time t = 0 the car is at A, 6 m from O as shown.
The velocity of the car, $v\ ms^{-1}$, is given by

$$v = 3 + 6t - 3t^2$$

Calculate the distance of the car from O when its acceleration is zero.

Q29 - ID: 4575 [8 marks, 10 minutes]

The displacement of a particle from a fixed point O at time t seconds is $t^4 - 18t^2 + 81$ metres, where $t \geq 0$.

(a) Verify that when $t = 3$ the particle is at rest at the point O.

(b) Calculate the acceleration of the particle when $t = 3$.

Q30 - ID: 5999 [7 marks, 8 minutes]

A particle moves along a straight line so that its velocity, $v\ ms^{-1}$ at time t seconds is given by $v = 18e^{6t} + 9$.
When $t = 0$, the displacement, s, of the particle is 7 metres.
Find an expression for s in terms of t.

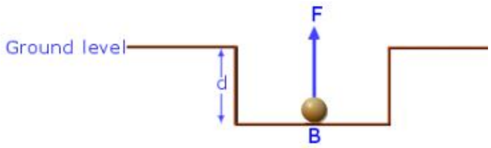
Q31 - ID: 7336 *[12 marks, 14 minutes]*

A model train travels along a straight track.
After time t seconds after setting out from station A the train has velocity $v\ ms^{-1}$ and displacement x metres from A.
It is given that for $0 \leq t \leq= 7$, $x = 0.06t^4 - 0.96t^3 + 4.32t^2$.
After leaving A the train comes to instantaneous rest at station B.
(a) Express v in terms of t. Verify that when $t = 2$ the velocity of the train is $7.68\ ms^{-1}$.
(b) Express the acceleration of the train in terms of t and hence show that when the acceleration of the train is zero $t^2 - 8t + 12 = 0$.
(c) Calculate the minimum value of v.
(d) Calculate the distance AB.

Q32 - ID: 7351 *[19 marks, 23 minutes]*

The displacement $x\ m$ from the origin O of a particle on the x-axis is given by
$$x = 11 + 48t + 18t^2 - 4t^3$$
where t is the time in seconds and $-3 \leq t \leq 8$.
(a) Write down the displacement of the particle when $t = 0$.
(b) Find an expression in terms of t for the velocity, $v\ ms^{-1}$, of the particle.
(c) Find an expression in terms of t for the acceleration of the particle.
(d) Find the maximum value of v in the interval $-3 \leq t \leq 8$.
(e) Show that $v = 0$ only when $t = -1$ and $t = 4$. Find the values of x at these times.
(f) Calculate the distance travelled by the particle from $t = 0$ to $t = 6$.
(g) Determine how many times the particle passes through O in the interval $-3 \leq t \leq 8$.

Q33 - ID: 7400 *[12 marks, 14 minutes]*



Shown is a stone of mass $0.8\ kg$ sitting at B, the bottom of a $d = 0.9\ m$ deep hole.
A workman applies a vertical force F to the stone and lifts it upwards.
The magnitude of F at time t seconds, $t \geq 0$, can be modelled by $F = (8.4 - 0.32t)$ newtons.
(a) Show that the acceleration of the stone can be expressed as $a = (0.7 - 0.4t)\ ms^{-2}$.
(b) Find an expression for the velocity v of the stone at time t.
(c) Find an expression for the displacement of the stone from B at time t.
(d) Show that when $v = 0.2ms^{-1}$ for the second time, the stone is at ground level.

Q34 - ID: 7406 *[11 marks, 13 minutes]*

The displacement of a ball from a fixed point O at time t seconds, $t \geq 0$,
is given by $s = (t^3 - 9t^2 + 24t)$ metres.
(a) Find an expression for the velocity of the ball at time t.
(b) Find the times when the ball is instantaneously at rest.
(c) Find the distance travelled by the ball in the first 4 seconds.

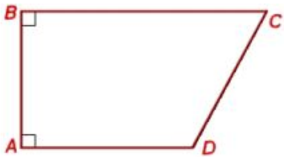
Q35 - ID: 7343 *[10 marks, 12 minutes]*

A car is travelling at 19 ms^{-1} along a straight road when it passes a point A
at time $t = 0$, where t is in seconds. For $0 \leq t \leq 3$, the car accelerates at $0.4t\ ms^{-2}$.
(a) Calculate the speed of the car when $t = 3$.
(b) Calculate the displacement of the car from A when $t = 3$.

Q36 - ID: 7359 *[16 marks, 19 minutes]*

A toy boat moves in a horizontal plane with position vector $r = xi + yj$,
where i and j are the standard unit vectors east and north respectively.
The origin of the position vectors is at O. The displacements x and y are in metres.
First consider only the motion of the boat parallel to the x-axis.
For this motion $x = 6t - 3t^2$. The velocity of the boat in the x-direction is $v_x\ ms^{-1}$.
(a) Find an expression in terms of t for v_x and determine when the boat
instantaneously has zero speed in the x-direction.
Now consider only the motion of the boat parallel to the y-axis. For this
motion $v_y = (t-1)(3t-1)$, where $v_y\ ms^{-1}$ is the velocity of the boat
in the y-direction at time t seconds.
(b) Given that $y = 6$ when $t = 2$, use integration to show that $y = t^3 - 2t^2 + 1t - -4$.
The position vector of the boat is given in terms of t by $r = (6t - 3t^2)i + (t^3 - 2t^2 + 1t + 4)j$
(c) Find the time(s) when the boat is due north of O and also the distance
of the boat from O at any such times.
(d) Find the time(s) when the boat is instantaneously at rest. Find the distance
of the boat from O at any such times.

Q37 - ID: 857 *[8 marks, 10 minutes]*



A thin uniform wire, of total length $20cm$, is bent to form a frame. The frame is in the shape of a trapezium $ABCD$, where $AB = 4cm, AD = 4, CD = 5cm$ and AB is perpendicular to BC and AD.

(a) Find the distance of the centre of mass of the frame from AB.

The frame has mass m. A particle of mass km is attached to the frame at C. When the frame is freely suspended from the mid-point of BC, the frame hangs in equilibrium with BC horizontal.

(b) Find the value of k.

Q38 - ID: 601 *[10 marks, 12 minutes]*

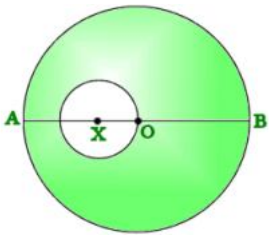
The diagram shows four uniform rods joined to form a rigid rectangular framework $ABCD$, where $AB = CD = 2a$, and $BC = AD = 6a$. Each rod has mass m. Particles, of mass $6m$ and $4m$, are attached to the framework at points C and D respectively.

(a) Find the distance of the centre of mass of the loaded framework from (i) AB, (ii) AD.

The loaded framework is freely suspended from B and hangs in equilibrium.

(b) Find the angle which BC makes with the vertical.

Q39 - ID: 3048 *[10 marks, 12 minutes]*



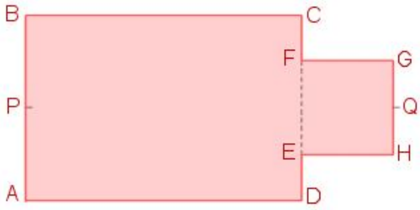
The diagram shows a template T made by removing a circular disc, of centre X and radius 8 cm, from a uniform circular lamina, of centre O and radius 24 cm. The point X lies on the diameter AOB of the lamina and AX = 16 cm. The centre of mass of T is at the point G.

(a) Find AG

The template T is free to rotate about a smooth fixed horizontal axis, perpendicular to the plane of T, which passes through the mid-point of OB. A small stud of mass $\frac{1}{5}m$ is fixed at B, and T and the stud are in equilibrium with AB horizontal. Modelling the stud as a particle,

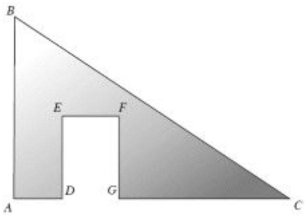
(b) find the mass of T in terms of m

Q40 - ID: 3268 *[10 marks, 12 minutes]*



A uniform lamina is in the shape of a rectangle ABCD and a square EFGH, as shown in the diagram. The length AB is 48 cm, the length BC is 56 cm, the length DE is 12 cm and the length EF is 24 cm.
The point P is the midpoint of AB and the point Q is the midpoint of HG.
(a) Explain why the centre of mass of the lamina lies on PQ.
(b) Find the distance of the centre of mass of the lamina from AB.
(c) The lamina is freely suspended from A. Find, to the nearest degree, the angle between AD and the vertical when the lamina is in equilibrium.

Q41 - ID: 7396 *[11 marks, 13 minutes]*



The diagram shows a uniform lamina ABCGFED consisting of triangle ABC with rectangle DEFG removed. Triangle ABC is right-angled at A with $AB = 15\,cm$ and $AC = 24\,cm$. Rectangle DEFG has sides $DE = 7\,cm$ and $EF = 6\,cm$. Distance $AD = 3\,cm$.
(a) Calculate the distances of the centre of mass of the lamina from AB and AC, giving your answers correct to 3 significant figures.
(b) The lamina is freely suspended from A and hangs in equilibrium. Calculate the angle AC makes with the vertical.

Q42 - ID: 870 *[6 marks, 7 minutes]*

A brick of mass $2kg$ slides in a straight line on a horizontal floor. The brick is modelled as a particle and the floor as a rough plane. The initial speed of the brick is $12ms^{-1}$. The brick is brought to rest after moving $13m$ by the constant frictional force between the brick and the floor.
(a) Calculate the kinetic energy lost by the brick in coming to rest, stating the units of your answer.
(b) Calculate the coefficient of friction between the brick and the floor.

Q43 - ID: 3267 *[10 marks, 12 minutes]*

A hot air balloon moves vertically upwards with a constant velocity.
When the balloon is at a height of 29 metres above ground level,
a box of mass 3 kg is released from the balloon. After the box is
released, it initially moves vertically upwards with speed $12 ms^{-1}$
(a) Find the initial kinetic energy of the box.
(b) Show that the kinetic energy of the box when it hits the ground is $1068.6 J$.
(c) Hence find the speed of the box when it hits the ground.
(d) State two modelling assumptions which you have made.

Q44 - ID: 3524 *[5 marks, 6 minutes]*

A parcel of mass $3.8 kg$ is moving in a straight line on a smooth horizontal
floor. Initially the parcel is moving with speed $7\ ms^{-1}$. The parcel is brought to
rest in a distance of $20\ m$ by a constant horizontal force of magnitude R newtons.
Modeling the parcel as a particle, find
(a) the kinetic energy lost by the parcel in coming to rest,
(b) the value of R.

Q45 - ID: 3261 *[9 marks, 11 minutes]*

A car of mass $1470 kg$ is travelling along a straight horizontal road.
When the car is travelling at a speed of $v\ ms^{-1}$, it experiences a
resistance force of magnitude $41v$ newtons.
(a) On this road, the car has a maximum speed of $56\ ms^{-1}$
Show that the maximum power of the car is 128576 watts.
(b) Find the maximum possible acceleration of the car when its
speed on the road is $26\ ms^{-1}$

Q46 - ID: 387 *[8 marks, 10 minutes]*

A car of mass $800kg$ is moving at a constant speed of $13ms^{-1}$ down a straight road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{29}$. The resistance to motion from non gravitational forces is modelled as a constant force of magnitude $800N$.
(a) Find, in kW, the rate of working of the engine of the car.
When the car is travelling down the road at $13ms^{-1}$, the engine is switched off. The car comes to rest in time T seconds after the engine is switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude $800N$.
(b) Find the value of T.

Q47 - ID: 779 *[6 marks, 7 minutes]*

A car of mass $900kg$ moves along a straight horizontal road with a constant speed of $28ms^{-1}$.
The resistance to motion of the car has magnitude $400N$.
(a) Find, in kW, the rate at which the engine of the car is working.
The car now moves up a hill inclined at α to the horizontal, where $\sin\alpha = \frac{1}{21}$. The resistance to motion of the car from non-gravitational forces remains of magnitude $400N$.
The engine of the car now works at a rate of $22kW$.
(b) Find the acceleration of the car when its speed is $22ms^{-1}$.

Q48 - ID: 553 *[7 marks, 8 minutes]*

A car of mass $900kg$ moves along a straight horizontal road. The resistance to motion of the car from non-gravitational forces is of constant magnitude $400N$.
The car moves with constant speed and the engine of the car is working at a rate of $23kW$.
(a) Find the speed of the car.
The car moves up a hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{13}$.
The car engine continues to work at $23kW$ and the resistance to motion from non-gravitational forces remains of magnitude $400N$.
b) Find the constant speed at which the car can move up the hill.

Q49 - ID: 761 *[6 marks, 7 minutes]*

A particle of mass $1.2kg$ is moving in a straight
line on a rough horizontal plane. The speed of the particle is
reduced from $22ms^{-1}$ to $17ms^{-1}$ as the particle moves $17m$.
Assuming the only resistance to motion is the friction between
the particle and the plane, find
(a) the work done by friction in reducing the speed of the
particle from $22ms^{-1}$ to $17ms^{-1}$,
(b) the coefficient of friction between the particle and the plane.

Q50 - ID: 3049 *[12 marks, 14 minutes]*

A particle P of mass m is moving in a straight line on a smooth
horizontal table. Another particle Q of mass km is at rest on the table.
The particle P collides directly with Q. The direction of motion of P is
reversed by the collision. After the collision, the speed of P is v and the
speed of Q is $5v$. The coefficient of restitution between P and Q is 0.5.
(a) Find, in terms of v only, the speed of P before the collision.
(b) Find the value of k.
After being struck by P, the particle Q collides directly with a particle R
of mass $13m$ which is at rest on the table. After this second collision, Q
and R have the same speed and are moving in opposite directions. Show that
(c) the coefficient of restitution between Q and R is 0.5 ,
(d) there will be a further collision between P and Q.

Q51 - ID: 7391 *[6 marks, 7 minutes]*

An object of mass $0.9\ kg$ is moving on a smooth horizontal floor towards a vertical wall.
The velocity of the object is $7\ ms^{-1}$ in a direction which is perpendicular to the wall.
Before the object reaches the wall, it is given an impulse after which it is moving in
the same direction with speed $3\ ms^{-1}$.
(a) Calculate the magnitude and direction of the impulse, stating your units clearly.
After the impulse, the object collides with the wall. The coefficient of restitution between
the object and the wall is 0.6.
(b) Find the speed of the object after the impact with the wall.

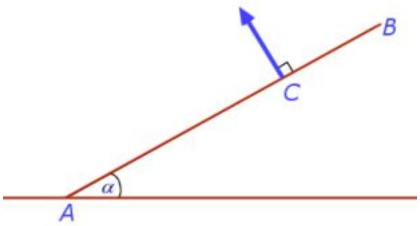
Q52 - ID: 7393 *[8 marks, 10 minutes]*

A sphere A, of mass 10 kg, moving with speed 5 ms^{-1}, collides directly with another sphere B, of mass 5 kg, moving in the opposite direction with speed 2 ms^{-1}. The coefficient of restitution between the spheres is 0.3. Calculate the speeds of the spheres after the collision, clearly indicating their directions of motion.

Q53 - ID: 544 *[8 marks, 10 minutes]*

A cricket ball of mass 0.9kg is struck by a bat. Immediately before being struck, the velocity of the ball is $(-20i)ms^{-1}$. Immediately after being struck, the velocity of the ball is $(13i + 10j)ms^{-1}$.
(a) Find the magnitude of the impulse exerted on the ball by the bat.
In the subsequent motion, the position vector of the ball is r metres at time t seconds. In a model of the situation, it is assumed that $r = [13ti + (10t - 2t^2)j]$.
Using this model,
(b) find the speed of the ball when $t = 4$.

Q54 - ID: 7409 *[12 marks, 14 minutes]*



A workman places end A of a uniform plank AB, length 4 m and mass x kg, on rough horizontal ground. He holds the plank with AB making an angle of $\alpha = 27°$ with the ground by applying a force of 195 N perpendicular to the plank at a point C on AB as shown, where $CB = 1$ m.
(a) Find the frictional force acting at A.
(b) By taking moments about A find x.
The plank is on the verge of slipping.
(c) Find the coefficient of friction between the plank and the ground.

My Full M2 Paper 1 - Mark Scheme

A1 - ID: 7686 *[17 marks, 20 minutes]*

(a) $x = ut$ | M1

$y = cut - 4.9t^2$ | M1A1

$t = \frac{x}{u} \Rightarrow y = cu\frac{x}{u} - 4.9(\frac{x}{u})^2 = cx - \frac{4.9x^2}{u^2}$ | M1A1

(b) $y = 0 \Rightarrow cR - \frac{4.9R^2}{u^2} = 0$ | M1

$\Rightarrow R\left(c - \frac{4.9R}{u^2}\right) = 0$

$u = 10 \Rightarrow R = \frac{u^2c}{4.9} = \frac{1000}{49}c$ | M1A1

$x = \frac{500}{49}c \Rightarrow y = H$ | M1

$\Rightarrow H = \frac{500}{49}c^2 - \frac{250}{49}c^2 = \frac{250}{49}c^2$ | M1A1

(c) $\frac{dy}{dx} = c - \frac{9.8x}{u^2} = c - \frac{49}{500}x$ | M1A1

$x = 0 \Rightarrow \frac{dy}{dx} = c$ | B1

$\Rightarrow c - \frac{49}{500}x = \frac{-1}{c}$ | M1A1

$\Rightarrow x = \frac{500}{49}(c + \frac{1}{c})$ | A1

A2 - ID: 7680 *[8 marks, 10 minutes]*

$\frac{dv}{dt} = 12t - 4$ | M1A1

$\frac{dv}{dt} = 0 \Rightarrow 12t - 4 = 0$

$\Rightarrow t = \frac{1}{3}s$ | M1A1

$s = \int 6t^2 - 4t + 4\,dt = 2t^3 - 2t^2 + 4t$ | M1A1

$t = \frac{1}{3} \Rightarrow s = 2(\frac{1}{3})^3 - 2(\frac{1}{3})^2 + 4(\frac{1}{3}) = \frac{32}{27}m$ | M1A1

A3 - ID: 7687 *[11 marks, 13 minutes]*

(a) Rectangle : Area = $24x$; cog = x

Semicircles : Area = 4.5π; cog = $\frac{4\times 3}{3\pi}$

Template : Area = $24x + 9\pi$; cog = $\bar{x}$ |B4

$\Rightarrow 24x^2 - (9\pi \times \frac{12}{3\pi}) = (24x + 9\pi)\bar{x}$ |M1A1

$\Rightarrow 24x^2 - 36 = (24x + 9\pi)\bar{x}$

$\Rightarrow \bar{x} = \dfrac{8x^2 - 12}{8x + 3\pi}$ |A1

(b) $x = 2 \Rightarrow \bar{x} = \dfrac{20}{16 + 3\pi}$ |B1

$\tan\theta = \dfrac{6}{4 - \bar{x}} = \dfrac{6}{4 - \frac{20}{16+3\pi}}$ |M1A1

$= \dfrac{6(16 + 3\pi)}{4(16 + 3\pi) - 20} = \dfrac{48 + 9\pi}{22 + 6\pi}$ |A1

A4 - ID: 7682 *[6 marks, 7 minutes]*

work done $= \frac{1}{2}0.9(22)^2$ |B1

P.E $= 0.9 \times 6g$ |B1

$\Rightarrow 6R = \frac{1}{2}0.9(22)^2 - 0.9 \times 6g$ |M1A1

$\Rightarrow R = (\frac{1}{2}0.9(22)^2 - 0.9 \times 6g) \div 6 = 27.48$ |M1A1

A5 - ID: 7683 *[11 marks, 13 minutes]*

(a) Resolve $\leftrightarrow \Rightarrow \frac{480}{7} - R = 0$ |B1M1A1

$\Rightarrow R = 68.6\,N$ |A1

(b) Resolve $\leftrightarrow \Rightarrow \frac{45}{U} + 80g \times \frac{1}{14} - 45U = 0$ |B1

$\Rightarrow 45 + 56U - 45U^2 = 0$ |M1A2

$\Rightarrow (9 - 5U)(5 + 9U) = 0$ |M1

$\Rightarrow U = \frac{9}{5}ms^{-1}$ |M1A1

A6 - ID: 7685 *[8 marks, 10 minutes]*

(a) $I \updownarrow = 0.28 \times 39\sin 57 = 9.158$ |M1

$I \leftrightarrow = 0.28(-39\cos 57 + 34) = 3.573$ |M1

$\Rightarrow |I| = \sqrt{3.573^2 + 9.158^2} = 9.83Ns$ |M1A1

(b) $\tan\theta = \pm\dfrac{9.158}{3.573}$ |M1A1

$\Rightarrow \theta = 111°$ |M1A1

A7 - ID: 7681 *[7 marks, 8 minutes]*

CLM $\Rightarrow 8mu - mu = 2mv_1 + mv_2$ |$M1A1$

$\Rightarrow 7u = 2v_1 + v_2$

NLR $\Rightarrow 7eu = -v_1 + v_2$ |$M1A1$

$\Rightarrow 7u - 7eu = 3v_1$

$\Rightarrow v_1 = \frac{7}{3}u(1-e)$ |$M1A1$

$\Rightarrow v_2 = 7eu + v_1 = 7eu + \frac{7}{3}u(1-e) = u(\frac{14}{3}e + \frac{7}{3})$ |$A1$

A8 - ID: 7684 *[7 marks, 8 minutes]*

moments at B $\Rightarrow R \times 7\cos\alpha = F \times 7\sin\alpha + 24g \times 3.5\cos\alpha$ |$M1A2$

$F = 0.4R \Rightarrow 5.6R = 0.4R \times 4.2 + 24g \times 2.8$ |$M1B1$

$\Rightarrow 3.92R = 67.2g$

$\Rightarrow R = 168\,N$ |$M1A1$

A9 - ID: 398 *[10 marks, 12 minutes]*

(a) $a = \frac{dv}{dt} = (3\underline{i} - 6\underline{j})$

$v = (3t + c)\underline{i} + (-6t + k)\underline{j}$ |$M1A1$

$t = 0 \Rightarrow v = (-3\underline{i} + 6\underline{j})$

$\Rightarrow v = (3t + -3)\underline{i} + (-6t + 6)\underline{j}$

$\Rightarrow (-6t + 6) = 0$

$\Rightarrow t = \frac{6}{6}$ |$M1A1$

(b) $t = 4 \Rightarrow v = 9\underline{i} + -18\underline{j}$ |$M1$

$\Rightarrow$ speed $= \sqrt{9^2 + -18^2} = 20.12m/s$ |$A2$

(c) $\tan\alpha = \frac{9}{-18}$ |$M1$

$\Rightarrow \alpha = -26.6$ |$A1$

$\Rightarrow$ angle $= 153.4$ |$A1$

A10 - ID: 483 *[9 marks, 11 minutes]*

(a) $v = \frac{dr}{dt} = (18 - 21t^2)i + 2ctj$ |$M1A2$

$t = 1 \Rightarrow v = -3i + 2cj$ |$M1$

$|v| = 14 \Rightarrow -3^2 + (2c)^2 = 14^2$ |$M1$

$\Rightarrow 9 + 4c^2 = 196$

$\Rightarrow c^2 = 46.75 \Rightarrow c = 6.837$ |$A1$

(b) $a = \frac{dv}{dt} = (-42t)i + 2cj$ |$M1$

$t = 1 \Rightarrow a = -42i + 13.675j$ |$M1A1$

A11 - ID: 477 [8 marks, 10 minutes]

(a) $v = \frac{dr}{dt} = (2t+4)i + (3-3t^2)j$ |M1A1

$t = 8 \Rightarrow v = 20i - 189j$ |M1

$\Rightarrow |v| = \sqrt{20^2 + 189^2} = \sqrt{36121} = 190.1$ |M1A1

(b) $t = 8 \Rightarrow 0.5(v - (20i - 189j)) = 9i - 15j$ |M1A1

$\Rightarrow 0.5v - 10i + 94.5j = 9i - 15j$

$\Rightarrow v = 38i - 219j$ |A1

A12 - ID: 3262 [11 marks, 13 minutes]

(a) $F = ma \Rightarrow (4400i - 5280tj) = 880a$ |M1

$\Rightarrow a = (5i - 6tj)$ |A1

(b) $v = \int a\,dt = (5it - 3t^2 j) + c$ |M1A1

$t = 0, v = (4i + 31j) \Rightarrow c = (4i + 31j)$ |M1

$\Rightarrow v = (5it - 3t^2 j) + (4i + 31j)$

$\Rightarrow v = (4 + 5t)i + (31 - 3t^2)j$ |A1

(c) $r = \int v\,dt = (4t + 2.5t^2)i + (31t - 1t^3)j + c$ |M1A2

$t = 0, r = (3i + 5j) \Rightarrow c = (3i + 5j)$ |M1

$\Rightarrow r = (4t + 2.5t^2)i + (31t - 1t^3)j + (3i + 5j)$

$\Rightarrow r = (3 + 4t + 2.5t^2)i + (5 + 31t - 1t^3)j$ |A1

A13 - ID: 3525 [9 marks, 11 minutes]

(a) $v = \frac{dp}{dt} = (12t - 4)i + (6t^2 - 5)j$ |M1A1

(b) $6t^2 - 5 = 0 \Rightarrow 6t^2 = 5$ |M1

$\Rightarrow t = 0.913\ s$ |M1A1

(c) $t = 5 \Rightarrow v = 56i + 145j$ |B1

$\Rightarrow 4i - 4j = 0.2(v - 56i - 145j)$ |M1

$\Rightarrow 15.2i + 25j = 0.2v$

$\Rightarrow v = 76i + 125j\ ms^{-1}$ |M1A1

A14 - ID: 569 [8 marks, 10 minutes]

(a) $x_A = 20t, x_B = 25\cos\alpha t$ |B1B1

meet $\Rightarrow 20t = 25\cos\alpha t \Rightarrow \cos\alpha = \frac{4}{5}$ |M1A1

(b) $y_A = 67.5 - \frac{1}{2}gt^2$ |B1

$y_B = 25\sin\alpha t - \frac{1}{2}gt^2 = 15t - \frac{1}{2}gt^2$ |B1

meet $\Rightarrow 67.5 - \frac{1}{2}gt^2 = 15t - \frac{1}{2}gt^2$

$\Rightarrow 67.5 = 15t$

$\Rightarrow t = 4.5s$ |M1A1

A15 - ID: 2959 *[9 marks, 11 minutes]*

(a) $s = ut + \frac{1}{2}at^2 \Rightarrow -2.46 = 0 - \frac{1}{2}9.8t^2 \Rightarrow t^2 = 0.502\,s$ |M1A1

$\Rightarrow t = 0.709\,s$ |A1

(b) $s = ut = 12.754\,m$ |M1A1

(c) $v_x = 18$

$v_y = u + at = 0 - 9.8 \times 0.709 = -6.944$ |M1A1

angle $= \tan^{-1}\frac{6.944}{18} = 21.1°$ |M1A1

A16 - ID: 7771 *[12 marks, 14 minutes]*

(a) $s = ut + \frac{1}{2}at^2 \Rightarrow 5 = 20\sin 54t - 4.9t^2$ |M1A1

$\Rightarrow 4.9t^2 - 20\sin 54t + 5 = 0$ |A1

$\Rightarrow t = 0.345, 2.957$

$\Rightarrow t = 0.345\,s$ |M1A1

(b) $s = ut = 20\cos 54 \times 0.345 = 4.057\,m$ |M1A1

(c) $v_x = 20\cos 54 = 11.8$ |B1

$v_y = u + at = 20\sin 54 - 9.8 \times 0.345 = 12.8$ |M1A1

$\Rightarrow v = \sqrt{11.8^2 + 12.8^2} = 17.4\,ms^{-1}$ |M1A1

A17 - ID: 2910 *[19 marks, 23 minutes]*

(a) $s_x = ut \Rightarrow 60 = 21t \Rightarrow t = 2.857\,s$ |M1A1

$v = u + at \Rightarrow 0 = u - 9.8 \times 2.857 \Rightarrow u = 28$ |M1A1

(b) $s = ut + \frac{1}{2}at^2 = 28t - 4.9t^2$ |B1

(c) : start at same height; same vertical speed |B1

distance $= 21t - 5.25t = 15.75t$ |B2

(d) distance $= 15.75 \times 2.857 = 45\,m$ |B2

(e) Time to fall is $\Rightarrow 40 - 10 = \frac{1}{2} \times 9.8 \times t^2$ |M1A1

$\Rightarrow t = 2.474$ |A1

$\Rightarrow$ distance $= 15.75 \times 2.474 = 38.971\,m$ |M1A1

(f) $x = 21t, \quad y = 28t - 4.9t^2$

$\Rightarrow y = 28\frac{x}{21} - 4.9\left(\frac{x}{21}\right)^2$ |B1M1A1

$\Rightarrow y = \frac{1}{441}(588x - 4.9x^2)$ |A1

A18 - ID: 5463 *[6 marks, 7 minutes]*

(a) $s = ut + 0.5at^2 \Rightarrow s = 46\sin\alpha t - 5t^2 = 36.8t - 5t^2$ |M1A1

(b) $36.8t - 5t^2 = 0 \Rightarrow t = 7.36$ |M1A1

range $= 46\cos\alpha t = 203.14\,m$ |M1A1

A19 - ID: 7306 [14 marks, 17 minutes]

(a) $s = ut + 0.5at^2 \Rightarrow -5 = 69\sin 13t - 4.9t^2$ |M1A1

$\Rightarrow 4.9t^2 - 69\sin 13t - 5 = 0$ |A1

$\Rightarrow t = \dfrac{69\sin 13 \pm \sqrt{4761\sin^2 13 + 98}}{9.8}$ |M1

$\Rightarrow t = 3.46, -0.29$ |M1

$\Rightarrow t = 3.46$ |A1

(b) $s = ut = 69\cos 13 \times 3.46 = 232.78\,m$ |M1A1

(c) min speed, $v_x = 69\cos 13 = 67.23\,ms^{-1}$ |M1A1

(d) $v_y = u + at = 69\sin 13 - 9.8 \times 3.46 = -18.41$ |M1A1

$\Rightarrow$ speed $= \sqrt{67.23^2 + -18.41^2} = 69.7\,ms^{-1}$ |M1A1

A20 - ID: 7312 [8 marks, 10 minutes]

(a) $s = ut + \frac{1}{2}at^2 \Rightarrow 0 = 13\sin\theta \times 2 - \frac{1}{2}g \times 4$ |M1A1

$\Rightarrow 26\sin\theta = 19.6$

$\Rightarrow \theta = \sin^{-1}\frac{19.6}{26} = 48.9°$ |A1

(b) $s = 3 \Rightarrow 13\sin 48.9 \times t - \frac{1}{2}gt^2 = 3$ |M1A1

$\Rightarrow gt^2 - 26\sin 48.9t + 6 = 0$

$\Rightarrow t = \dfrac{26\sin 48.9 \pm \sqrt{676\sin^2 48.9 - 24g}}{2g}$ |M1

$\Rightarrow t = 1.62, 0.38$ |A1

$\Rightarrow$ time above 3 metres = 1.25 s |A1

A21 - ID: 7328 [14 marks, 17 minutes]

(a) $v^2 = u^2 + 2as \Rightarrow 0 = (25\sin 56)^2 - 2gs$ |M1A1

$\Rightarrow 2gs = 625\sin^2 56$

$\Rightarrow s = 21.9\,m$ |M1A1

(b) $s = ut + \frac{1}{2}at^2 \Rightarrow 2.1 = (25\sin 56)t - \frac{1}{2}gt^2$ |M1A1

$\Rightarrow gt^2 - 50\sin 56t + 4.2 = 0$

$\Rightarrow t = \dfrac{50\sin 56 \pm \sqrt{2500\sin^2 56 - 16.8g}}{2g}$ |M1

$\Rightarrow t = 4.13, 0.1$ |A1

$\Rightarrow$ time = 4.13 s |A1

(c) $v_x = 25\cos 56 = 13.98$ |B1

$v_y = u + at = 25\sin 56 - g \times 4.13 = -19.71\,ms^{-1}$ |M1A1

$\Rightarrow$ speed $= \sqrt{13.98^2 + -19.71^2} = 24.2\,ms^{-1}$ |M1A1

A22 - ID: 7355 *[6 marks, 7 minutes]*

$4.704 = 9.8t - 4.9t^2 \Rightarrow 4.9t^2 - 9.8t + 4.704 = 0$ | M1

$\Rightarrow t^2 - 2t + 0.96 = 0$ | M1

$\Rightarrow t = 0.8, 1.2$ | A1

descending $\Rightarrow t = 1.2\,m$ | A1

$s = ut + \frac{1}{2}at^2 = 35\cos\alpha \times 1.2 = 39.06\,m$ | M1A1

A23 - ID: 7768 *[10 marks, 12 minutes]*

(a) $h = \frac{1}{2} \times 9.8 \times 0.6^2 = 1.76\,m$ | M1A1

(b) $distance = 22 \times 0.6 = 13.2\,m$ | M1A1

(c) $v_x = 22ms^{-1}$

$v_y = u + at = 0 + 9.8 \times 0.6 = 5.88ms^{-1}$ | B1

$\Rightarrow v = \sqrt{5.88^2 + 22^2} = 22.8\,ms^{-1}$ | M1A1

$direction = \tan^{-1}\frac{5.88}{22} = 15°$ below horizontal | M1A1B1

A24 - ID: 3902 *[11 marks, 13 minutes]*

(a) $s(t)$ continuous at $t = 7 \Rightarrow s(7) = 49$ | M1

$\Rightarrow 49 = -2(7)^2 + A(7) + B$

$\Rightarrow 7A + B = 147$ | M1A1

differentiate $\Rightarrow s'(x) = \begin{cases} 2t & 0 \le t < 7 \\ -4t + A & t > 7 \end{cases}$ | M1A1

$s'(t)$ continuous at $t = 7 \Rightarrow s'(7) = 14$

$\Rightarrow 14 = -4(7) + A$

$\Rightarrow A = 42 \Rightarrow B = -147$ | M1A1

(b) $s(T) = 73 \Rightarrow -2T^2 + 42T + -147 = 73$

$\Rightarrow T^2 - 21T + 110 = 0$

$\Rightarrow (T - 10)(T - 11) = 0$

$\Rightarrow T = 10, 11$ | M1A1

crashes once only! $\Rightarrow T = 10$

$\Rightarrow s'(10) = 2\,ms^{-1}$ | M1A1

A25 - ID: 392 *[6 marks, 7 minutes]*

$a = 35 - 10t \Rightarrow v = \int 35 - 10t\,dt = 35t - 5t^2 + c$ | M1A1

$t = 0, v = 90 \Rightarrow v = 35t - 5t^2 + 90$ | A1

$v = 0 \Rightarrow 5t^2 - 35t - 90 = 0$ | M1

$\Rightarrow t^2 - 7t - 18 = 0$

$\Rightarrow (t - 9)(t + 2) = 0$ | M1

$\Rightarrow t = 9$ | A1

A26 - ID: 3580 *[6 marks, 7 minutes]*

(a) $v = \frac{ds}{dt} = 70 - 20t$ |$M1A1$

(b) max displacement $\Rightarrow v = 0$ |$M1$

$\Rightarrow 70 - 20t = 0 \Rightarrow t = 3.5$ |$A1$

$\Rightarrow s = 70(3.5) - 10(3.5)^2 + 1200$

$\Rightarrow s = 1322.5\,m$ |$M1A1$

A27 - ID: 2917 *[14 marks, 17 minutes]*

(a) $t = 0 \Rightarrow v = -12\,ms^{-1}$ |$A1$

(b) $v = 0 \Rightarrow t^2 - 4t - 12 = 0 \Rightarrow (t-6)(t+2) = 0 \Rightarrow t = 6, -2$ |$M1A1$

(c) $a = \frac{dv}{dt} = 2t - 4$ |$M1A1$

$a = 0 \Rightarrow t = 2 \Rightarrow v = -16ms^{-1}$ |$M1A1$

(d) $A = (2, -16)$ |$A1$

(e) distance $= \int_2^6 t^2 - 4t - 12\,dt$ |$M1$

$= \left[\frac{1}{3}t^3 - 2t^2 - 12t\right]_2^6 = \left(\frac{216}{3} - 72 - 72\right) - \left(\frac{8}{3} - 8 - 24\right)$ |$M1A1$

$= -42.67 \Rightarrow$ distance $= 42.67\,m$ |$M1A1$

(f) distance $= 2 \times 42.67 = 85.33\,m$ |$A1$

A28 - ID: 2908 *[8 marks, 10 minutes]*

$a = \frac{dv}{dt} = 6 - 6t$ |$M1A1$

$a = 0 \Rightarrow t = 1$ |$A1$

$s = \int v\,dt = 3t + 3t^2 - 1t^3 + c$ |$M1A1$

$t = 0, s = 6 \Rightarrow 6 = c$

$\Rightarrow s = 3t + 3t^2 - 1t^3 + 6$ |$M1A1$

$t = 1 \Rightarrow s = 3(1) + 3(1)^2 - 1(1)^3 + 6 = 11\,m$ |$A1$

A29 - ID: 4575 *[8 marks, 10 minutes]*

(a) $t = 3 \Rightarrow d = (3)^4 - 18(3)^2 + 81 = 0$ |$B1$

$\Rightarrow v = 4t^3 - 36t$ |$M1A1$

$t = 3 \Rightarrow v = 4(3)^3 - 36(3) = 0$ |$M1A1$

(b) $a = 12t^2 - 36$ |$M1A1$

$t = 3 \Rightarrow a = 12(3)^2 - 36 = 72\,ms^{-2}$ |$A1$

A30 - ID: 5999 *[7 marks, 8 minutes]*

$s = \int 18e^{6t} + 9\,dt$ |M1

$= 3e^{6t} + 9t + c$ |A3

$t = 0, s = 7 \Rightarrow 7 = 3 + c$

$\Rightarrow c = 4$ |M1A1

$\Rightarrow s = 3e^{6t} + 9t + 4$ |A1

A31 - ID: 7336 *[12 marks, 14 minutes]*

(a) $v = \frac{dx}{dt} = 0.24t^3 - 2.88t^2 + 8.64t$ |M1A1

$t = 2 \Rightarrow v = 0.24(2)^3 - 2.88(2)^2 + 8.64(2) = 7.68\,ms^{-1}$ |A1

(b) $a = \frac{dv}{dt} = 0.72t^2 - 5.76t + 8.64$ |M1A1

$a = 0 \Rightarrow 0.72t^2 - 5.76t + 8.64 = 0$

$\Rightarrow t^2 - 8t + 12 = 0$ |A1

(c) $a = 0 \Rightarrow (t-2)(t-6) = 0$ |M1A1

$\Rightarrow t = 2, 6$

$t = 6 \Rightarrow v = 0\,ms^{-1}$ |M1A1

(d) $t = 6 \Rightarrow x = 0.06(6)^4 - 0.96(6)^3 + 4.32(6)^2 = 25.92\,m$ |M1A1

A32 - ID: 7351 *[19 marks, 23 minutes]*

(a) $t = 0 \Rightarrow x = 11\,m$ |B1

(b) $v = \frac{dx}{dt} = 48 + 36t - 12t^2$ |M1A1

(c) $a = \frac{dv}{dt} = 36 - 24t$ |M1A1

(d) $a = 0 \Rightarrow t = 1.5$ |M1A1

$\Rightarrow v = 48 + 36(1.5) - 12(1.5)^2 = 75\,ms^{-1}$ |A1

(e) $v = 0 \Rightarrow 48 + 36t - 12t^2 = 0$ |M1A1

$\Rightarrow 4 + 3t - t^2 = 0$

$\Rightarrow (4-t)((1+t) = 0 \Rightarrow t = -1, 4$ |A1

$t = -1 \Rightarrow x = -15\,m$ |B1

$t = 4 \Rightarrow x = 235\,m$ |B1

(f) $t = 6 \Rightarrow x = 83\,m$ |M1

$\Rightarrow$ distance for $0 \le t \le 4 = 235 - 11 = 224$

$\Rightarrow$ distance for $4 \le t \le 6 = 235 - 83 = 152$

$\Rightarrow$ total = 376 m |A2

(g) $t = -3 \Rightarrow x = 137\,m, \quad t = -1 \Rightarrow x = -15\,m$

$t = 0 \Rightarrow x = 11\,m, \quad t = 8 \Rightarrow x = -501\,m$

$\Rightarrow$ sign changes mean $x = 0$ at three different times |M1B2

A33 - ID: 7400 *[12 marks, 14 minutes]*

(a) $force = ma \Rightarrow 8.4 - 0.32t - 0.8g = 0.8a$ |$M1A1$

$\Rightarrow 0.56 - 0.32t = 0.8a$

$\Rightarrow a = (0.7 - 0.4t)\,ms^{-2}$ |$A1$

(b) $v = \int a\,dt = 0.7t - 0.2t^2 + c$ |$M1A1$

$t = 0, v = 0 \Rightarrow v = 0.7t - 0.2t^2$ |$A1$

(c) $s = \int v\,dt = 0.35t^2 - 0.07t^3 + c$ |$M1A1$

$t = 0, s = 0 \Rightarrow s = 0.35t^2 - 0.07t^3$ |$A1$

(d) $v = 0.2 \Rightarrow 0.2 = 0.7t - 0.2t^2$

$\Rightarrow 0.2t^2 - 0.7t + 0.2 = 0$

$\Rightarrow t = 3.19, 0.31$ |$M1A1$

$t = 3.19 \Rightarrow s = 0.35(3.19)^2 - 0.07(3.19)^3 = 1.4\,m$ |$A1$

A34 - ID: 7406 *[11 marks, 13 minutes]*

(a) $v = \frac{ds}{dt} = 3t^2 - 18t + 24$ |$M1A2$

(b) $v = 0 \Rightarrow 3t^2 - 18t + 24 = 0$ |$M1$

$\Rightarrow t^2 - 6t + 8 = 0$

$\Rightarrow (t-2)(t-4) = 0$

$\Rightarrow t = 2, 4$ |$M1A1$

(c) $t = 0 \Rightarrow s = 0$ |$A1$

$t = 2 \Rightarrow s = (2)^3 - 9(2)^2 + 24(2) = 20$ |$A1$

$t = 4 \Rightarrow s = (4)^3 - 9(4)^2 + 24(4) = 16$ |$A1$

$\Rightarrow$ distance $= 20 + 4 = 24\,m$ |$M1A1$

A35 - ID: 7343 *[10 marks, 12 minutes]*

(a) $v = \int 0.4t\,dt = 0.2t^2 + c$ |$M1A1$

$t = 0, v = 19 \Rightarrow v = 0.2t^2 + 19$ |$A1$

$t = 3 \Rightarrow v = 20.8\,ms^{-1}$ |$M1A1$

(b) $s = \int 0.2t^2 + 19\,dt$

$= \frac{1}{3}0.2t^3 + 19t + c$ |$M1A1$

$t = 0, s = 0 \Rightarrow s = \frac{1}{3}0.2t^3 + 19t$ |$A1$

$t = 3 \Rightarrow s = 58.8\,m$ |$M1A1$

A36 - ID: 7359 [16 marks, 19 minutes]

(a) $v_x = \frac{dx}{dt} = 6 - 6t$ |$M1A1$

$v_x = 0 \Rightarrow t = 1\,s$ |$A1$

(b) $y = \int v_y\,dt = \int 3t^2 - 4t + 1\,dt$

$= t^3 - 2t^2 + 1t + c$ |$M1A1$

$y = 6, t = 2 \Rightarrow 6 = 8 - 8 + 2 + c$

$\Rightarrow c = 4$

$\Rightarrow y = t^3 - 2t^2 + 1t - -4$ |$M1A1$

(c) due north $\Rightarrow 6t - 3t^2 = 0$

$\Rightarrow t = 0, 2$ |$M1A1$

$t = 0 \Rightarrow y = 4\,m$ |$A1$

$t = 2 \Rightarrow y = (2)^3 - 2(2)^2 + 1(2) + 4 = 6\,m$ |$A1$

(d) $v_x = 0 \Rightarrow t = 1$ |$M1B1$

$t = 1 \Rightarrow v_y = 0$ |$B1$

$\Rightarrow r = 3i + 4j$

$\Rightarrow$ distance $= \sqrt{3^2 + 4^2} = 5\,m$ |$M1A1$

A37 - ID: 857 [8 marks, 10 minutes]

(a) $BC = 7$

$M(AB) : (7 \times 3.5) + (5 \times 5.5) + (4 \times 2) = 20 \times x$ |$M1A2$

$\Rightarrow 60 = 20x \Rightarrow x = 3$ |$M1A1$

(b) $M(midpoint) : m \times (3.5 - 3) = km \times 3.5$ |$M1A1$

$\Rightarrow k = \frac{1}{7}$ |$A1$

A38 - ID: 601 [10 marks, 12 minutes]

(a)(i) $M(AB) : m.\frac{6a}{2} + m.\frac{6a}{2} + m.6a + 6m.6a + 4m.6a = 14m.x$ |$M2A1$

$\Rightarrow 72a = 14x \Rightarrow x = \frac{72}{14}a$ |$A1$

(a)(ii) $M(AD) : m.\frac{2a}{2} + m.\frac{2a}{2} + m.2a + 6m.2a = 14m.y$ |$M1A1$

$\Rightarrow 16a = 14y \Rightarrow y = \frac{16}{14}a$ |$A1$

(b) $\tan\alpha = \dfrac{2a - \frac{16}{14}a}{\frac{72}{14}a}$ |$M1A1$

$= \frac{28 - 16}{72} = \frac{12}{72} \Rightarrow \alpha = \tan^{-1}\frac{12}{72} = 9.5°$ |$A1$

A39 - ID: 3048 *[10 marks, 12 minutes]*

(a) mass ratios : large: 24^2
: large: 8^2
: template: 512 | $B2$
moments at A $\Rightarrow 24^2 \times 24 = 8^2 \times 16 + 512 \times AG$ | $M1A1$
$\Rightarrow AG = 25\,cm$ | $M1A1$
(b) $M(axis) \Rightarrow (36 - AG) \times M = 12 \times \frac{1}{5}m$ | $M1A1$
$\Rightarrow 11M = 2.4m$
$\Rightarrow M = 0.218m$ | $M1A1$

A40 - ID: 3268 *[10 marks, 12 minutes]*

(a) Reason : PQ is line of symmetry | $B1$
(b) Moments about AB : $2688\rho \times 28 + 576\rho \times 68 = 3264\rho\bar{x}$ | $M1A2$
$\Rightarrow 114432 = 3264\bar{x}$
$\Rightarrow \bar{x} = 35.1$ | $A1$
(c) $\tan\theta = \dfrac{24}{35.1}$ | $M1A1$
$\Rightarrow \theta = 34.4°$ | $M1A1$

A41 - ID: 7396 *[11 marks, 13 minutes]*

(a) Triangle ABC : Area = 180; cog (x) = 8; cog(y) = 5 | $B1$
Rectangle DEFG : Area = 42; cog (x) = 6; cog(y) = 3.5 | $B1$
Lamina : Area = 138; cog (x) = x; cog(y) =y | $B1$
Moments about AB : $42 \times 6 + 138 \times x = 180 \times 8$ | $M1A1$
$\Rightarrow 252 + 138x = 1440$
$\Rightarrow x = 8.61\,cm$ | $A1$
Moments about AC : $42 \times 3.5 + 138 \times y = 180 \times 5$ | $M1A1$
$\Rightarrow 147 + 138y = 900$
$\Rightarrow y = 5.46\,cm$ | $A1$
(b) $\tan\theta = \dfrac{5.46}{8.61}$
$\Rightarrow \theta = 32.4°$ | $M1A1$

A42 - ID: 870 *[6 marks, 7 minutes]*

(a) energy $= \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 12^2 = 144J$ | $B1B1$
(b) $F = \mu \times 2 \times g$ | $B1$
work-energy $\Rightarrow \mu \times 2 \times g \times 13 = 144$ | $M1A1$
$\Rightarrow \mu = 0.57$ | $A1$

A43 - ID: 3267 *[10 marks, 12 minutes]*

(a) KE $= \frac{1}{2}mv^2 = \frac{1}{2} \times 3 \times 12^2 = 216J$ |$M1A1$

(b) KE at ground = initial KE + change in PE

$= 216 + 3 \times g \times 29 = 1068.6J$ |$M1A2$

(c) $\frac{1}{2}mV^2 = 1068.6 \Rightarrow V = 26.7\,ms^{-1}$ |$M1A2$

(d) assumptions : No air resistance; box is a particle |$B2$

A44 - ID: 3524 *[5 marks, 6 minutes]*

(a) KE lost $= \frac{1}{2}mv^2 = \frac{1}{2} \times 3.8 \times 7^2 = 93.1J$ |$M1A1$

(b) work-energy $\Rightarrow 93.1 = R \times 20$ |$M1A1$

$\Rightarrow R = 4.655$ |$A1$

A45 - ID: 3261 *[9 marks, 11 minutes]*

(a) Power = Force × velocity |$M1$

$= 41 \times 56 \times 56$ |$M1A1$

$= 128576\,watts$ |$A1$

(b) $v = 26 \Rightarrow$ resistant force $= 41 \times 26 = 1066\,N$ |$M1$

Force exerted by engine $= \frac{128576}{26} = 4945.231$ |$M1A1$

$F = ma \Rightarrow 4945.231 - 1066 = 1470a$ |$M1$

$\Rightarrow a = 2.639\,ms^{-1}$ |$A1$

A46 - ID: 387 *[8 marks, 10 minutes]*

(a) $F + 800g\sin\alpha = 800$ |$M1$

$\Rightarrow F = 529.655$ |$A1$

work $= 529.655 \times 13 = 6885.517W$ |$M1$

$= 6.886\,kW$ |$A1$

(b) $N2L \Rightarrow 800g\sin\alpha - 800 = 800a$ |$M1$

$\Rightarrow a = -0.592$ |$A1$

$v = u + at \Rightarrow 0 = 13 - 0.592T$ |$M1$

$\Rightarrow T = 22\,s$ |$A1$

A47 - ID: 779 *[6 marks, 7 minutes]*

(a) $\frac{P}{28} = 400 \Rightarrow P = 11.2\,kW$ |$M1A1$

(b) resolve ↗ $\Rightarrow \frac{22000}{22} - 900 \times 9.8 \times \sin\alpha - 400 = 900a$ |$M1A2$

$\Rightarrow 1000 - 420 - 400 = 900a$

$\Rightarrow 180 = 900a$

$\Rightarrow a = 0.2ms^{-2}$ |$A1$

A48 - ID: 553 *[7 marks, 8 minutes]*

(a) Driving force $= \frac{P}{v} \Rightarrow \frac{23000}{v} = 400$ | $B1M1$

$\Rightarrow v = 57.5ms^{-1}$ | $A1$

(b) $\frac{P}{v} = 400 + 900 \times g \times \frac{1}{13}$ | $M1A1$

$\Rightarrow \frac{23000}{v} = 1078.462$

$\Rightarrow v = 21.3ms^{-1}$ | $M1A1$

A49 - ID: 761 *[6 marks, 7 minutes]*

(a) work done $= \frac{1}{2}1.2(22^2 - 17^2)$ | $M1$

$= 117J$ | $A1$

(b) $F = \mu R = \mu 1.2g$ | $M1$

work-energy : $\mu 1.2g \times 17 = 117$ | $M1A1$

$\Rightarrow \mu = 0.59$ | $A1$

A50 - ID: 3049 *[12 marks, 14 minutes]*

(a) NLR $\Rightarrow e = \frac{\text{speed of separation}}{\text{speed of approach}}$

$\Rightarrow 0.5 = \frac{5v - (-v)}{u}$ | $M1A1$

$\Rightarrow u = 12v$ | $A1$

(b) CM $\Rightarrow m \times u = m \times -v + km \times 5v$

$\Rightarrow 12mv = -mv + 5kmv$ | $M1A1$

$\Rightarrow 12 = -1 + 5k \Rightarrow k = 2.6$ | $A1$

(c) CM $\Rightarrow 2.6m \times 5v = 2.6m \times -w + 13m \times w$

$\Rightarrow w = 1.25v$ | $M1A1$

$e = \frac{\text{speed of separation}}{\text{speed of approach}} = \frac{2w}{5v} = \frac{2.5v}{5v} = 0.5$ | $M1A1$

(d) $1.25v > v \Rightarrow$ speed of Q greater than speed of P | $M1A1$

A51 - ID: 7391 *[6 marks, 7 minutes]*

(a) Impulse $= 0.9 \times 3 - 0.9 \times 7 = -3.6\,Ns$ | $M1B1A1$

direction = opposite to direction of motion | $A1$

(b) speed $= 3 \times 0.6 = 1.8\,ms^{-1}$ | $M1A1$

A52 - ID: 7393 *[8 marks, 10 minutes]*

$\text{CLM} \Rightarrow 10 \times 5 + 5 \times -2 = 10v_a + 5v_b$ |*M1A1*

$\Rightarrow 40 = 10v_a + 5v_b$

$v_b - v_a = 0.3(5+2) = 2.1$ |*M1A1*

$\Rightarrow 40 = 10v_a + 5(2.1 + v_a)$

$\Rightarrow 29.5 = 15v_a$

$\Rightarrow v_a = 1.967\, ms^{-1}$ |*M1A1*

$\Rightarrow v_b = 4.067\, ms^{-1}$ |*A1*

$\Rightarrow$ A in same direction as before

B has reversed direction |*A1*

A53 - ID: 544 *[8 marks, 10 minutes]*

(a) $I = \pm 0.9(13i + 10j - (-20i))$ |*M1*

$= \pm(29.700000000000003i + 9j)$ |*M1*

$\Rightarrow |I| = \sqrt{29.700000000000003^2 + 9^2} = \sqrt{963.09} = 31Ns$ |*M1A1*

(b) $v = \frac{dr}{dt} = 13i + (10 - 4t)j$ |*M1*

$t = 4 \Rightarrow v = 13i + (10 - 16)j = 13i + -6j$ |*M1*

$\text{speed} = \sqrt{13^2 + -6^2} = \sqrt{205} = 14.3ms^{-1}$ |*M1A1*

A54 - ID: 7409 *[12 marks, 14 minutes]*

(a) Resolve $\leftrightarrow \Rightarrow F = 195 \sin 27 = 88.53\, N$ |*M1A1*

(b) moments at A $\Rightarrow \frac{4}{2} \cos 27 \times xg = 3 \times 195$ |*M2A2*

$\Rightarrow x = \frac{1170}{4 \cos 27g} = 33.498\, kg$ |*A1*

(c) Resolve $\updownarrow \Rightarrow R + 195 \cos 27 = 33.498g$ |*M1A1*

$\Rightarrow R = 154.534\, N$ |*A1*

$\mu = \frac{F}{R} = \frac{88.53}{154.534} = 0.57$ |*M1A1*