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Steve Blades

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Full FP1 Paper

Syllabus: **EdExcel** Questions: **124** Time: **16 hours 55 minutes** Total Marks: **849**

This paper contains a set of questions followed by the corresponding mark schemes. The time you should spend on each question together with its worth in marks is also given. The content of this paper is based on material from a wide selection of national and international examination boards and organisations.

You are advised to have:
a set of geometrical equipment, pen, HB pencil, eraser. Check if you are allowed a calculator. Some examinations, but not all, allow calculators, including graphical models.

NOTES: The following browsers have been tested with this facility: Mozilla Firefox 3.x, 4.x; Microsoft Internet Explorer versions 6, 7, 8 and 9 RC (see the website for the small font problem with IE7 and IE8 was tested in IE7 compatibility mode), Apple Safari and Google Chrome. Best results are when the background printing of images and colours is enabled (not available in Chrome on Windows/Mac or Safari on Windows). There are known printing format issues with the Opera web browser and we do not recommend using this browser.

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Full FP1 Paper

Questions: **124** Time: **16 hours 55 minutes** Total Marks: **849**

Q1 - ID: 596 *[5 marks, 6 minutes]*

Given that $z = 16 + 5i$ and $\frac{z}{w} = 5 - 13i$,

(a) Find w in the form $a + bi$, where a and b are real.

(b) Find the argument of z, in radians to 2 decimal places.

Q2 - ID: 566 *[9 marks, 11 minutes]*

Given that $z = 4 + 9i$ and $w = -5 + 13i$,

(a) Find $|w|$.

The complex numbers z and w are represented by the points A and B on an argand diagram.

(b) Show that triangle OAB is an isosceles right angled triangle.

(c) Find the exact value of $arg\left(\frac{z}{w}\right)$.

Q3 - ID: 952 *[9 marks, 11 minutes]*

$$z = -6 + 4i$$

(a) Calculate $arg\, z$, giving your answer in radians to 3 decimal places.

The complex number w is given by $w = \frac{A}{3 - i}$, where A is a positive constant. Given that $|w| = \sqrt{250}$,

(b) find w in the form $a + ib$, where a and b are constants,

(c) calculate $arg\, \frac{w}{z}$.

Q4 - ID: 437 *[10 marks, 12 minutes]*

Given that $\frac{z + 6i}{z - \lambda i} = i$, where λ is a positive, real constant,

(a) show that $z = \left(\frac{\lambda}{2} + 3\right) + i\left(\frac{\lambda}{2} - 3\right)$.

Given also that $\arg z = \tan^{-1}\frac{1}{4}$, calculate

(b) the value of λ,

(c) the value of $|z|^2$.

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Q5 - ID: 436 *[11 marks, 13 minutes]*

$$z = \sqrt{8} - i$$

$z*$ is the complex conjugate of z.

(a) Show that $\frac{z}{z*} = \frac{7}{9} - \frac{2\sqrt{8}i}{9}$.

(b) find the value of $\left|\frac{z}{z*}\right|$

(c) Verify, for $z = \sqrt{8} - i$, that $\arg\frac{z}{z*} = \arg z - \arg z*$.

(d) Find a quadratic equation with roots z and $z*$ in the form $ax^2 + bx + c = 0$, where a, b and c are real constants to be found.

Q6 - ID: 463 *[4 marks, 5 minutes]*

(a) Find the roots of the equation

$$z^2 + 4z + 8 = 0$$

giving your answers in the form $a + ib$, where a and b are integers.

(b) Show these roots on an Argand diagram.

Q7 - ID: 747 *[5 marks, 6 minutes]*

The complex numbers z_1 and z_2 are given by

$$z_1 = 2 + 3i, \quad z_2 = 8 + pi$$

where p is an integer.

(a) Find $\frac{z_2}{z_1}$, in the form $a + ib$, where a and b are expressed in terms of p.

Given that $arg\left(\frac{z_2}{z_1}\right) = \frac{\pi}{4}$,

(b) find the value of p.

Q8 - ID: 3030 *[5 marks, 6 minutes]*

You are given that $\alpha = -9 + 7i$,

(a) calculate α^2.

(b) express α in modulus-argument form.

Q9 - ID: 3476 *[8 marks, 10 minutes]*

$$z = -6 + i$$

Express in the form $a + bi$.

(a) $\frac{1}{z}$, (b) z^2.

(c) Show that $|z^2 - z| = \sqrt{1850}$.

(d) Find $\arg(z^2 - z)$.

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Q10 - ID: 4237 *[7 marks, 8 minutes]*

The complex number z is defined by

$$z = \frac{a + 3i}{a - i}, \quad a \in \Re, a > 0$$

Given that the real part of z is $\frac{1}{2}$, find

(a) the value of a,

(b) the argument of z, giving your answer in radians to 2 decimal places.

Q11 - ID: 5684 *[10 marks, 12 minutes]*

Given that $z_1 = 5 + 5i$ and $z_2 = \frac{50 - 0i}{z_1}$,

(a) find z_2 in the form $a + ib$, where a and b are real.

(b) Show, on an Argand diagram, the point P representing z_1 and the point Q representing z_2.

(c) Given that O is the origin, show that angle POQ = $\frac{\pi}{2}$.

The circle passing through the points O, P and Q has centre C. Find

(d) the complex number represented by C,

(e) the exact value of the radius of the circle.

Q12 - ID: 5720 *[8 marks, 10 minutes]*

The complex numbers z_1 and z_2 are given by

$$z_1 = 8 - i \text{ and } z_2 = -3 + 8i$$

(a) Show z_1 and z_2 on a single Argand diagram.

Find, showing your working,

(b) the value of $|z_1|$,

(c) the value of $arg\, z_1$, giving your answer in radians to 2 decimal places,

(d) $\frac{z_1}{z_2}$ in the form $a + bi$, where a and b are real.

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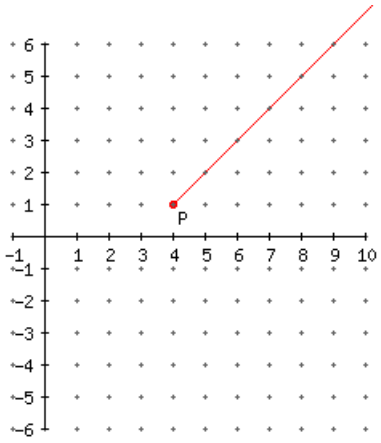
Q13 - ID: 5857 *[3 marks, 4 minutes]*



Write down the equation of the locus represented in the Argand diagram shown.

Q14 - ID: 427 *[6 marks, 7 minutes]*

(a) Given that $z = 3 - 3i$ express in the form $a + ib$, where a and b are real numbers

(a) z^2 (b) $\frac{1}{z}$.

(c) Find the exact value of each of $|z|$, $|z^2|$, and $\left|\frac{1}{z}\right|$.

Q15 - ID: 2559 *[5 marks, 6 minutes]*

Two complex numbers, α and β, are given by:

$\alpha = 8 - 3i$ and $\beta = 2 - 4i$,

(a) Express $\alpha\beta$ in the form $a + bi$.

(b) Express $\frac{\alpha + \beta}{\beta}$ in the form $a + bi$.

Q16 - ID: 4664 *[7 marks, 8 minutes]*

Given that $(a + bi)^2 = 7 + 24i$ obtain a pair of simultaneous equations involving a and b. Hence find the two square roots of $7 + 24i$.

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Q17 - ID: 5338 *[6 marks, 7 minutes]*

The complex numbers u, v and w are related by the equation

$$\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$$

Given that $v = 6 + i$ and $w = 2 - 7i$, find u in the form $x + iy$.

Q18 - ID: 5884 *[4 marks, 5 minutes]*

It is given that $z_1 = 7 + i$ and that z_1^* is the complex conjugate of z_1.
Find the real numbers x and y such that $x + 2iy = z_1 + 3iz_1^*$

Q19 - ID: 5935 *[6 marks, 7 minutes]*

It is given that $z = x + iy$ where x and y are real numbers.
(a) Find, in terms of x and y, the real and imaginary parts of $7iz + 5z^*$,
where z^* is the complex conjugate of z.
(b) Find the complex number z such that $7iz + 5z^* = 29 + 31i$.

Q20 - ID: 5965 *[5 marks, 6 minutes]*

Express $\frac{1}{(2 - i\sqrt{12})^3}$ in the form $\frac{a}{b}$ where $a, b \in Z$.

Q21 - ID: 5862 *[4 marks, 5 minutes]*

Express $\frac{7 + 5i}{6 - i}$ in the form $x + iy$, showing clearly how you obtain your answer.

Q22 - ID: 491 *[4 marks, 5 minutes]*

The complex number z satisfies the equation

$$|z| = |z + 4|$$

Show that the real part of z is -2

Q23 - ID: 588 *[7 marks, 8 minutes]*

$f(x) = 3x^3 - 26x^2 + px - 104$.
The equation $f(x) = 0$ has $(4 - 6i)$ as a root.
Solve the equation and find the value of p.

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Q24 - ID: 599 *[13 marks, 16 minutes]*

Given that $z = -5 + 4i$ and $zw = -10 + 4i$, find
(a) w in the form $p + iq$, where p and q are real,
(b) the modulus of z and the argument of z in radians,
(c) the values of the real constants m and n such that:
$mz + nzw = -10 - 19i$.

Q25 - ID: 737 *[10 marks, 12 minutes]*

(a) Given that $3 + 1i$ is a root of the equation
$z^2 + bz + c = 0$ where b and c are real constants,
(i) write down the other root of the equation.
(ii) find the value of b and the value of c.
(b) Given that $3 + i$ is a root of the equation
$z^3 + mz^2 + nz - 4 = 0$ where m and n are real constants,
find the value of m and the value of n.

Q26 - ID: 940 *[7 marks, 8 minutes]*

$f(x) = x^4 - 10x^3 + 70x^2 - 90x + 549$.
(a) Show that $3i$ is a root of the equation $f(x) = 0$.
(b) Hence solve $f(x) = 0$ completely.

Q27 - ID: 663 *[5 marks, 6 minutes]*

Given that $4 + 12i$ is a root of the equation $z^3 + 96z + 1280 = 0$,
(a) find the other two roots of the equation.
(b) prove that the points represented by these roots on an Argand diagram are the vertices of a right-angled triangle.

Q28 - ID: 373 *[7 marks, 8 minutes]*

Given that $3 - 4i$ is a solution of the equation
$x^4 - 6x^3 + 32x^2 - 42x + 175 = 0$,
solve the equation completely.

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Q29 - ID: 909 *[6 marks, 7 minutes]*

The complex numbers z and w satisfy the simultaneous equations

$7z + iw = -2$,

$z - w = 5 + 2i$.

(a) Use algebra to find z, giving your answers in the form $a + ib$, where a and b are real.

(b) Calculate $\arg z$, giving your answer in radians to 2 decimal places.

Q30 - ID: 4927 *[6 marks, 7 minutes]*

It is given that $z = x + iy$, where x and y are real numbers.

(a) Find, in terms of x and y , the real and imaginary parts of $z - 7iz^*$ where z^* is the complex conjugate of z.

(b) Find the complex number z such that $z - 7iz^* = 336$

Q31 - ID: 4235 *[5 marks, 6 minutes]*

(a) Write down the value of the real root of the equation $x^3 - 216 = 0$

(b) Find the complex roots of $x^3 - 216 = 0$, giving your answers in the form $a + ib$, where a and b are real.

Q32 - ID: 4929 *[4 marks, 5 minutes]*

It is given that $z_1 = 7 + i$ and that z_1^* is the complex conjugate of z_1.

Find the real numbers x and y such that

$x + 4iy = z_1 + 5iz_1^*$.

Q33 - ID: 5404 *[7 marks, 8 minutes]*

The cubic equation $x^3 + ax^2 + bx + c = 0$ where a, b and c are real, has roots $(2 + i)$ and 6.

(a) Write down the other root of the equation.

(b) Find the values of a, b and c.

Q34 - ID: 5405 *[11 marks, 13 minutes]*

(a) Use an algebraic method to find the square roots of the complex number $9 + 40i$

(b) Find $(5 - 4i)^2$.

(c) Hence solve the quartic equation $x^4 - 18x^2 + 1681 = 0$.

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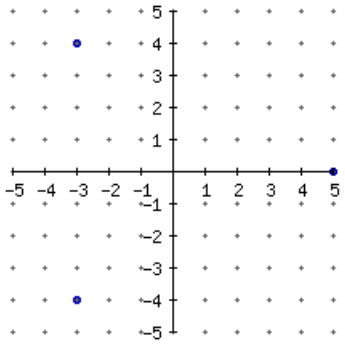
Q35 - ID: 5453 *[11 marks, 13 minutes]*

Two complex numbers α and β are given by $\alpha = 4 - 4j$ and $\beta = -1 + j$.
α and β are both roots of a quartic equation $x^4 + Ax^3 + Bx^2 + Cx + D = 0$.
(a) Write down the other two roots.
(b) Represent these four roots on an Argand diagram.
(c) Find the values of A, B, C and D.

Q36 - ID: 5687 *[6 marks, 7 minutes]*

Given that $x = -\frac{1}{3}$ is the real solution of the equation
$3x^3 - 35x^2 + 123x + 45 = 0$.
find the two complex solutions of this equation.

Q37 - ID: 5959 *[7 marks, 8 minutes]*



(a) Show that $z = 5$ is a root of the cubic equation
$z^3 + z^2 - 5z - 125 = 0$ and find the other roots.
(b) Show the roots on an Argand diagram.

Q38 - ID: 4868 *[5 marks, 6 minutes]*

(a) Find the roots of the quadratic equation $z^2 - 4z + 8 = 0$ in the form $a + bj$.
(b) Express these roots in modulus-argument form.

Q39 - ID: 5031 *[5 marks, 6 minutes]*

$f(x) = 2x^3 - 4x^2 + 7x - 39$.
Given that $x = 3$ is a solution of the equation $f(x) = 0$, solve $f(x) = 0$ completely.

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Q40 - ID: 5722 *[7 marks, 8 minutes]*

$f(x) = (x^2 + 4)(x^2 + 16x + 68)$

(a) Find the four roots of $f(x) = 0$.

(b) Find the sum of these four roots.

Q41 - ID: 5860 *[12 marks, 14 minutes]*

Two complex numbers, α and β, are given by $\alpha = 2 + j$ and $\beta = 6 - j$.

(a) Express $\alpha + \beta$, $\alpha\alpha^*$ and $\frac{\alpha + \beta}{\alpha}$ in the form $a + bj$.

(b) Find a quadratic equation with roots α and α^*.

(c) α and β are roots of a quartic equation with real coefficients. Write down the two other roots and find this quartic equation in the form $z^4 + Az^3 + Bz^2 + Cz + D = 0$.

Q42 - ID: 5869 *[10 marks, 12 minutes]*

(a) Use an algebraic method to find the square roots of the complex number $3 + i\sqrt{7}$. Give your answers in the form $x + iy$, where x and y are exact real numbers.

(b) Hence find, in the form $x + iy$ where x and y are exact real numbers, the roots of the equation $z^4 - 6z^2 + 16 = 0$.

Q43 - ID: 7026 *[5 marks, 6 minutes]*

The complex number $6 + 5i$ is a root of the quadratic equation $x^2 + bx + c = 0$, where b and c are real numbers.

(a) Write down the other root of this equation.

(b) Find the values of b and c.

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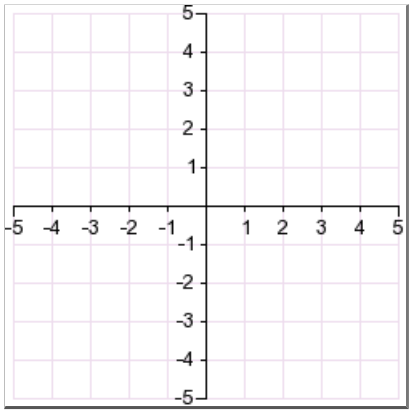
Q44 - ID: 8026 *[10 marks, 12 minutes]*



(a) Solve the equation
$(3 + i)z = (5 + in)$
Give your answer in the form $a + ib$,
expressing a and b in terms of the real constant n.
The roots of the equation $z^2 + 4z + 13 = 0$
are denoted by z_1 and z_2.
(b) Find z_1 and z_2 and show these roots on
an Argand diagram.
(c) Find the modulus and argument in radians
of each of $(z_1 - 1)$ and $(z_2 - 1)$.

Q45 - ID: 625 *[9 marks, 11 minutes]*

$f(x) = 1 - e^x + 5\sin 2x$.
The equation $f(x) = 0$ has a root α in the interval
$1 < x < 1.4$.
(a) Starting with the interval $[1, 1.4]$, use interval bisection
three times to find the value of α to 1 decimal place.
(b) Taking your answer to part (a) as a first approximation
to α, apply the Newton-Raphson procedure once to $f(x)$
to obtain a second approximation to α.
(c) By considering the change of sign of $f(x)$ over
an appropriate interval, show that your answer to
part (b) is accurate to 2 decimal places.

Q46 - ID: 723 *[5 marks, 6 minutes]*

$f(x) = 0.26x - 2 + 4\sin\sqrt{x}$.
(a) Show that the equation $f(x) = 0$ has a root α between
$x = 0.23$ and $x = 0.27$.
(b) Starting with the interval $[0.23, 0.27]$, use interval bisection
three times to find an interval of width 0.005 which contains α.

Q47 - ID: 2689 *[6 marks, 7 minutes]*

Show that the equation $x^2 + \sqrt{3 + x} = 3$ has a root in the interval $[0.8, 1.2]$.
Use interval bisection to obtain an estimate of the root with maximum possible error 0.025.

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Q48 - ID: 4926 *[7 marks, 8 minutes]*

(a) Show that the equation $x^3 + x - 8 = 0$ has a root between 1.8 and 2.
(b) Use interval bisection twice, starting with the interval in part (a), to give this root to one decimal place.

Q49 - ID: 4255 *[8 marks, 10 minutes]*

(a) Show that the equation $x^3(4 - x) = 2$ has a root in the interval $[0.7, 1.2]$.
(b) Use interval bisection to obtain an estimate of the root with maximum possible error 0.0625.
(c) Determine how many more iterations are required to reduce the maximum possible error to less than 0.005.

Q50 - ID: 2139 *[8 marks, 10 minutes]*

Show that the equation $x^5 - 5x + 1 = 0$ has a root in the interval $[0, 1]$
Apply two iterations of the method of false position to obtain an estimate of this root. Give you answer to 3 decimal places. Determine whether or not your answer is correct to 3 decimal places.

Q51 - ID: 5688 *[4 marks, 5 minutes]*

$f(x) = 2x^2 + x - \tan\frac{x}{3} - 1$
The equation $f(x) = 0$ has a root α in the interval $[0.5, 0.6]$
Use linear interpolation, on the values at the end points of this interval, to obtain an approximation to α. Give your answer to 3 decimal places.

Q52 - ID: 695 *[7 marks, 8 minutes]*

$$f(x) = x^3 + x - 3.$$

The equation f(x) = 0 has a root, α, between 1 and 2
(a) By considering $\frac{df(x)}{dx}$, show that α is the only real root of $f(x) = 0$.
(b) By taking 1.2 as your first approximation to α, apply the Newton Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 significant figures.
(c) Show that your answer to part (b) gives the value of α correct to 3 significant figures.

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Q53 - ID: 729 *[11 marks, 13 minutes]*

$f(x) = x^3 + 14x - 13$.

(a) Show that the equation $f(x) = 0$ has only one real root.
(b) Show that the real root of $f(x) = 0$ lies between 0 and 1
(c) Obtain an approximation to the real root of $f(x) = 0$ by performing two applications of the Newton-Raphson procedure to $f(x)$ using $x = 1$ as the first approximation. Give your answer to 3 decimal places.
(d) By considering the change of sign of $f(x)$ over an appropriate interval, show that your answer to part (c) is accurate to 3 decimal places.

Q54 - ID: 2697 *[6 marks, 7 minutes]*

Given that the derivative of $\tan x$ is $1 + \tan^2 x$ show that the Newton Raphson interation for the solution of the equation $\tan x - 2.25x = 0$ is

$$x_{r+1} = x_r - \frac{\tan x_r - 2x_r}{\tan^2 x_r - 1.25}$$

Use this iteration with $x_0 = -6.9$ to determine the root α correct to 4 decimal places.

Q55 - ID: 5422 *[7 marks, 8 minutes]*

It is given that $f(x) = 2 - \frac{6}{x^2}$.

(a) Use the Newton-Raphson method, with first approximation $x_1 = 1.6$ to find the next approximations x_2 and x_3 to a root of $f(x) = 0$. Give your answers to 6 decimal places.
(b) The root of $f(x) = 0$ for which x_1, x_2 and x_3 are approximations is α. Write down the value of α.
(c) The error e_n is defined by $e^n = \alpha - x_n$. Find e_1, e_2 and e_3, giving your answers to 5 decimal places.
(d) Verify that $e^3 = \frac{e_2^3}{e_1^2}$.

Q56 - ID: 5886 *[2 marks, 2 minutes]*

The equation $x^3 - x + 9 = 0$ has one real root α.
Taking $x_1 = -2$ as a first approximation to α, use the Newton-Raphson method to find a second approximation, x_2 to α,

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Q57 - ID: 5036 *[9 marks, 11 minutes]*

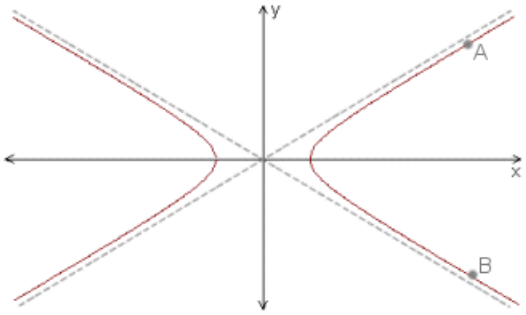
$f(x) = 5\sqrt{x} + \frac{18}{\sqrt{x}} - 21$

(a) Show that the equation $f(x) = 0$ has a root α between $x = 1.4$ and $x = 1.5$.
(b) Find $f'(x)$.
(c) Taking 1.4 as a first approximation to α, use the Newton-Raphson process on $f(x)$ once to obtain a second approximation to α.
Give your answer to 2 decimal places.

Q58 - ID: 5725 *[10 marks, 12 minutes]*

Given that α is the only real root of the equation $x^3 - x^2 - 12 = 0$,
(a) show that the equation $2.6 < \alpha < 2.7$
(b) Taking 2.6 as a first approximation to α, apply the Newton-Raphson procedure once to $x^3 - x^2 - 12 = 0$ to obtain a second approximation to α, giving your answer to 3 decimal places.
(c) Use linear interpolation once on the interval [2.6, 2.7] to find another approximation to α, giving your answer to 3 decimal places.

Q59 - ID: 4932 *[8 marks, 10 minutes]*



The diagram shows the hyperbola $\frac{x^2}{9} - y^2 = 1$ and its asymptotes.
(a) Write down the equations of the two asymptotes.
(b) The points on the hyperbola for which $x = 9$ are denoted by A and B .
Find, in surd form, the y-coordinates of A and B.
The hyperbola and its asymptotes are translated by 9 units in the positive y direction.
(c) Write down the y-coordinates of the image points of A and B under this translation
(d) write down the equations of the hyperbola and the asymptotes after the translation.

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Q60 - ID: 4934 *[9 marks, 11 minutes]*

A curve C has equation $y = \dfrac{2}{x(x-2)}$

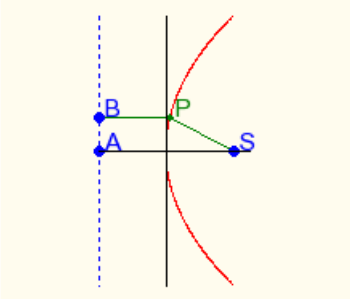
(a) Write down the equations of the the three asymptotes of C.
(b) The curve C has one stationary point. By considering an appropriate quadratic equation, find the coordinates of this stationary point.
(No credit will be given for solutions based on differentiation.)

Q61 - ID: 5934 *[7 marks, 8 minutes]*

A curve C has equation $y = 13 + \dfrac{1}{x+4}$

(a) Define the translation which transforms the curve with equation $y = \dfrac{1}{x}$ onto the curve C.
(b) Write down the equations of the two asymptotes of C.
(c) Find the coordinates of the points where the curve C intersects the coordinate axes.

Q62 - ID: 7642 *[5 marks, 6 minutes]*



The diagram shows a sketch of part of the parabola with equation $y^2 = 16x$.
The point P on the parabola has x-coordinate $\frac{1}{4}$. The point S is the focus of the parabola.
(a) Write down the coordinates of S.
The points A and B lie on the directrix of the parabola. The point A is on the x-axis and the y-coordinate of B is positive. Given that ABPS is a trapezium,
(b) calculate the perimeter of ABPS.

Q63 - ID: 586 *[13 marks, 16 minutes]*

The point $P\left(4p, \frac{4}{p}\right)$ and the point $Q\left(4q, \frac{4}{q}\right)$, where $p \neq -q$, lie on the rectangular hyperbola with equation $xy = 16$. The tangents to the curve at the points P and Q meet at the point R.
(a) Show that at the point R,

$x = \frac{8pq}{p+q}$ and $y = \frac{8}{p+q}$

As p and q vary, the locus of R has equation $xy = 7$.
(b) Find the relationship between p and q in the form $q = f(p)$.

Q64 - ID: 5032 *[4 marks, 5 minutes]*

The rectangular hyperbola, H, has parametric equations $x = 9t, y = \dfrac{9}{t}, t \neq 0$.
(a) Write the cartesian equation of H in the form $xy = c^2$.
Points A and B on the hyperbola have parameters $t = 1$ and $t = 9$ respectively.
(b) Find the coordinates of the mid-point of AB.

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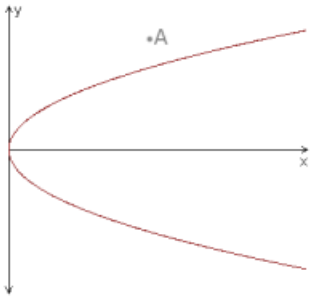
Q65 - ID: 5939 *[14 marks, 17 minutes]*



The diagram shows the parabola $y^2 = 4x$ and the point A with coordinates $(3, 4)$.
(a) Find an equation of the straight line having gradient m and passing through the point $A(3, 4)$.
(b) Show that, if this straight line intersects the parabola, then the y-coordinates of the points of intersection satisfy the equation $my^2 - 4y + (16 - 12m) = 0$
(c) By considering the discriminant of the equation in part (b), find the equations of the two tangents to the parabola which pass through A.
perpendicular to the tangent at Q.
(d) Find the coordinates of the points at which these tangents touch the parabola.

Q66 - ID: 5685 *[10 marks, 12 minutes]*

A parabola has equation $y^2 = 4ax, a > 0$. The point Q $(aq^2, 2aq)$ lies on the parabola.
(a) Show that an equation of the tangent to the parabola at Q is $qy = x + aq^2$
This tangent meets the y-axis at the point R.
(b) Find an equation of the line l which passes through R and is perpendicular to the tangent at Q.
(c) Show that l passes through the focus of the parabola.
d) Find the coordinates of the point where l meets the directrix of the parabola.

Q67 - ID: 5726 *[11 marks, 13 minutes]*

The parabola C has equation $y^2 = 18x$.
(a) Verify that the point P $(4.5t^2, 9t)$ is a general point on C
(b) Write down the coordinates of the focus S of C.
(c) Show that the normal to C at P has equation
$y + tx = 9t + 4.5t^3$
The normal to C at P meets the x-axis at the point N.
d) Find the area of the triangle PSN in terms of t, giving your answer in its simplest form.

Q68 - ID: 913 *[5 marks, 6 minutes]*

A, B and C are 2×2 matrices.
(a) Given that $AB = AC$, and that A is not singular, prove that $B = C$.
(b) Given that $AB = AC$, where $A = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 7 \\ 0 & 4 \end{pmatrix}$,
find a matrix C whose elements are all non-zero.

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Q69 - ID: 2072 *[2 marks, 2 minutes]*

Given that $M = \begin{pmatrix} -3 & 3 \\ -2 & -3 \end{pmatrix}$
find M^{-1}.

Q70 - ID: 2340 *[5 marks, 6 minutes]*

Let the matrix T be defined by $\begin{pmatrix} x & x+3 \\ x-1 & -x \end{pmatrix}$ such that $\det T = -9$.
(a) Show that the equation for x is $2x^2 - -2x - 12 = 0$.
(b) The solutions for this equation are a and b. Find a and b.

Q71 - ID: 2557 *[3 marks, 4 minutes]*

You are given the matrix $M = \begin{pmatrix} -1 & -3 \\ 2 & -4 \end{pmatrix}$
(a) Find the inverse of M.
(b) A triangle of area 6 square units undergoes the transformation represented by the matrix M. Find the area of the image of the triangle following this transformation.

Q72 - ID: 5401 *[4 marks, 5 minutes]*

The matrix A is given by $\begin{pmatrix} 1 & 4 \\ -3 & 4 \end{pmatrix}$ and I is the 2×2 identity matrix.
(a) Find $A - 7I$.
(b) Find A^{-1}

Q73 - ID: 6006 *[6 marks, 7 minutes]*

Let $M = \begin{pmatrix} -3 & 0 \\ -1 & -2 \end{pmatrix}$
(a) Write down the determinant of M.
(b) Write down M^{-1}
(b) Hence solve $M \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Q74 - ID: 5033 *[6 marks, 7 minutes]*

Given that $X = \begin{pmatrix} 9 & a \\ -1 & -8 \end{pmatrix}$ where a is a constant, and $a \neq 72$,
(a) find X^{-1} in terms of a.
Given that $X + X^{-1} = I$, where I is the 2×2 identity matrix,
(b) find the value of a.

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Q75 - ID: 5724 *[8 marks, 10 minutes]*

$A = \begin{pmatrix} a & -2 \\ -1 & 6 \end{pmatrix}$ where a is a constant.

(a) Find the value of a for which the matrix A is singular.

$B = \begin{pmatrix} 3 & -2 \\ -1 & 6 \end{pmatrix}$ where a is a constant.

(b) Find B^{-1}.

The transformation represented by B maps the point P onto the point Q.

Given that Q has coordinates $(k - 5, 5k + 15)$, where k is a constant,

(c) show that P lies on the line with equation $y = x + \frac{5}{2}$.

Q76 - ID: 5863 *[4 marks, 5 minutes]*

The matrix A is given by $A = \begin{pmatrix} 5 & 0 \\ a & 4 \end{pmatrix}$. Find

(a) A^{-1}.

(b) $5A - \begin{pmatrix} 3 & 3 \\ 0 & 5 \end{pmatrix}$.

Q77 - ID: 5865 *[4 marks, 5 minutes]*

Given that A and B are 2×2 non-singular matrices
and I is the 2×2 identity matrix, simplify

$$B(AB)^{-1}A - I.$$

Q78 - ID: 2770 *[7 marks, 8 minutes]*

$M = \begin{pmatrix} 4 & -4 \\ 3 & -2 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(a) Show that $M^2 = 2M - 4I$

(b) Hence or otherwise express the matrix M^3 in the form $\alpha M + \beta I$,
where α, β are real numbers.

Q79 - ID: 4666 *[5 marks, 6 minutes]*

If $A = \begin{pmatrix} 8 & 2 \\ k & -8 \end{pmatrix}$ and A^2 is a matrix whose entries are all 0, find k.

Q80 - ID: 4925 *[5 marks, 6 minutes]*

The matrices A and B are given by

$$A = \begin{pmatrix} 20 & 9 \\ 5 & 8 \end{pmatrix}, B = \begin{pmatrix} 5 & 3 \\ 2 & 2 \end{pmatrix}$$

The matrix $M = A - 4B$

(a) Show that $M = n$, where n is a positive integer.

(b) The matrix M represents a combination of an enlargement of scale factor
p and a reflection in a line L. State the value of p and write down the equation of L.

(c) Show that $M^2 = qI$ where q is an integer and I is the 2×2 identity matrix.

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Q81 - ID: 5888 *[3 marks, 4 minutes]*

If $M = \begin{pmatrix} \sqrt{7} & 7 \\ 7 & -\sqrt{7} \end{pmatrix}$ show that $M^2 = pI$.
where p is an integer and I is the 2×2 identity matrix.

Q82 - ID: 5938 *[7 marks, 8 minutes]*

The matrices A and B are given by $A = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}, B = \begin{pmatrix} 4 & 0 \\ 0 & -4 \end{pmatrix}$.

(a) Calculate the matrix AB.
(b) Show that A^2 is of the form kI where k is an integer and I is the 2×2 identity matri›
(c) Show that $(AB)^2 \neq A^2B^2$.

Q83 - ID: 5995 *[7 marks, 8 minutes]*

Let $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 0 \\ -4 & 3 \end{pmatrix}$. Find

(a) $A + B$.
(b) $-3A$.
(c) AB.

Q84 - ID: 5861 *[12 marks, 14 minutes]*

You are given that $A = \begin{pmatrix} 2 & 3 & -3 \\ 2 & -2 & k \\ -2 & 8 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & 12 & 0 \\ -4 & 14 & 8+k \\ -12 & 22 & 10 \end{pmatrix}$
and that AB is of the form $\begin{pmatrix} 40 & \alpha & 3k-6 \\ 24-12k & -4+22k & -16+8k \\ 0 & 0 & \beta \end{pmatrix}$.

(a) Show that $\alpha = 0$ and $\beta = 24 + 8k$
(b) Find AB when $k = 2$
(c) For the case when $k = 2$ write down the matrix A^{-1}.
(d) Use the result from part (c) to solve the following simultaneous equations.
$2x + 3y - 3z = 10$
$2x - 2y + 2z = -20$
$-2x + 8y - 4z = 44$

Q85 - ID: 3572 *[6 marks, 7 minutes]*

(a) Given $A = \begin{pmatrix} 3 & 3 \\ 3 & 4 \end{pmatrix}$, find A^{-1}
(b) Hence solve the system of simultaneous equations:
$3x + 3y = 2$
$3x + 4y = -1$

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Q86 - ID: 4963 *[8 marks, 10 minutes]*

A system of equations is given by

$(\lambda + 17)x + 12y = k$

$5x + \lambda y = 3$

(a) Find the values of λ for which the system of equations does not have a unique solution.

(b) If $\lambda = 3$ find the values of k for which there are no real solutions.

Q87 - ID: 2326 *[7 marks, 8 minutes]*

The square matrix M is such that $M^3 = 0$.

Show that the inverse of $(I - M)$ is $I + M + M^2$.

Q88 - ID: 3570 *[5 marks, 6 minutes]*

Let $M = \begin{pmatrix} 4 & -1 \\ 4 & -1 \end{pmatrix}$.

(a) Find M^2

(b) Let $N = \begin{pmatrix} -1 & 2 \\ -2 & -4 \end{pmatrix}$. Solve the matrix equation $2X + M = N$

Q89 - ID: 3571 *[6 marks, 7 minutes]*

Let $M = \begin{pmatrix} 9 & 1 \\ 1 & 3 \end{pmatrix}$ and $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

Given that $M^2 - 12M + kI = O$, find k.

Q90 - ID: 3029 *[5 marks, 6 minutes]*

You are given the matrix $A = \begin{pmatrix} -3 & -1 \\ 1 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & -3 \\ -1 & -2 \end{pmatrix}$

(a) Find BA

(b) A plane shape of area 4 square units is transformed using matrix A. The image is transformed using matrix B. What is the area of the resulting shape?

Q91 - ID: 5883 *[7 marks, 8 minutes]*

$M = \begin{pmatrix} 5 & 4 \\ 6 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(a) Show that $M^2 = 4M - -29I$

(b) Hence or otherwise express the matrix M^3 in the form $\alpha M + \beta I$ where α, β are real numbers.

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Q92 - ID: 5447 *[4 marks, 5 minutes]*

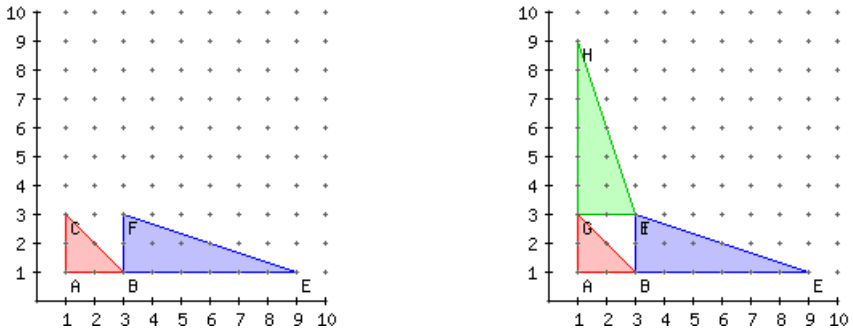
(a) Write down the matrix for reflection in the y-axis.
(b) Write down the matrix for enlargement, scale factor 5, centred on the origin.
(c) Find the matrix for reflection in the y-axis, followed by enlargement, scale factor 5, centred on the origin.

Q93 - ID: 5402 *[7 marks, 8 minutes]*

Describe fully the geometrical transformation represented by each of the following matrices:

(a) $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 0.42 & 0.91 \\ -0.91 & 0.42 \end{pmatrix}$

Q94 - ID: 5933 *[7 marks, 8 minutes]*



(a) Write down the matrix of the stretch which maps ABC to DEF
(b) The triangle DEF is reflected in the line $y = x$ to give a third triangle, GHI. Draw the triangle GHI.
(c) Find the matrix of the transformation which maps ABC to GHI.

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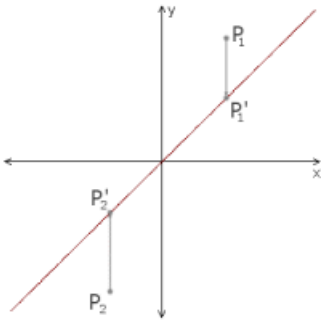
Q95 - ID: 5964 *[13 marks, 16 minutes]*



A transformation T acts on all points in the plane.
The image of a general point P is denoted by P'. P' always lies on the line $y = x$ and has the same x-coordinate as P, as shown.
(a) Write down the image of the point $(-5, 4)$ under the transformation T.
(b) Write down the image of the point (x, y) under the transformation T.
(c) Find the 2×2 matrix which represents the transformation.
(d) Describe the transformation M represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.
(e) Find the matrix representing the composite transformation of T followed by M.
(f) Find the image of the point (x, y) under this composite transformation.
(g) State the equation of the line on which all these images lie.

Q96 - ID: 5683 *[14 marks, 17 minutes]*

$A = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

(a) Describe fully the transformations described by each of the matrices A, B and C.
It is given that the matrix $D = CA$, and that the matrix $E = DB$.
(b) Find D.
(c) Show that $E =$
The triangle ORS has vertices at the points with coordinates $(0, 0)$, $(-18, 18)$ and $(9, 23)$.
This triangle is transformed onto the triangle $OR'S'$ by the transformation described by E.
(d) Find the coordinates of the vertices of triangle $OR'S'$.
e) Find the area of triangle $OR'S'$ and deduce the area of triangle ORS.

Q97 - ID: 5723 *[8 marks, 10 minutes]*

$R = \begin{pmatrix} a & 7 \\ a & b \end{pmatrix}$, where a and b are constants and $a > 0$
(a) Find R^2 in terms of a and b.
Given that R^2 represents an enlargement with centre $(0, 0)$ and scale factor 44,
(b) find the value of a and the value of b.

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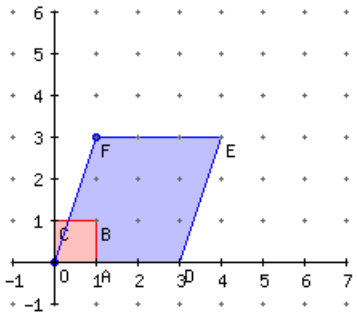
Q98 - ID: 5856 *[5 marks, 6 minutes]*



The diagram shows the unit square OABC and its image ODEF.
after undergoing a transformation.
(a) Write down the matrix P representing this transformation.
(b) The parallelogram ODEF is transformed by the matrix $Q = \begin{pmatrix} 6 & -2 \\ 0 & 2 \end{pmatrix}$.
Find the coordinates of the vertices of its image, OGHI, following this transformation.
(c) Describe fully the transformation represented by QP.

Q99 - ID: 5867 *[9 marks, 11 minutes]*

(a) The transformation P is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.
Give a geometrical description of transformation P.
(b) The transformation Q is represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.
Give a geometrical description of transformation Q.
(c) The transformation R is equivalent to transformation P followed by transformation Q. Find the matrix that represents R.
(d) Give a geometrical description of the single transformation that is represented by your answer to part (c).

Q100 - ID: 2071 *[4 marks, 5 minutes]*

A transformation $T : R^2 \longrightarrow R^2$ is represented by the matrix M, where

$$M = \begin{pmatrix} 4 & -4 \\ 4 & -6 \end{pmatrix}$$

There are two lines, passing through the origin, each of which is mapped onto itself under the transformation T.
Find cartesian equations for each of these lines.

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Q101 - ID: 2074 *[2 marks, 2 minutes]*

A transformation $T : R^2 \longrightarrow R^2$ is represented by the matrix

$A = \begin{pmatrix} k & 4 \\ 2 & -2 \end{pmatrix}$ where k is a constant.

For the case $k = -4$, find the image under T of the line with equation $y = 1x + 2$.

Q102 - ID: 5448 *[3 marks, 4 minutes]*

Find the equation of the line of invariant points under the transformation given by the matrix $\begin{pmatrix} -3 & -2 \\ 6 & 4 \end{pmatrix}$

Q103 - ID: 692 *[5 marks, 6 minutes]*

Prove that:

$$\sum_{r=1}^{n} (r^2 - 5r - 5) = \frac{1}{6}n(2n^2 - 12n - 44)$$

Q104 - ID: 785 *[4 marks, 5 minutes]*

Prove that:

$$\sum_{r=1}^{n} 24(r^2 - 4) = n(8n^2 + 12n - 92)$$

Q105 - ID: 957 *[4 marks, 5 minutes]*

Prove that $\sum_{r=1}^{n} (r - 2)(r + 3) = \frac{1}{6}n[2n^2 + 6n - 32]$.

Q106 - ID: 4931 *[7 marks, 8 minutes]*

(a) Find $\sum_{1}^{n} (r^3 - 10r)$ expressing your answer in the form

$kn(n + 1)(n + p)(n + q)$

where k is a fraction and p and q are integers.

It is given that $S = \sum_{1}^{1000} (r^3 - 10r)$

Without calculating the value of S, show that S is a multiple of 2010.

Q107 - ID: 5955 *[5 marks, 6 minutes]*

Using the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n} r^2(r + 1) = \frac{1}{12}n(n + 1)(n + 2)(3n + 1).$$

Q108 - ID: 5960 *[6 marks, 7 minutes]*

Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=1}^{n} [(r + 6)(r - 7)] = \frac{1}{3}n[n^2 - 127].$$

Q109 - ID: 5035 *[7 marks, 8 minutes]*

(a) Show, using the formulae for $\sum r$ and $\sum r^2$, that

$$\sum_{r=1}^{n} (18r^2 + 8r - 5) = n(n + 2)(6n + 1)$$

(b) Hence, or otherwise, find the value of $\sum_{r=13}^{22} (18r^2 + 8r - 5)$

Q110 - ID: 5721 *[9 marks, 11 minutes]*

(a) Using the formulae for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n} r(r + 1)(r + 3) = \frac{1}{12}n(n + 1)(n + 2)(3n + k),$$

where k is a constant to be found.

(b) Hence evaluate $\sum_{r=17}^{35} r(r + 1)(r + 3)$

Q111 - ID: 5859 *[6 marks, 7 minutes]*

Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n} r(r^2 - 10) = \frac{1}{4}n(n + 1)(n + 5)(n - 4).$$

Q112 - ID: 5864 *[6 marks, 7 minutes]*

Find $\sum_{r=1}^{n} (20r^3 + 30r^2 + 10r)$, expressing your answer in fully factorised form.

Q113 - ID: 591 *[5 marks, 6 minutes]*

(a) Express $\frac{5x + 8}{x + 3}$ in the form $p + \frac{q}{x + 3}$,

where p and q are integers to be found.

The sequence of real numbers $u_1, u_2, u_3, \ldots$ is such that

$$u_1 = 4.2 \text{ and } u_{n+1} = \frac{5u_n + 8}{u_n + 3}.$$

(b) Prove by induction that $u_n > 4$, for positive integers n.

Q114 - ID: 767 *[5 marks, 6 minutes]*

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Prove by induction that for all positive integers, n

$$A^n = \begin{pmatrix} 1 & n & \frac{1}{2}n(n + 3) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

Q115 - ID: 5102 *[7 marks, 8 minutes]*

Prove by induction that for all integers $n \geq 1$

$$\sum_{r=1}^{n} (r^2 + 1)(r!) = n(n + 1)!$$

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Q116 - ID: 5956 *[8 marks, 10 minutes]*

Prove by induction that for all integers $n \geq 1$

$$\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(3n+1)$$

Q117 - ID: 5961 *[8 marks, 10 minutes]*

A sequence is defined by $a_1 = 11$ and $a_{k+1} = 11a_k - 5$.
(a) Calculate the value of the third term a_3.
(b) Prove by induction that $a_n = \frac{(21 \times 11^{n-1}) + 1}{2}$.

Q118 - ID: 5978 *[9 marks, 11 minutes]*

The function f is defined by $f(x) = xe^{8x}$.
Prove by induction that $f^{(n)}(x) = (8^n x + n8^{n-1})e^{8x}$ for all $n \in Z^+$,
where $f^{(n)}(x)$ represents the nth derivative of $f(x)$.

Q119 - ID: 5990 *[10 marks, 12 minutes]*

(a) Find the sum of the infinite geometric sequence 54, -18 , 6, -2, ...
(b) Use mathematical induction to prove that for $n \in Z^+$,

$$a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

Q120 - ID: 5034 *[5 marks, 6 minutes]*

A series of positive integers $u_1, u_2, u_3, \ldots$ is defined by
$u_1 = 9$ and $u_{n+1} = 9u_n - 8$, for $n \geq 1$
Prove by induction that $u_n = 8 \times 9^{n-1} + 1$, for $n \geq 1$.

Q121 - ID: 5686 *[5 marks, 6 minutes]*

Prove by induction that for all integers $n \geq 1$

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$$

Q122 - ID: 5727 *[14 marks, 17 minutes]*

Prove by induction that, for $n \in Z^+$
(a) $f(n) = 5^n + 5n + 6$ is divisible by 4.
(b) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$.

Q123 - ID: 5858 *[7 marks, 8 minutes]*

Prove by induction that $14 + 28 + 56 + \ldots + 7 \times 2^n = 14(2^n - 1)$

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Q124 - ID: 5870 *[7 marks, 8 minutes]*

It is given that $u_n = 13^n + 6^{n-1}$

(a) Show that $u_n + u_{n+1} = 14 \times 13^n + 7 \times 6^{n-1}$

(b) Prove by induction that u_n is a multiple of 7.

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Full FP1 Paper - Mark Scheme

A1 - ID: 596 *[5 marks, 6 minutes]*

(a) $w = \frac{z}{5-13i} = \frac{16+5i}{5-13i}$

$= \frac{16+5i}{5-13i} \times \frac{5+13i}{5+13i}$ |M1

$= \frac{15+233i}{194}$

$= \frac{15}{194} + \frac{233}{194}i$ |A2

(b) $arg(z) = \arctan\frac{5}{16}$ |M1

$= 0.3$ |A1

A2 - ID: 566 *[9 marks, 11 minutes]*

(a) $|w| = \sqrt{25+169} = \sqrt{194}$ |B1

(b) $|OA| = \sqrt{16+81} = \sqrt{97}$ |B1

$\vec{BA} = \begin{pmatrix} 9 \\ -4 \end{pmatrix}$

$|BA| = \sqrt{81+16} = \sqrt{97}$ |M1

$\Rightarrow$ Triangle is isosceles. |A1

$OB^2 = 194$

$OA^2 + AB^2 = 97 + 97 = 194 \Rightarrow$ Right angled |M1A1

(c) $arg\left(\frac{z}{w}\right) = arg(z) - arg(w)$ |M1

$= -\angle AOB = -\frac{\pi}{4}$ |M1A1

A3 - ID: 952 *[9 marks, 11 minutes]*

(a) $arg\, z = \tan^{-1}\frac{4}{-6} = 2.554$ |M1A1

(b) $|w| = \sqrt{250} \Rightarrow \sqrt{250} = \frac{A}{\sqrt{10}} \Rightarrow A = 50$ |M1A1

$\Rightarrow w = \frac{50}{3-i} \times \frac{3+i}{3+i} = 15 + 5$ |M1A1

(c) $arg\frac{w}{z} = arg\, w - arg\, z = \tan^{-1}\frac{5}{15} - 2.554$ |M1A1

$= -2.232$ |A1

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A4 - ID: 437 *[10 marks, 12 minutes]*

(a) $\dfrac{z+6i}{z-\lambda i} = i \Rightarrow z+6i = i(z-\lambda i)$

$\Rightarrow (1-i)z = \lambda - 6i$ |M1A1

$\Rightarrow z = \dfrac{\lambda-6i}{1-i} = \dfrac{\lambda-6i}{1-i}\cdot\dfrac{1+i}{1+i} = \dfrac{\lambda+6-(6-\lambda)i}{2}$

$\Rightarrow z = \left(\dfrac{\lambda}{2}+3\right) + i\left(\dfrac{\lambda}{2}-3\right)$ |M1A2

(b) $\arg z = \tan^{-1}\dfrac{1}{4} \Rightarrow \dfrac{\frac{\lambda}{2}-3}{\frac{\lambda}{2}+3} = \dfrac{1}{4} \Rightarrow \dfrac{\lambda-6}{\lambda+6} = \dfrac{1}{4}$ |M2

$\Rightarrow 4(\lambda-6) = \lambda+6 \Rightarrow \lambda = 10$ |A1

(c) $z = 8+2i \Rightarrow |z|^2 = 8^2+2^2 = 68$ |M1A1

A5 - ID: 436 *[11 marks, 13 minutes]*

(a) $\dfrac{z}{z*} = \dfrac{\sqrt{8}-i}{\sqrt{8}+i}\cdot\dfrac{\sqrt{8}-i}{\sqrt{8}-i}$ |M1

$= \dfrac{7-2\sqrt{8}i}{9} = \dfrac{7}{9} - \dfrac{2\sqrt{8}i}{9}$ |M1A1

(b) $\left|\dfrac{z}{z*}\right| = \sqrt{\left(\dfrac{7}{9}\right)^2+\left(\dfrac{2\sqrt{8}}{9}\right)^2} = \sqrt{\dfrac{81}{81}} = 1$ |M1A1

(c) $\arg\left(\dfrac{z}{z*}\right) = \tan^{-1}\dfrac{-2\sqrt{8}}{7} = -0.6797$ |M1A1

$\arg z - \arg z* = \tan^{-1}\dfrac{-1}{\sqrt{8}} - \tan^{-1}\dfrac{1}{\sqrt{8}}$

$= -0.3398 - 0.3398 = -0.6797$ |M1A1

(d) roots z and $z*$ $\Rightarrow (x-(\sqrt{8}-i))(x-(\sqrt{8}+i)) = 0$

$\Rightarrow x^2 - 2\sqrt{8}x + 9 = 0$ |M1A1

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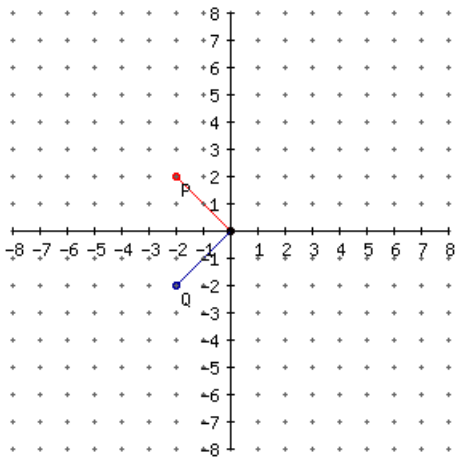
A6 - ID: 463 *[4 marks, 5 minutes]*



(a) $z = \dfrac{-4 \pm \sqrt{16 - 32}}{2} = \dfrac{-4 \pm \sqrt{-16}}{2}$ |*M*1*A*1

$= \dfrac{-4 \pm 4i}{2} = -2 \pm 2i$ |*A*1

(b) See below: plot P at $(-2, 2)$, Q at $(-2, -$ |*A*1

A7 - ID: 747 *[5 marks, 6 minutes]*

(a) $\dfrac{z_2}{z_1} = \dfrac{8 + pi}{2 + 3i} \cdot \dfrac{2 - 3i}{2 - 3i}$ |*M*1

$= \dfrac{(8 + pi)(2 - 3i)}{13}$

$= \dfrac{16 + 3p + (2p - 24)i}{13}$

$= \dfrac{16 + 3p}{13} + \dfrac{2p - 24}{13}i$ |*M*1*A*1

(b) $arg\left(\dfrac{z_2}{z_1}\right) = \dfrac{\pi}{4} \Rightarrow 16 + 3p = 2p - 24$ |*M*1

$\Rightarrow p = -40$ |*A*1

A8 - ID: 3030 *[5 marks, 6 minutes]*

(a) $\alpha^2 = (-9 + 7i)(-9 + 7i)$

$= 81 + -126i - 49 = 32 - 126i$ |*M*1*A*1

(b) $|\alpha| = \sqrt{81 + 49} = \sqrt{130}$ |*B*1

$\arg\alpha = \tan^{-1}\frac{7}{-9} = \pi - 0.661 = 2.481$ |*B*1

$\Rightarrow \alpha = \sqrt{130}(\cos 2.481 + i \sin 2.481)$ |*A*1

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A9 - ID: 3476 *[8 marks, 10 minutes]*

(a) $\frac{1}{z} = \frac{1}{-6+i} = \frac{-6-i}{(-6+i)(-6-i)}$

$= \frac{-6-i}{37} = \frac{-6}{37} - \frac{1}{37}i$ |M1A1

(b) $z^2 = (-6+i)(-6+i) = 35 + -12i$ |M1A1

(c) $|z^2 - z| = |35 + -12i - (-6+i)| = |41 + -13i|$

$= \sqrt{41^2 + -13^2} = \sqrt{1850}$ |M1A1

(d) $\arg(z^2 - z) = \tan^{-1}\frac{-13}{41} = -17.6°$ |M1A1

A10 - ID: 4237 *[7 marks, 8 minutes]*

(a) $\frac{a+3i}{a-i} = \frac{a+3i}{a-i} \times \frac{a+i}{a+i} = \frac{a^2+4ia-3}{a^2+1}$ |M1A1

real part $= \frac{1}{2} \Rightarrow \frac{a^2-3}{a^2+1} = \frac{1}{2}$

$\Rightarrow 2a^2 - 6 = a^2 + 1$

$\Rightarrow 1a^2 = 7$

$\Rightarrow a = \sqrt{7}$ |M1A1

(b) $\tan\theta = \frac{4a}{a^2-3}$ |B1

$\Rightarrow \theta = \tan^{-1}\frac{4a}{a^2-3} = \tan^{-1}\frac{4\times\sqrt{7}}{7-3}$

$\Rightarrow \theta = 0.92$ |M1A1

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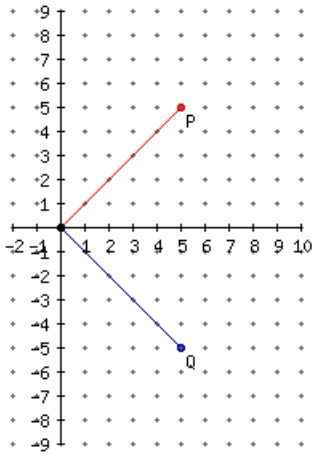
A11 - ID: 5684 *[10 marks, 12 minutes]*



(a) $z_2 = \dfrac{50 - 0i}{5 + 5i} = \dfrac{(50 - 0i)(5 - 5i)}{(5 + 5i)(5 - 5i)}$

$= \dfrac{250 - 250i}{50} = 5 - 5i$ |M1A1

(b) argand diagram : see below |M1A1

(c) gradient OP = 1, gradient OQ = −1

$\Rightarrow OP \perp OQ$ |M1A1

(d) PQ = diameter $\Rightarrow C(\dfrac{5 + 5}{2}, \dfrac{5 - 5}{2}) = C(5, 0)$

$\Rightarrow z = 5 - 0i$ |M1A1

(e) radius $= \sqrt{5^2 + 0^2} = \dfrac{\sqrt{100}}{2}$ |M1A1

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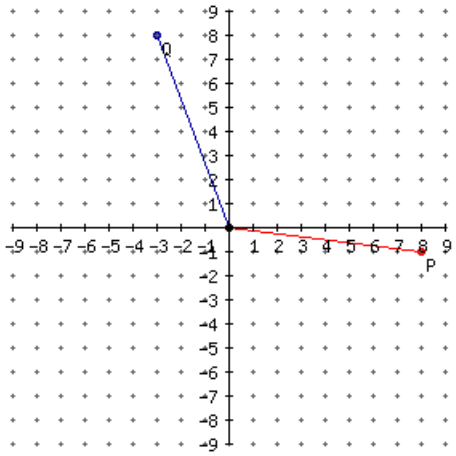
A12 - ID: 5720 *[8 marks, 10 minutes]*



(a) see below : See below: plot P at $(8, -1)$, Q at $(-3, 8)$ |B1

(b) $|z_1| = \sqrt{8^2 + 1} = \sqrt{65}$ |M1A1

(c) $arg\, z_1 = \tan^{-1} \frac{-1}{8} = -0.12$ |M1A1

(d) $\frac{z_1}{z_2} = \frac{8 - i}{-3 + 8i} = \frac{(8 - i)(-3 - 8i)}{(-3 + 8i)(-3 - 8i)}$ |M1

$= \frac{-32 - 61i}{73} = -\frac{32}{73} - \frac{61}{73}i$ |M1A1

A13 - ID: 5857 *[3 marks, 4 minutes]*

coordinates of P : $(4, 1)$ |B1

locus : $\arg(z - (4 - -1i)) = \frac{\pi}{4}$ |B2

A14 - ID: 427 *[6 marks, 7 minutes]*

(a) $z^2 = (3 - 3i)(3 - 3i) = 0 - 18i$ |M1A1

(b) $\frac{1}{z} = \frac{1}{3 - 3i} = \frac{3 + 3i}{(3 - 3i)(3 + 3i)} = \frac{3 + 3i}{18} = \frac{3}{18} + \frac{3}{18}i$ |M1A1

(c) $|z| = \sqrt{18} = 3\sqrt{2}$

$|z^2| = 18$

$\left|\frac{1}{z}\right| = \sqrt{\left(\frac{3}{18}\right)^2 + \left(\frac{3}{18}\right)^2}$

$= \frac{\sqrt{18}}{18} = \frac{3\sqrt{2}}{18} = \frac{\sqrt{2}}{6}$ |M1A1

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A15 - ID: 2559 *[5 marks, 6 minutes]*

(a) $\alpha\beta = (8 - 3i)(2 - 4i) = 4 + -38i$ |M1A1

(b) $\frac{\alpha + \beta}{\beta} = \frac{(\alpha + \beta)\beta*}{\beta\beta*}$ |M1

$= \frac{\alpha\beta* + \beta\beta*}{20}$ |A1

$= \frac{28 + 26i + 20}{20}$

$= 2.4 + 1.3i$ |A1

A16 - ID: 4664 *[7 marks, 8 minutes]*

$(a + bi)^2 = 7 + 24i \Rightarrow a^2 - b^2 + 2abi = 7 + 24i$

$\Rightarrow a^2 - b^2 = 7, 2ab = 24$ |M1A1

$\Rightarrow b = \frac{12}{a}$

$\Rightarrow a^2 - \frac{144}{a^2} = 7$ |M1

$\Rightarrow a^4 - 7a^2 - 144 = 0$ |A1

$\Rightarrow (a^2 + 9)(a^2 - 16) = 0$ |M1

$\Rightarrow a = \pm 4$

$\Rightarrow b = \pm 3$

$\Rightarrow \sqrt{7 + 24i} = \pm(4 + 3i)$ |A2

A17 - ID: 5338 *[6 marks, 7 minutes]*

$\frac{1}{u} = \frac{1}{v} + \frac{1}{w} = \frac{w + v}{vw}$

$\Rightarrow u = \frac{vw}{w + v}$ |M1

$\Rightarrow u = \frac{(6 + i)(2 - 7i)}{2 - 7i + 6 + i} = \frac{19 - 40i}{8 - 6i}$ |M1A1

$\Rightarrow u = \frac{19 - 40i}{8 - 6i} \times \frac{8 + 6i}{8 + 6i}$

$\Rightarrow u = \frac{(19 - 40i)(8 + 6i)}{100}$ |M1A1

$\Rightarrow u = \frac{392 - 206i}{100} = \frac{392}{100} - \frac{206i}{100}$ |A1

A18 - ID: 5884 *[4 marks, 5 minutes]*

$z_1 + 3iz_1^* = 7 + i + 3i(7 - i)$ |M1

$= 7 + i + 21i + 3$

$= 10 + 22i$ |M1

$\Rightarrow x = 10, y = 11$ |M1A1

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A19 - ID: 5935 *[6 marks, 7 minutes]*

(a) $7iz + 5z^* = 7i(x + iy) + 5(x - iy)$ |M1
$= 7ix - 7y + 5x - 5iy$
$= (5x - 7y) + (7x - 5y)i$ |M1A1

(b) $7iz + 5z^* = 29 + 31i \Rightarrow (5x - 7y) + (7x - 5y)i = 29 + 31i$
$\Rightarrow 5x - 7y = 29, 7x - 5y = 31$ |M2
$\Rightarrow x = 3, y = -2$
$\Rightarrow z = 3 - 2i$ |A1

A20 - ID: 5965 *[5 marks, 6 minutes]*

$(2 - i\sqrt{12})(2 - i\sqrt{12}) = -8 - i4\sqrt{12}$ |M1A1
$(2 - i\sqrt{12})^3 = (-8 - i4\sqrt{12})(2 - i\sqrt{12})$
$= -56 - i8\sqrt{12}i - -8\sqrt{12}i$
$= -56$ |M1A1
$\Rightarrow \frac{1}{(2 - i\sqrt{12})^3} = -\frac{1}{56}$ |A1

A21 - ID: 5862 *[4 marks, 5 minutes]*

$\frac{7 + 5i}{6 - i} = \frac{(7 + 5i)(6 + i)}{(6 - i)(6 + i)}$ |M1
$= \frac{37 + 37i}{37}$ |A2
$= 1 + 1i$ |A1

A22 - ID: 491 *[4 marks, 5 minutes]*

$z = x + iy \Rightarrow |z| = \sqrt{x^2 + y^2}$
$\Rightarrow |z + 4| = \sqrt{(x + 4)^2 + y^2}$
$\Rightarrow \sqrt{x^2 + y^2} = \sqrt{(x + 4)^2 + y^2}$ |M1A1
$\Rightarrow x^2 + y^2 = (x + 4)^2 + y^2$
$\Rightarrow x^2 + y^2 = x^2 + 8x + 16 + y^2$ |A1
$\Rightarrow 8x + 16 = 0$
$\Rightarrow x = -2$ |M1

A23 - ID: 588 *[7 marks, 8 minutes]*

$(4 - 6i)$ root $\Rightarrow (4 + 6i)$ root |B1
$\Rightarrow (x - 4 + 6i)(x - 4 - 6i)$ factors of f(x) |M1
$\Rightarrow (x^2 - 8x + 52)$ factor of f(x) |A1
$\Rightarrow f(x) = (x^2 - 8x + 52)(3x - 2)$ |M1A1
$\Rightarrow f(x) = 3x^3 - 26x^2 + 172x - 104$
$\Rightarrow$ third root is 0.667, $p = 172$ |M1A1

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A24 - ID: 599 *[13 marks, 16 minutes]*

(a) $w = \frac{-10+4i}{-5+4i} = \frac{(-10+4i)(-5-4i)}{(-5+4i)(-5-4i)}$ |M1

$= \frac{(50+16)+(-20--40)i}{(-5)^2+(4)^2}$ |A2

$= \frac{66+20i}{41} = 1.61+0.49i$ |A1

(b) $|z| = \sqrt{25+16} = 6.4$ |M1A1

$arg(z) = \pi - \tan^{-1}\frac{4}{5} = 2.47$ |M1A1

(c) $(R) \Rightarrow -5m + -10n = -10$ |M1A1

$(I) \Rightarrow 4m + 4n = -19$ |A1

$\Rightarrow m = -11.5, n = 6.75$ |M1A1

A25 - ID: 737 *[10 marks, 12 minutes]*

(a) (i) 2nd root $= 3-1i$ |B1

(ii) $z^2+bz+c=0 \Rightarrow (3+1i)^2+b(3+1i)+c=0$ |M1

$\Rightarrow 9+6i-1+3b+1bi+c=0$

imaginary parts $\Rightarrow 6+1b=0 \Rightarrow b=-6$ |B1

real parts $\Rightarrow 8+3b+c=0 \Rightarrow c=10$ |M1A1

(b) $3+1i$ root $\Rightarrow (3+1i)^3+m(3+1i)^2+n(3+1i)-4=0$

$\Rightarrow (18+26i)+m(8+6i)+n(3+1i)-4=0$ |M1B1

real parts $\Rightarrow 18+8m+3n-4=0$

$\Rightarrow 8m+3n=-14$

imaginary parts $\Rightarrow 26+6m+1n=0$

$\Rightarrow 6m+1n=-26$ |M1

$\Rightarrow 8m+3(-26-6m)=-14$

$\Rightarrow m=-6.4 \Rightarrow n=12.4$ |A2

A26 - ID: 940 *[7 marks, 8 minutes]*

(a) $f(3i) = (3i)^4-10(3i)^3+70(3i)^2-90(3i)+549$

$= 81+270i-630-270i+549=0$ |B1

(b) 2nd root $= -3i$ |B1

$\Rightarrow (x+3i)(x-3i)=x^2+9$ |A1

$\Rightarrow f(x)=(x^2+9)(x^2-10x+61)$ |M1A1

$\Rightarrow x = \frac{10\pm\sqrt{-144}}{2} = 5\pm 6i$ |M1A1

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A27 - ID: 663 *[5 marks, 6 minutes]*

(a) $4 + 12i$ is a root $\Rightarrow$ $4 - 12i$ is a root | B1

$z^3 + 96z + 1280 = (z - 4 - 12i)(z - 4 + 12i)(z - \alpha)$

$= (z^2 - 8z + 160)(z - \alpha)$

$\Rightarrow \alpha = -8$ | M1A1

(b) points are : $A(4, 12), B(4, -12), C(-8, 0)$

Gradient $CA = 1$

Gradient $CB = -1$

$\Rightarrow$ CA and CB are perpendicular

$\Rightarrow$ angle $ACB = 90°$ | M1A1

A28 - ID: 373 *[7 marks, 8 minutes]*

$3 + 4i$ is a soln $\Rightarrow (x - 3 - 4i)(x - 3 + 4i) = x^2 - 6x + 25$ | B1M1

$\Rightarrow f(x) = (x^2 - 6x + 25)(x^2 + ax + b)$

(*Constant*) $\Rightarrow 175 = 25b \Rightarrow b = 7$ | B1

(x^3) $\Rightarrow -6 = -6 + a$ | M1

$\Rightarrow a = 0$ | A1

$x^2 + 7 = 0 \Rightarrow x = \pm\sqrt{7}i$ | M1A1

A29 - ID: 909 *[6 marks, 7 minutes]*

(a) $z - w = 5 + 2i \Rightarrow zi - wi = 5i - 2$

adding $\Rightarrow 7z + zi = -4 + 5i$ | M1

$\Rightarrow z = \dfrac{-4 + 5i}{7 + i} = \dfrac{-4 + 5i}{7 + i} \times \dfrac{7 - i}{7 - i}$ | M1A1

$\Rightarrow z = \dfrac{-23 + 39i}{50} = -\dfrac{23}{50} + \dfrac{39}{50}i$ | A1

(b) $\arg z = \tan^{-1}\dfrac{39}{-23} = 2.1$ | M1A1

A30 - ID: 4927 *[6 marks, 7 minutes]*

(a) $z - 7iz^* = x + iy - 7i(x - iy)$ | B1

$= x - 7y + (y - 7x)i$ | M1

$\Rightarrow$ real part $= x - 7y$, imaginary part $= y - 7x$ | A1

(b) $z - 7iz^* = 336 \Rightarrow x - 7y + (y - 7x)i = 336$

$\Rightarrow x - 7y = 336$

$\Rightarrow y - 7x = 0$ | M1

$\Rightarrow x - 49x = 336 \Rightarrow x = -7$ | M1

$\Rightarrow y = -49$

$\Rightarrow z = -7 - 49i$ | A1

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A31 - ID: 4235 *[5 marks, 6 minutes]*

(a) Real root : $x = 6$ |B1

(b) $x^3 - 216 = (x - 6)(x^2 + 6x + 36)$ |M1A1

$\Rightarrow x = \dfrac{-6 \pm \sqrt{6^2 - 4 \times 36}}{2}$

$\Rightarrow x = \dfrac{-6 \pm \sqrt{-108}}{2}$

$\Rightarrow x = -3 \pm \dfrac{\sqrt{108}}{2} i$ |M1A1

A32 - ID: 4929 *[4 marks, 5 minutes]*

$x + 4iy = z_1 + 5iz_1 \Rightarrow x + 4iy = 7 + i + 5i(7 - i)$ |M1

$\Rightarrow x + 4iy = 12 + 36i$ |M1

$\Rightarrow x = 12$

$\Rightarrow y = 9$ |M1A1

A33 - ID: 5404 *[7 marks, 8 minutes]*

(a) root $= (2 - i)$ |B1

(b) $\alpha + \beta + \gamma = -a \Rightarrow 2 + i + 2 - i + 6 = -a$

$\Rightarrow a = -10$ |M1A1

$\alpha\beta\gamma = -c \Rightarrow (2 + i)(2 - i)6 = -c$

$\Rightarrow c = -30$ |M1A1

$\alpha\beta + \alpha\gamma + \beta\gamma = b \Rightarrow (2 + i)(2 - i) + 6(2 + i) + 6(2i) = b$

$\Rightarrow b = 5 + (12 + 6i) + (12 - 6i) = 29$ |M1A1

A34 - ID: 5405 *[11 marks, 13 minutes]*

(a) $(a + ib)^2 = 9 + 40i \Rightarrow a^2 - b^2 = 9, ab = 20$ |M1A1

$\Rightarrow a^2 - \dfrac{400}{a^2} = 9$

$\Rightarrow a^4 - 9a^2 - 400 = 0$ |M1

$\Rightarrow (a^2 - 25)(a^2 + 16) = 0$

$\Rightarrow a^2 = 25 \Rightarrow a = \pm 5 \Rightarrow b = \pm 4$ |M1

$\Rightarrow$ square root $= \pm(5 + 4i)$ |A1

(b) $(5 - 4i)^2 = 9 - 40i$ |B2

(c) $x^4 - 18x^2 + 1681 = 0 \Rightarrow x^2 = \dfrac{18 \pm \sqrt{324 - 6724}}{2} = 9 \pm 40i$ |M1A1

$\Rightarrow x = \pm(5 + 4i), \pm(5 - 4i)$ |A2

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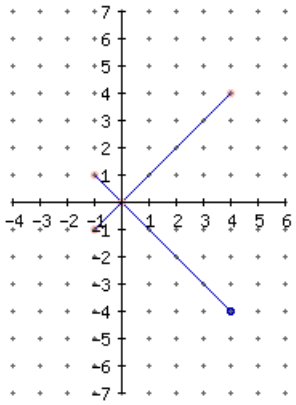
A35 - ID: 5453 *[11 marks, 13 minutes]*



(a) roots $= 4 + 4j, -1 - j$ |B2

(b) diagram : see below |B2

(c) equation : $(x - 4 + 4j)(x - 4 - 4j)(x + 1 - j)(x + 1 + j) = 0$ |M1B2

: $(x^2 - 8x + 32)(x^2 + 2x + 2) = 0$ |A1

: $x^4 - 6x^3 + 18x^2 + 48x + 64$ |A1

$\Rightarrow A = -6$

$\Rightarrow B = 18$

$\Rightarrow C = 48$

$\Rightarrow D = 64$ |A2

A36 - ID: 5687 *[6 marks, 7 minutes]*

$3x^3 - 35x^2 + 123x + 45 = (3x + 1)(x^2 - 12x + 45) = 0$ |M1A1

$x = \dfrac{12 \pm \sqrt{144 - 180}}{2}$ |M1A1

$= \dfrac{12 \pm \sqrt{-36}}{2}$

$= \dfrac{12 \pm 6i}{2}$

$= 6 \pm 3i$ |M1A1

A37 - ID: 5959 *[7 marks, 8 minutes]*

(a) $z = 5 \Rightarrow (5)^3 + (5)^2 - 5(5) - 125 = 0$ |B1

$z^3 + z^2 - 5z - 125 = (z - 5)(z^2 + 6z + 25)$ |M1A1

$\Rightarrow z = \dfrac{-6 \pm \sqrt{64}i}{2}$

$\Rightarrow z = -3 \pm 4i$ |M1A1

(b) argand diagram: see below |B2

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A38 - ID: 4868 *[5 marks, 6 minutes]*

(a) $z = \dfrac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 8}}{2}$ |M1

$= 2 \pm \dfrac{\sqrt{-16}}{2} = 2 \pm 2j$ |A1

(b) modulus $= \sqrt{2^2 + 2^2} = \sqrt{8}$ |A1

argument $= \tan^{-1}\dfrac{2}{2} = 1.6$ |A1

$\Rightarrow z = \sqrt{8}(\cos 1.6 \pm \sin 1.6)$ |M1

A39 - ID: 5031 *[5 marks, 6 minutes]*

$f(x) = (x - 3)(2x^2 + 2x + 13)$ |M1A1

$\Rightarrow x = 3$ |A1

$\Rightarrow x = \dfrac{-2 \pm \sqrt{-100}}{4} = \dfrac{-2 \pm \sqrt{100}i}{4}$ |M1A1

A40 - ID: 5722 *[7 marks, 8 minutes]*

(a) $x^2 + 4 = 0 \Rightarrow x = \pm 2i$ |M1A1

$x^2 + 16x + 68 = 0 \Rightarrow x = \dfrac{-16 \pm \sqrt{-16}}{2}$ |M1A1

$\Rightarrow x = -8 \pm 2i$ |A1

(b) sum $= (2i) + (-2i) + (-8 + 2i) + (-8 - 2i)$

$= -16$ |M1A1

A41 - ID: 5860 *[12 marks, 14 minutes]*

(a) $\alpha + \beta = 8$ |B1

$\alpha\alpha^* = (2 + j)(2 - j) = 5$ |M1A1

$\dfrac{\alpha + \beta}{\alpha} = \dfrac{8}{2 + j} = \dfrac{8(2 - j)}{5} = \dfrac{16}{5} - \dfrac{8}{5}j$ |M1A1

(b) equation : $(z - (2 + j))(z - (2 - j)) = 0$

: $z^2 - 4z + 5 = 0$ |M1A1

(c) other roots : $2 - j, 6 + j$ |B1

$(z - (6 - j))(z - (6 + j)) = z^2 - 12z + 37$ |M1

equation : $(z^2 - 4z + 5)(z^2 - 12z + 37) = 0$ |M1

: $z^4 - 16z^3 + 90z^2 - 208z + 185$ |A2

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A42 - ID: 5869 *[10 marks, 12 minutes]*

(a) $(x + iy)^2 = 3 + i\sqrt{7} \Rightarrow x^2 - y^2 = 3, 2xy = \sqrt{7}$ |M1A1

$\Rightarrow x^2 - \frac{7}{4x^2} = 3$

$\Rightarrow 4x^4 - 12x^2 - 7 = 0$ |M1

$\Rightarrow (2x^2 - 7)(2x^2 + 1) = 0$

$\Rightarrow x = \pm\frac{\sqrt{7}}{\sqrt{2}} = \pm\frac{\sqrt{14}}{2}$

$\Rightarrow y = \pm\frac{\sqrt{7}}{\sqrt{14}} = \pm\frac{\sqrt{2}}{2}$ |M1A1

$\Rightarrow$ square roots $= \pm(\frac{\sqrt{14}}{2} + \frac{\sqrt{2}}{2}i)$ |A1

(b) $z^4 - 6z^2 + 16 = 0 \Rightarrow z^2 = \frac{6 \pm \sqrt{-28}}{2} = 3 \pm \sqrt{7}i$ |M1A1

$\Rightarrow z = \pm(\frac{\sqrt{14}}{2} \pm \frac{\sqrt{2}}{2}i)$ |M1A1

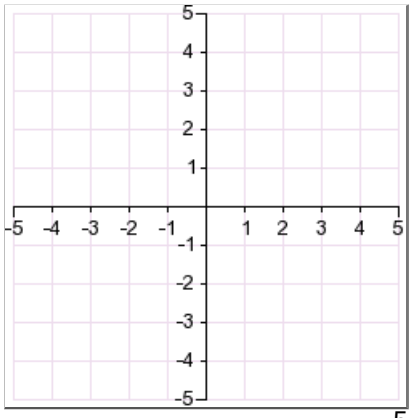
A43 - ID: 7026 *[5 marks, 6 minutes]*

(a) other root $= 6 - 5i$ |B1

(b) equation : $(x - (6 + 5i))(x - (6 - 5i)) = 0$ |M1A1

: $x^2 - 12x + 61 = 0$ |M1A1

A44 - ID: 8026 *[10 marks, 12 minutes]*



(a) $(3 + i)z = (5 + in) \Rightarrow z = \frac{5 + in}{3 + i} = \frac{(5 + in)(3 - i)}{(3 + i)(3 - i)}$ |M1

$\Rightarrow z = \frac{15 + 3in - 5i + n}{10} = \frac{15 + n}{10} + i\frac{3n - 5}{10}$ |B1M1A1

(b) $z^2 + 4z + 13 = 0 \Rightarrow z = \frac{-4 \pm \sqrt{-36}}{2} = -2 \pm 3i$

$\Rightarrow z_1 = -2 - 3i, z_2 = -2 + 3i$ |M1A1

Argand diagram see diagram |B1

(c) $|z_1 - 1| = |-3 - 3i| = \sqrt{(3)^2 + (-3)^2} = \sqrt{18}$

$\Rightarrow |z_2 - 1| = \sqrt{18}$ |M1

$\arg(z_1 - 1) = \tan^{-1}\frac{-3}{-3} = -\frac{3\pi}{4}$

$\arg(z_2 - 1) = \tan^{-1}\frac{3}{-3} = \frac{3\pi}{4}$ |M1A1

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A45 - ID: 625 *[9 marks, 11 minutes]*

(a) $f(1) = 2.828,\quad f(1.2) = 1.057,\quad f(1.4) = -1.38$
$\Rightarrow$ next interval = $[1.2, 1.4]$ |M1
$f(1.2) = 1.057,\quad f(1.3) = -0.092,\quad f(1.4) = -1.38$
$\Rightarrow$ next interval = $[1.2, 1.3]$
$f(1.2) = 1.057,\quad f(1.25) = 0.502,\quad f(1.3) = -0.092$
$\Rightarrow$ required interval = $[1.25, 1.3]$
$\Rightarrow \alpha = 1.3$ [to 1 dp] |M1A1

(b) $f'(x) = -e^x + 10\cos 2x$ |M1A1
$\Rightarrow x_2 = 1.3 - \dfrac{f(1.3)}{f'(1.3)} = 1.3 - \dfrac{-0.0918}{-12.2382} = 1.29$ |M1A1

(c) $f(1.285) = 0.09,\quad f(1.295) = -0.031$
change of sign $\Rightarrow$ root = 1.29 [to 2 dp] |M1A1

A46 - ID: 723 *[5 marks, 6 minutes]*

(a) $f(0.23) = -0.095,\quad f(0.27) = 0.056$
$\Rightarrow$ change of sign, root in $[0.23, 0.27]$ |M1A1

(b) $f(0.23) = -0.095,\quad f(0.25) = -0.017,\quad f(0.27) = 0.056$
$\Rightarrow$ next interval = $[0.25, 0.27]$ |M1
$f(0.25) = -0.017,\quad f(0.26) = 0.02,\quad f(0.27) = 0.056$
$\Rightarrow$ next interval = $[0.25, 0.26]$
$f(0.25) = -0.017,\quad f(0.255) = 0.001,\quad f(0.26) = 0.02$
$\Rightarrow$ required interval = $[0.25, 0.255]$ |M1A1

A47 - ID: 2689 *[6 marks, 7 minutes]*

(a) $f(0.8) = -0.411,\quad f(1.2) = 0.489$
$\Rightarrow$ change of sign, root in $[0.8, 1.2]$ |M1A1

(b) $f(0.8) = -0.411,\quad f(1) = 0,\quad f(1.2) = 0.489$
$\Rightarrow$ next interval = $[1, 1.2]$ |M1
$f(1) = 0,\quad f(1.1) = 0.235,\quad f(1.2) = 0.489$
$\Rightarrow$ next interval = $[1.1, 1.2]$
$f(1.1) = 0.235,\quad f(1.15) = 0.36,\quad f(1.2) = 0.489$
$\Rightarrow$ required interval = $[1.15, 1.2]$ |A2
$\Rightarrow$ estimate = 1.175 |A1

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A48 - ID: 4926 *[7 marks, 8 minutes]*

(a) $f(x) = x^3 + x - 8$
$f(1.8) = -0.368,\ f(2) = 2$ |B2
$\Rightarrow$ change of sign, root in $[1.8, 2]$ |M1
(b) $f(1.8) = -0.368,\ f(1.9) = 0.759,\ f(2) = 2$
$\Rightarrow$ next interval = $[1.8, 1.9]$ |M1A1
$f(1.8) = -0.368,\ f(1.85) = 0.182,\ f(1.9) = 0.759$
$\Rightarrow$ interval = $[1.8, 1.85]$
$\Rightarrow$ estimate = 1.8 |M1A1

A49 - ID: 4255 *[8 marks, 10 minutes]*

(a) $f(x) = x^3(4 - x) - 2$
$f(0.7) = -0.868,\ f(1.2) = 2.838$
$\Rightarrow$ change of sign, root in $[0.7, 1.2]$ |M1A1
(b) $f(0.7) = -0.868,\ f(0.95) = 0.615,\ f(1.2) = 2.838$
$\Rightarrow$ next interval = $[0.7, 0.95]$ |M1A1
$f(0.7) = -0.868,\ f(0.825) = -0.217,\ f(0.95) = 0.615$
$\Rightarrow$ required interval = $[0.825, 0.95]$ |A1
$\Rightarrow$ estimate = 0.8875 |A1
(c) mpe = $0.03125, 0.15625, 0.0078125, 0.00390625$
$\Rightarrow$ 4 iterations |M1A1

A50 - ID: 2139 *[8 marks, 10 minutes]*

$x_1 = 0,\ y_1 = 1,\ x_2 = 1,\ y_2 = -3$
$\Rightarrow$ change of sign so root in $[0, 1]$ |M1A1
$\Rightarrow x = x_1 - \dfrac{y_1(x_2 - x_1)}{(y_2 - y_1)}$
$= 0 - \dfrac{1(1 - 0)}{(-3 - 1)} = 0 - \dfrac{1}{-4} = 0.25$ |M1A1
$f(0.25) = -0.24902 \Rightarrow$ root in $[0, 0.25]$
$x_1 = 0,\ y_1 = 1,\ x_2 = 0.25,\ y_2 = -0.24902$
$\Rightarrow x = x_1 - \dfrac{y_1(x_2 - x_1)}{(y_2 - y_1)}$
$= 0 - \dfrac{1(0.25 - 0)}{(-0.24902 - 1)} = 0 - \dfrac{0.25}{-1.249} = 0.2$ |M1A1
$f(0.2005) = -0.00218\quad f(0.1995) = 0.00282$
$\Rightarrow$ change of sign so root correct to 3dp |M1A1

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A51 - ID: 5688 *[4 marks, 5 minutes]*

$x_1 = 0.5, \quad y_1 = -0.168227218$

$x_2 = 0.6, \quad y_2 = 0.117289964$ |M1

$\Rightarrow x = x_1 - \dfrac{y_1(x_2 - x_1)}{(y_2 - y_1)}$

$= 0.5 - \dfrac{-0.168227218(0.6 - 0.5)}{(0.117289964 - -0.168227218)}$ |A1

$= 0.5 - \dfrac{-0.017}{0.286} = 0.558920173$

$= 0.559$ (3 dp) |M1A1

A52 - ID: 695 *[7 marks, 8 minutes]*

(a) $\frac{df(x)}{dx} = 3x^2 + 1 \Rightarrow \frac{df(x)}{dx} \neq 0$ |M1A1

$\Rightarrow$ no turning points

$\Rightarrow f(x)$ crosses x-axis once only |A1

(b) $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = x_1 - \frac{x_1^3 + x_1 - 3}{3x_1^2 + 1}$

$= 1.2 - \frac{1.2^3 + 1.2 - 3}{3(1.2)^2 + 1}$

$= 1.21$ |M1A1

(c) $f(1.215) = 0.01$

$f(1.205) = -0.05$

$\Rightarrow$ Sign change so root = 1.21 |M1A1

A53 - ID: 729 *[11 marks, 13 minutes]*

(a) $f'(x) = 3x^2 + 14 \Rightarrow f'(x) \neq 0$ |M1A1

$\Rightarrow$ no turning points , so 1 root only |A1

(b) $f(0) = -13, \quad f(1) = 2$

$\Rightarrow$ Sign change so $0 < \alpha < 1$ |M1A1

(c) $x_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{2}{17}$

$= 0.882$ |M1A1

$x_2 = 0.882 - \frac{f(0.882)}{f'(0.882)} = 0.882 - \frac{0.0341}{16.3338}$

$= 0.88$ |M1A1

(d) $f(0.8795) = -0.00669, \quad f(0.8805) = 0.00963$ |M1

$\Rightarrow$ Sign change so $\alpha = 0.88$ to 3 dp |A1

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A54 - ID: 2697 *[6 marks, 7 minutes]*

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)} = x_r - \frac{\tan x_r - 2.25x_r}{1 + \tan^2 x_r - 2.25}$$

$$= x_r - \frac{\tan x_r - 2.25x_r}{\tan^2 x_r - 1.25} \quad |M1A1$$

$$x_1 = -6.9 - \frac{f(-6.9)}{f'(-6.9)} = 12.92957 \quad |M1A1$$

$$x_2 = 12.92957 - \frac{f(12.92957)}{f'(12.92957)} = -13.040634$$

$$x_3 = -13.040634 - \frac{f(-13.040634)}{f'(-13.040634)} = 16.182501$$

$$x_4 = 16.182501 - \frac{f(16.182501)}{f'(16.182501)} = -20.219532 \quad |A1$$

$$\Rightarrow \alpha = -20.2195 \text{ to 4 dp} \quad |A1$$

A55 - ID: 5422 *[7 marks, 8 minutes]*

(a) $f'(x) = \frac{12}{x^3}$ $|B1$

$\Rightarrow x_2 = 1.6 - \frac{f(1.6)}{f'(1.6)} = 1.717333$ $|A1$

$\Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.731864$ $|A1$

(b) $\alpha = \sqrt{3}$ $|B1$

(c) $e_1 = 0.13205,\ e_2 = 0.01472,\ e_3 = 0.00019$ $|B2$

$\frac{e_2^3}{e_1^2} = 0.0001828 \sim 0.00019$ $|B1$

A56 - ID: 5886 *[2 marks, 2 minutes]*

$$f'(x) = 3x^2 - 1$$
$$f'(-2) = 11$$
$$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -2 - \frac{3}{11} = -2.273 \quad |M1A1$$

A57 - ID: 5036 *[9 marks, 11 minutes]*

(a) $f(1.4) = 0.1289$

$f(1.5) = -0.1793$

$\Rightarrow$ change of sign so root between $x = 1.4$ and $x = 1.5$ $|M1A1$

(b) $f'(x) = 2.5x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}}$ $|M1A2$

(c) $f'(1.4) = -3.3202$ $|M1A1$

$\Rightarrow \alpha = 1.4 - \frac{f(1.4)}{f'(1.4)} = 1.4 - \frac{0.1289}{-3.3202}$

$\Rightarrow \alpha = 1.44$ $|M1A1$

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A58 - ID: 5725 *[10 marks, 12 minutes]*

(a) $f(2.6) = -1.184$
$f(2.7) = 0.393$
$\Rightarrow$ change of sign so root between $x = 2.6$ and $x = 2.7$ |M1A1

(b) $f'(x) = 3x^2 - 2x$ |A1
$f'(2.6) = 15.08$ |M1A1
$\Rightarrow \alpha = 2.6 - \dfrac{f(2.6)}{f'(2.6)} = 2.6 - \dfrac{-1.184}{15.08} = 2.679$ |M1A1

(c) $x = x_1 - \dfrac{y_1(x_2 - x_1)}{(y_2 - y_1)}$
$= 2.6 - \dfrac{-1.184(2.7 - 2.6)}{(0.393 - -1.184)}$ |A1
$= 2.6 - \dfrac{-0.118}{1.577} = 2.675$ (3 dp) |M1A1

A59 - ID: 4932 *[8 marks, 10 minutes]*

(a) asymptotes : $y = \pm\frac{1}{3}x$ |M1A1

(b) $x = 9 \Rightarrow y^2 = 9 - 1$
$\Rightarrow y = \pm\sqrt{8}$ |M1A1

(c) $A : y = 9 + \sqrt{8}, B : y = 9 - \sqrt{8}$ |B1

(d) equation : $\dfrac{x^2}{9} - (y - 9)^2 = 1$ |M1A1

asymptotes : $y = 9 \pm \dfrac{1}{3}x$ |B1

A60 - ID: 4934 *[9 marks, 11 minutes]*

(a) asymptotes : $y = 0, x = 0, x = 2$ |B3

(b) Let $\dfrac{2}{x(x-2)} = c \Rightarrow 2 = cx(x - 2)$
$\Rightarrow cx^2 - 2cx - 2 = 0$ |M1A1
$b^2 - 4ac = 0 \Rightarrow 4c^2 + 8c = 0$
$\Rightarrow c = 0, -\frac{2}{1}$
$\Rightarrow y = -\frac{2}{1}$ |M1A1
$\Rightarrow x = 1$ |M1A1

A61 - ID: 5934 *[7 marks, 8 minutes]*

(a) translation : $\begin{pmatrix} -4 \\ 13 \end{pmatrix}$ |B2

(b) asymptotes : $x = -4, y = 13$ |B2

(c) $x = 0 \Rightarrow y = 13 + \dfrac{1}{4} = \dfrac{53}{4}$ |B1
$y = 0 \Rightarrow 13 + \dfrac{1}{x + 4} = 0$
$\Rightarrow \dfrac{1}{x + 4} = -13$
$\Rightarrow x + 4 = -\dfrac{1}{13} \Rightarrow x = -\dfrac{53}{13}$ |M1A1

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A62 - ID: 7642 *[5 marks, 6 minutes]*

(a) coordinates of S = $(4, 0)$ | B1

(b) $AB = 2$ | B1

$AS = 8$

$BP = 4 + \frac{1}{4}$ | B1

$PS = BP$ | M1

$\Rightarrow$ perimeter $= 2 + 8 + 2(4 + \frac{1}{4}) = \frac{37}{2}$ | M1A1

A63 - ID: 586 *[13 marks, 16 minutes]*

(a) $\frac{dy}{dx} = -\frac{16}{x^2} = -\frac{1}{p^2}$ at $x = 4p$ | M1A1

tangent at P : $y - \frac{4}{p} = -\frac{1}{p^2}(x - 4p)$ or $p^2y + x = 8$ | M1

tangent at Q : $q^2y + x = 8q$ | A1

$\Rightarrow (p^2 - q^2)y = 8(p - q)$ | M1

$\Rightarrow y = \frac{8(p - q)}{(p^2 - q^2)} = \frac{8}{p + q}$ | A1

$\Rightarrow x = 8p - p^2y = 8p - \frac{8p^2}{p + q}$

$\Rightarrow x = \frac{8p(p + q) - 8p^2}{p + q} = \frac{8pq}{p + q}$ | M1A1

(b) $xy = 7 \Rightarrow \frac{8pq}{p + q} \times \frac{8}{p + q} = 7$ | M1

$\Rightarrow 64pq = 7(p + q)^2$

$\Rightarrow 7p^2 - 50pq + 7q^2 = 0$ | A1

$\Rightarrow (7p - q)(p - 7q) = 0$ | M1

$\Rightarrow q = 7p$ or $q = \frac{p}{7}$ | A2

A64 - ID: 5032 *[4 marks, 5 minutes]*

(a) $xy = 9t \times \frac{9}{t} = 9^2$ | B1

(b) $t = 1 \Rightarrow x = 9, \quad y = 9$ | A1

$t = 9 \Rightarrow x = 81, \quad y = 1$ | A1

midpoint $= \left(\frac{9 + 81}{2}, \frac{9 + 1}{2}\right) = (45, 5)$ | A1

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A65 - ID: 5939 *[14 marks, 17 minutes]*

(a) equation $\Rightarrow y - 4 = m(x - 3)$ | M1A1

(b) at intersection $\Rightarrow y^2 = 4\dfrac{y - 4 + 3m}{m}$ | M1

$\Rightarrow my^2 = 4y - 16 + 12m$

$\Rightarrow my^2 - 4y + 16 - 12m = 0$ | M1A1

(c) $b^2 - 4ac = 0 \Rightarrow 16 - 4m(16 - 12m) = 0$ | M1

$\Rightarrow 48m^2 - 64m + 16 = 0$

$\Rightarrow 12m^2 - 16m + 4 = 0$

$\Rightarrow (12m - 4)(m - 1) = 0$ | M1A1

$\Rightarrow m = \dfrac{1}{3}, 1$

tangents $\Rightarrow y - 4 = \dfrac{1}{3}(x - 3), y - 4 = 1(x - 3)$ | M1A1

(d) $m = 1 \Rightarrow 1y^2 - 4y + 4 = 0$

$\Rightarrow y = 2,\ x = 1$ | M1A1

$m = \dfrac{1}{3} \Rightarrow \dfrac{1}{3}y^2 - 4y + 16 - 4 = 0$

$\Rightarrow y = 6,\ x = 9$ | M1A1

A66 - ID: 5685 *[10 marks, 12 minutes]*

(a) $y^2 = 4ax \Rightarrow 2y\dfrac{dy}{dx} = 4a$ | M1

at Q $\dfrac{dy}{dx} = \dfrac{1}{q}$ | A1

tangent : $y - 2aq = \dfrac{1}{q}(x - aq^2)$

: $qy - 2aq^2 = x - aq^2$

: $qy = x + aq^2$ | M1A1

(b) $x = 0 \Rightarrow y = aq$ | A1

gradient $l = -q$

equation l : $y - aq = -qx$ | M1A1

(c) $y = 0 \Rightarrow x = a$, so focus $(a, 0)$ on line | B1

(d) $x = -a \Rightarrow y = -qx + aq = 2aq$ | M1A1

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A67 - ID: 5726 *[11 marks, 13 minutes]*

(a) $y^2 = (9t)^2 = 81t^2$

$18x = 18(4.5t^2) = 81t^2$ |A1

(b) focus $= (4.5, 0)$ |B1

(c) differentiate $\Rightarrow 2y\dfrac{dy}{dx} = 18$ |M1A1

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{t}$

gradient normal $= -t$ |A1

equation normal : $y - 9t = -t(x - 4.5t^2)$

: $y + tx = 9t + 4.5t^3$ |M1A1

(d) $y = 0 \Rightarrow x = 9 + 4.5t^2$ |A1

area PSN $= \dfrac{1}{2}(9 + 4.5t^2 - 4.5) \times 9t$ |M1A1

$= 20.25t(1 + t^2)$ |A1

A68 - ID: 913 *[5 marks, 6 minutes]*

(a) A is not singular $\Rightarrow A^{-1}$ exists

$AB = AC \Rightarrow A^{-1}AB = A^{-1}AC \Rightarrow B = C$ |M1A1

(b) $AB = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 4 & 7 \\ 0 & 4 \end{pmatrix}$

$= \begin{pmatrix} 16 & 36 \\ 8 & 18 \end{pmatrix}$ |B1

$\Rightarrow \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 16 & 36 \\ 8 & 18 \end{pmatrix}$

$\Rightarrow 2a + 1c = 8, 2b + 1d = 18$ |M1

choose $a = 1 \Rightarrow c = 6$

choose $b = 4 \Rightarrow d = 10$

or any other non-zero values |A1

A69 - ID: 2072 *[2 marks, 2 minutes]*

$\det(M) = (-3 \times -3) - (3 \times -2) = 9 - -6 = 15$ |B1

$M^{-1} = \frac{1}{15}\begin{pmatrix} -3 & -3 \\ 2 & -3 \end{pmatrix}$ |B1

A70 - ID: 2340 *[5 marks, 6 minutes]*

(a) $\det(T) = -x^2 - (x + 3)(x - 1)$

$= -2x^2 + -2x + 3$ |M1A1

$\Rightarrow -2x^2 + -2x + 3 = -9$

$\Rightarrow 2x^2 - -2x - 12 = 0$ |A1

(b) $2x^2 - -2x - 12 = 0 \Rightarrow (2x + 6)(x - 2) = 0$

$\Rightarrow x = -\dfrac{6}{2}, 2$ |A2

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A71 - ID: 2557 *[3 marks, 4 minutes]*

(a) $\det(M) = (-1 \times -4) - (-3 \times 2) = 4 - -6 = 10$

$M^{-1} = \frac{1}{10}\begin{pmatrix} -4 & 3 \\ -2 & -1 \end{pmatrix}$ |M1A1

(b) area $= 6 \times 10 = 60$ |A1

A72 - ID: 5401 *[4 marks, 5 minutes]*

(a) $A - 7I = \begin{pmatrix} 1 & 4 \\ -3 & 4 \end{pmatrix} - \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = \begin{pmatrix} -6 & 4 \\ -3 & -3 \end{pmatrix}$ |B2

(b) $\det(A) = (1 \times 4) - (4 \times -3) = 4 - -12 = 16$ |B1

$A^{-1} = \frac{1}{16}\begin{pmatrix} 4 & -4 \\ 3 & 1 \end{pmatrix}$ |B1

A73 - ID: 6006 *[6 marks, 7 minutes]*

(a) determinant $= (-3 \times -2) - (0 \times -1) = 6$ |A1

(b) $M^{-1} = \frac{1}{6}\begin{pmatrix} -2 & 0 \\ 1 & -3 \end{pmatrix}$ |M1A1

(c) $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{6}\begin{pmatrix} -2 & 0 \\ 1 & -3 \end{pmatrix} \times \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ |M1

$= \frac{1}{6}\begin{pmatrix} -6 \\ -3 \end{pmatrix} = \begin{pmatrix} -1 \\ -\frac{1}{2} \end{pmatrix}$ |M1A1

A74 - ID: 5033 *[6 marks, 7 minutes]*

(a) $\det(X) = (9 \times -8) - (a \times -1) = 1a - 72$ |B1

$M^{-1} = \frac{1}{1a-72}\begin{pmatrix} -8 & -a \\ 1 & 9 \end{pmatrix}$ |M1A1

(b) $X + X^{-1} = I \Rightarrow \begin{pmatrix} 9 & a \\ -1 & -8 \end{pmatrix} + \frac{1}{1a-72}\begin{pmatrix} -8 & -a \\ 1 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ |A1

$\Rightarrow 1a - 72 = 1$

$\Rightarrow a = 73$ |M1A1

A75 - ID: 5724 *[8 marks, 10 minutes]*

(a) $\det(A) = 0 \Rightarrow 6a - 2 = 0$

$\Rightarrow a = \frac{1}{3}$ |M1A1

(b) $\det(B) = 16$ |B1

$B^{-1} = \frac{1}{16}\begin{pmatrix} 6 & 2 \\ 1 & 3 \end{pmatrix}$ |M1A1

(c) $\begin{pmatrix} x \\ y \end{pmatrix} = B^{-1}\begin{pmatrix} k - 5 \\ 5k + 15 \end{pmatrix}$ |M1

$= \frac{1}{16}\begin{pmatrix} 6(k - 5) - -2(5k + 15) \\ 1(k - 5) + 3(5k + 15) \end{pmatrix}$

$= \frac{1}{16}\begin{pmatrix} 16k \\ 16k + 40 \end{pmatrix}$

$\Rightarrow y = x + \frac{5}{2}$ |M1A1

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A76 - ID: 5863 *[4 marks, 5 minutes]*

(a) $A^{-1} = \frac{1}{20}\begin{pmatrix} 4 & 0 \\ -a & 5 \end{pmatrix}$ |B2

(b) $5A - \begin{pmatrix} 3 & 3 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 25 & 0 \\ 5a & 20 \end{pmatrix} - \begin{pmatrix} 3 & 3 \\ 0 & 5 \end{pmatrix}$

$= \begin{pmatrix} 22 & -3 \\ 5a & 15 \end{pmatrix}$ |B2

A77 - ID: 5865 *[4 marks, 5 minutes]*

$B(AB)^{-1}A - I = BB^{-1}A^{-1}A - I$ |B1M1

$= I \cdot I - I$ |A1

$= I^2 - I$

$= I - I$

$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ |A1

A78 - ID: 2770 *[7 marks, 8 minutes]*

(a) $M^2 = \begin{pmatrix} 4 & -4 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 4 & -4 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 4 & -8 \\ 6 & -8 \end{pmatrix}$ |M1A1

$2M - 4I = \begin{pmatrix} 8 & -8 \\ 6 & -4 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 4 & -8 \\ 6 & -8 \end{pmatrix}$ |M1A1

(b) $M^3 = 2M^2 - 4M = 2(2M - 4I) - 4M$ |M1A1

$= 0M - 8I$ |A1

A79 - ID: 4666 *[5 marks, 6 minutes]*

$A^2 = \begin{pmatrix} 8 & 2 \\ k & -8 \end{pmatrix}\begin{pmatrix} 8 & 2 \\ k & -8 \end{pmatrix}$ |M1

$= \begin{pmatrix} 64 + 2k & 0 \\ 0 & 2k + 64 \end{pmatrix}$ |A2

$\Rightarrow 64 + 2k = 0$

$\Rightarrow k = -32$ |M1A1

A80 - ID: 4925 *[5 marks, 6 minutes]*

(a) $M = A - 4B = \begin{pmatrix} 20 & 9 \\ 5 & 8 \end{pmatrix} - 4\begin{pmatrix} 5 & 3 \\ 2 & 2 \end{pmatrix}$

$= \begin{pmatrix} 0 & -3 \\ -3 & 0 \end{pmatrix} = 3\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ |B2

(b) $p = 3, L : y = -x$ |B2

(c) $M^2 = \begin{pmatrix} 0 & -3 \\ -3 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & -3 \\ -3 & 0 \end{pmatrix}$

$= \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix} = 9\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ |B2

A81 - ID: 5888 *[3 marks, 4 minutes]*

$$M^2 = \begin{pmatrix} \sqrt{7} & 7 \\ 7 & -\sqrt{7} \end{pmatrix} \times \begin{pmatrix} \sqrt{7} & 7 \\ 7 & -\sqrt{7} \end{pmatrix} \quad |M1$$

$$= \begin{pmatrix} 7 + 7^2 & 0 \\ 0 & 7^2 + 7 \end{pmatrix} \quad |M1$$

$$= 56I \quad |A1$$

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A82 - ID: 5938 *[7 marks, 8 minutes]*

(a) $AB = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} \times \begin{pmatrix} 4 & 0 \\ 0 & -4 \end{pmatrix} = \begin{pmatrix} 0 & -12 \\ 12 & 0 \end{pmatrix}$ |M1A1

(b) $A^2 = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix} = 9\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ |M1A1

(c) $(AB)^2 = \begin{pmatrix} 0 & -12 \\ 12 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & -12 \\ 12 & 0 \end{pmatrix} = \begin{pmatrix} -144 & 0 \\ 0 & -144 \end{pmatrix}$ |B1

$B^2 = \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix}$ |B1

$A^2B^2 = \begin{pmatrix} 144 & 0 \\ 0 & 144 \end{pmatrix} \neq (AB)^2$ |B1

A83 - ID: 5995 *[7 marks, 8 minutes]*

(a) $A + B = \begin{pmatrix} 1+4 & 2+0 \\ 3+-4 & -1+3 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ -1 & 2 \end{pmatrix}$ |M1A1

(b) $-3A = \begin{pmatrix} -3\times 1 & -3\times 2 \\ -3\times 3 & -3\times -1 \end{pmatrix} = \begin{pmatrix} -3 & -6 \\ -9 & 3 \end{pmatrix}$ |M1A1

(c) $AB = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \times \begin{pmatrix} 4 & 0 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ 16 & -3 \end{pmatrix}$ |M1A2

A84 - ID: 5861 *[12 marks, 14 minutes]*

(a) $\alpha = 2\cdot 12 + 3\cdot 14 + -3\cdot 22 = 0$ |B1

$\beta = -2\cdot 0 + 8\cdot(8+k) + -4\cdot 10 = 24 + 8k$ |M1A1

(b) $AB = \begin{pmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{pmatrix}$ |M1A1

(c) $A^{-1} = \frac{1}{40}\begin{pmatrix} 8 & 12 & 0 \\ -4 & 14 & 10 \\ -12 & 22 & 10 \end{pmatrix}$ |M1B1A1

(d) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{40}\begin{pmatrix} 8 & 12 & 0 \\ -4 & 14 & 10 \\ -12 & 22 & 10 \end{pmatrix} \times \begin{pmatrix} 10 \\ -20 \\ 44 \end{pmatrix}$ |M1

$= \frac{1}{40}\begin{pmatrix} -160 \\ 120 \\ -120 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ -3 \end{pmatrix}$ |A3

A85 - ID: 3572 *[6 marks, 7 minutes]*

(a) $A^{-1} = \frac{1}{3}\begin{pmatrix} 4 & -3 \\ -3 & 3 \end{pmatrix}$ |M1A1

(b) $\begin{pmatrix} 3 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 4 & -3 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ |M1A1

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 11 \\ -9 \end{pmatrix} = \begin{pmatrix} \frac{11}{3} \\ \frac{-9}{3} \end{pmatrix}$

$\Rightarrow x = 3.67, y = -3$ |A2

A86 - ID: 4963 *[8 marks, 10 minutes]*

(a) determinant $= 0 \Rightarrow (\lambda + 17)\lambda - 60 = 0$ |*M*1*A*1

$\Rightarrow \lambda^2 + 17\lambda - 60 = 0$

$\Rightarrow (\lambda + 20)(\lambda - 3) = 0$ |*M*1*A*1

$\Rightarrow \lambda = -20, 3$ |*M*1*A*1

(b) $\lambda = 3 \Rightarrow \begin{pmatrix} 20 & 12 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} k \\ 3 \end{pmatrix}$

no real values $\Rightarrow k \neq \frac{12}{3} \times 3$

$\Rightarrow k \neq 12$ |*M*1*A*1

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A87 - ID: 2326 *[7 marks, 8 minutes]*

$$\begin{aligned}(I-M)(I+M+M^2) &= I+M+M^2-M-M^2-M^3 && |M1A2\\ &= I-M^3 = I && |A2\\ &\Rightarrow (I-M)(I+M+M^2)=I\\ &\Rightarrow (I-M)^{-1}(I-M)(I+M+M^2)=(I-M)^{-1}I\\ &\Rightarrow (I+M+M^2)=(I-M)^{-1} && |A2\end{aligned}$$

A88 - ID: 3570 *[5 marks, 6 minutes]*

(a)
$$\begin{aligned}M^2 &= \begin{pmatrix}4&-1\\4&-1\end{pmatrix}\times\begin{pmatrix}4&-1\\4&-1\end{pmatrix} && |M1\\ &= \begin{pmatrix}12&-3\\12&-3\end{pmatrix} && |A1\end{aligned}$$

(b)
$$\begin{aligned}2X+M=N \Rightarrow 2X+\begin{pmatrix}4&-1\\4&-1\end{pmatrix} &= \begin{pmatrix}-1&2\\-2&-4\end{pmatrix} && |M1\\ \Rightarrow 2X &= \begin{pmatrix}-5&3\\-6&-3\end{pmatrix} && |A1\\ \Rightarrow X &= \tfrac{1}{2}\begin{pmatrix}-5&3\\-6&-3\end{pmatrix} && |A1\end{aligned}$$

A89 - ID: 3571 *[6 marks, 7 minutes]*

$$\begin{aligned}M^2 &= \begin{pmatrix}9&1\\1&3\end{pmatrix}\times\begin{pmatrix}9&1\\1&3\end{pmatrix} && |M1\\ &= \begin{pmatrix}82&12\\12&10\end{pmatrix} && |A1\\ 12M &= \begin{pmatrix}108&12\\12&36\end{pmatrix} && |A1\\ \Rightarrow \begin{pmatrix}82&12\\12&10\end{pmatrix}-\begin{pmatrix}108&12\\12&36\end{pmatrix}+\begin{pmatrix}k&0\\0&k\end{pmatrix} &= 0 && |A1\\ \Rightarrow \begin{pmatrix}-26&0\\0&-26\end{pmatrix}+\begin{pmatrix}k&0\\0&k\end{pmatrix} &= 0 && |A1\\ \Rightarrow k &= 26 && |A1\end{aligned}$$

A90 - ID: 3029 *[5 marks, 6 minutes]*

(a)
$$BA = \begin{pmatrix}-3&-3\\-1&-2\end{pmatrix}\begin{pmatrix}-3&-1\\1&-3\end{pmatrix} = \begin{pmatrix}6&12\\1&7\end{pmatrix} \quad |M1A1$$

(b)
$$\begin{aligned}\det BA &= 30 && |M1A1\\ \Rightarrow \text{area} &= 4\times 30 = 120 && |A1\end{aligned}$$

A91 - ID: 5883 *[7 marks, 8 minutes]*

(a)
$$\begin{aligned}M^2 &= \begin{pmatrix}5&4\\6&-1\end{pmatrix}\times\begin{pmatrix}5&4\\6&-1\end{pmatrix} = \begin{pmatrix}49&16\\24&25\end{pmatrix} && |M1A1\\ 4M--29I &= \begin{pmatrix}20&16\\24&-4\end{pmatrix}+\begin{pmatrix}29&0\\0&29\end{pmatrix} = \begin{pmatrix}49&16\\24&25\end{pmatrix} && |M1A1\end{aligned}$$

(b)
$$\begin{aligned}M^3 &= 4M^2--29M && |M1\\ &= 4(4M--29I)--29M\\ &= 45M--116I && |M1A1\end{aligned}$$

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A92 - ID: 5447 *[4 marks, 5 minutes]*

(a) matrix $= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ |*B*1

(b) matrix $= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$ |*B*1

(c) matrix $= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \times \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ |*M*1

$= \begin{pmatrix} -5 & 0 \\ 0 & 5 \end{pmatrix}$ |*A*1

A93 - ID: 5402 *[7 marks, 8 minutes]*

(a) transformation = enlargement, scale factor 4 |*B*1

(b) transformation = reflection in $y = x$ |*B*2

(c) transformation = stretch in y direction, scale factor 2 |*B*2

(d) transformation = rotation, 65.2° clockwise |*B*2

A94 - ID: 5933 *[7 marks, 8 minutes]*

(a) matrix $= \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ |*M*1*A*1

(b) triangle GHI = see below |*M*1*A*1

(c) matrix $= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ |*B*1*M*1

$= \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix}$ |*A*1

A95 - ID: 5964 *[13 marks, 16 minutes]*

(a) image $= (-5, -5)$ |*B*1

(b) image $= (x, x)$ |*B*2

(c) matrix $= \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ |*B*3

(d) transformation = Rotation 90° anticlockwise about origin |*B*2

(e) matrix $= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$ |*M*1*A*1

(f) image $= (-x, x)$ |*M*1*A*1

(g) equation: $y = -x$ |*B*1

A96 - ID: 5683 *[14 marks, 17 minutes]*

(a) A = enlargement, scale factor $3\sqrt{2}$ | M1A1

B = reflection in $y = x$ | B1

C = rotation anticlockwise, $\frac{\pi}{4}$ | B1

(b) $D = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \times \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix} = \begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}$ | M1A1

(c) $E = \begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix} \times \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$ | B1

(d) $= \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 0 & -18 & 9 \\ 0 & 18 & 23 \end{pmatrix} = \begin{pmatrix} 0 & 108 & 42 \\ 0 & 0 & 96 \end{pmatrix}$

(e) area $OR'S' = \frac{1}{2} \times 108 \times 96 = 5184$ | B1

area $OR'S' = 5184 \div 18 = 288$ | M1A1

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A97 - ID: 5723 *[8 marks, 10 minutes]*

(a) $R^2 = \begin{pmatrix} a & 7 \\ a & b \end{pmatrix} \times \begin{pmatrix} a & 7 \\ a & b \end{pmatrix}$

$= \begin{pmatrix} a^2 + 7a & 7a + 7b \\ a^2 + ab & 7a + b^2 \end{pmatrix}$ |M1A2

(b) sf 44 $\Rightarrow a^2 + 7a = 44$ |M1

$\Rightarrow a^2 + 7a - 44 = 0$

$\Rightarrow (a + 11)(a - 4) = 0$

$\Rightarrow a = 4$ |M1A1

$7a + 7b = 0 \Rightarrow b = -a = -4$ |M1A1

A98 - ID: 5856 *[5 marks, 6 minutes]*

(a) $P = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$ |B1

(b) coordinates $= \begin{pmatrix} 6 & -2 \\ 0 & 2 \end{pmatrix} \times \begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 3 & 3 \end{pmatrix}$

$= \begin{pmatrix} 0 & 18 & 18 & 0 \\ 0 & 0 & 6 & 6 \end{pmatrix}$ |M1A1

(c) transformation: stretch factor 18 in x directior |B1

: stretch factor 6 in y directior |B1

A99 - ID: 5867 *[9 marks, 11 minutes]*

(a) Transformation P= Reflection, in x-axis |B2

(b) Transformation Q= Reflection, in $y = -x$ |B2

(c) matrix $= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ |B2

(d) Transformation= Rotation, 90° clockwise, about (0,0) |B3

A100 - ID: 2071 *[4 marks, 5 minutes]*

$\begin{pmatrix} 4 & -4 \\ 4 & -6 \end{pmatrix} \times \begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} x' \\ mx' \end{pmatrix} \Rightarrow 4x + -4mx = x', 4x + -6mx = mx'$ |M1

$\Rightarrow \frac{4 + -6m}{4 + -4m} = m$ |A1

$\Rightarrow 4m^2 - 10m + 4 = 0$

$\Rightarrow (4m - 2)(m - 2) = 0$

$\Rightarrow m = 0.5, 2$ |M1

$\Rightarrow y = 0.5x, y = 2x$ |A1

A101 - ID: 2074 *[2 marks, 2 minutes]*

$\begin{pmatrix} -4 & 4 \\ 2 & -2 \end{pmatrix} \times \begin{pmatrix} x \\ 1x + 2 \end{pmatrix} = \begin{pmatrix} 0x + 8 \\ 0x + -4 \end{pmatrix}$ |M1

$\Rightarrow$ image is $(8, -4)$ |A1

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A102 - ID: 5448 *[3 marks, 4 minutes]*

$$\begin{pmatrix} -3 & -2 \\ 6 & 4 \end{pmatrix} \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow -3x + -2y = x, 6x + 4y = y \quad |M2$$

$$\Rightarrow y = -2x \quad |A1$$

A103 - ID: 692 *[5 marks, 6 minutes]*

$$\sum_{r=1}^{n} (r^2 - 5r - 5) = \sum r^2 - 5\sum r - \sum 5 \quad |M1$$

$$= \frac{1}{6}n(n+1)(2n+1) - \frac{5}{2}n(n+1) - 5n \quad |A1$$

$$= \frac{1}{6}\left(n(n+1)(2n+1) - 15n(n+1) - 30n\right) \quad |M1$$

$$= \frac{1}{6}n\left((n+1)(2n+1) - 15(n+1) - 30\right) \quad |A1$$

$$= \frac{1}{6}n(2n^2 - 12n - 44) \quad |A1$$

A104 - ID: 785 *[4 marks, 5 minutes]*

$$\sum_{r=1}^{n} 24(r^2 - 4) = \sum 24r^2 - \sum 96 \quad |M1$$

$$= 24\frac{1}{6}n(n+1)(2n+1) - 96n \quad |A1$$

$$= n(24\frac{1}{6}(2n^2 + 3n + 1) - 96)$$

$$= n(8n^2 + 12n - 92) \quad |M1A1$$

A105 - ID: 957 *[4 marks, 5 minutes]*

$$\sum_{r=1}^{n} (r-2)(r+3) = \sum_{r=1}^{n} r^2 + 1r - 6 = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} 1r - \sum_{r=1}^{n} \quad |M1$$

$$= \frac{1}{6}n(n+1)(2n+1) + 1\frac{1}{2}n(n+1) - 6n \quad |A2$$

$$= \frac{1}{6}n[(n+1)(2n+1) + 3(n+1) - 36]$$

$$= \frac{1}{6}n[2n^2 + 6n - 32] \quad |M1$$

A106 - ID: 4931 *[7 marks, 8 minutes]*

(a) $\sum_{r=1}^{n}(r^3 - 10r) = \sum r^3 - \sum 10r$

$= \frac{1}{4}n^2(n+1)^2 - \frac{10}{2}n(n+1)$ |M1

$= \frac{1}{4}n(n+1)(n(n+1) - 20)$ |M2A1

$= \frac{1}{4}n(n+1)(n+5)(n-4)$ |A1

(b) $(n+5) = 1005$

$\frac{n}{4} = 250$, which is even

$\Rightarrow S$ has factors 1005 and 250

$\Rightarrow S$ is a multiple of 2010 |M1A1

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A107 - ID: 5955 *[5 marks, 6 minutes]*

$$\sum_{r=1}^{n} r^2(r+1) = \sum_{r=1}^{n} (r^3 + 1r^2) \quad |M1$$

$$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{6}n(n+1)(2n+1) \quad |M1$$

$$= \frac{1}{12}n(n+1)\left(3n(n+1) + 2(2n+1)\right) \quad |A2$$

$$= \frac{1}{12}n(n+1)\left(3n^2 + 7n + 2\right)$$

$$= \frac{1}{12}n(n+1)(n+2)(3n+1) \quad |A1$$

A108 - ID: 5960 *[6 marks, 7 minutes]*

$$\sum_{r=1}^{n} [(r+6)(r-7)] = \sum_{r=1}^{n} [r^2 - r - 42] \quad |M1$$

$$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - 42n \quad |A2$$

$$= \frac{1}{6}n[(n+1)(2n+1) - 3(n+1) - 252] \quad |M1$$

$$= \frac{1}{6}n[2n^2 + 3n + 1 - 3n - 3 - 252]$$

$$= \frac{1}{6}n[2n^2 - 254] = \frac{1}{3}n[n^2 - 127] \quad |M1A1$$

A109 - ID: 5035 *[7 marks, 8 minutes]*

(a) $$\sum_{r=1}^{n} (18r^2 + 8r - 5) = 18\sum_{r=1}^{n} r^2 + 8\sum_{r=1}^{n} r - \sum_{r=1}^{n} 5 \quad |B1$$

$$= \frac{18}{6}n(n+1)(2n+1) + \frac{8}{2}n(n+1) - 5n \quad |M1$$

$$= n(3(2n^2 + 3n + 1) + 4(n+1) - 5)$$

$$= n(6n^2 + 13n + 2) \quad |M1$$

$$= n(n+2)(6n+1) \quad |M1A1$$

(b) $$\sum_{r=13}^{22} (18r^2 + 8r - 5) = 22(24)(133) - 12(14)(73)$$

$$= 57960 \quad |M1A1$$

A110 - ID: 5721 *[9 marks, 11 minutes]*

(a) $\sum_{r=1}^{n} r(r+1)(r+3) = \sum_{r=1}^{n} r(r^2+4r+3) = \sum_{r=1}^{n} (r^3+4r^2+3r)$ | *B*1

$= \frac{1}{4}n^2(n+1)^2 + \frac{4}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1)$ | *M*1

$= \frac{1}{12}n(n+1)(3n(n+1)+8(2n+1)+18)$ | *M*1

$= \frac{1}{12}n(n+1)(3n^2+19n+26)$ | *M*1

$= \frac{1}{12}n(n+1)(n+2)(3n+13)$ | *M*1*A*1

(b) $\sum_{r=17}^{35} r(r+1)(r+3) = \frac{1}{12}(35)(36)(37)(118) - \frac{1}{12}(16)(17)(18)(61)$

$= 433542$ | *M*1*A*1

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A111 - ID: 5859 *[6 marks, 7 minutes]*

$$\sum_{r=1}^{n} r(r^2 - 10) = \sum_{r=1}^{n} (r^3 - 10r) \quad |M1$$

$$= \frac{1}{4}n^2(n+1)^2 - \frac{10}{2}n(n+1) \quad |M1$$

$$= \frac{1}{4}n(n+1)\left(n(n+1) - 20\right) \quad |A2$$

$$= \frac{1}{4}n(n+1)\left(n^2 + n - 20\right)$$

$$= \frac{1}{4}n(n+1)(n+5)(n-4) \quad |M1A1$$

A112 - ID: 5864 *[6 marks, 7 minutes]*

$$\sum_{r=1}^{n} (20r^3 + 30r^2 + 10r) = \frac{20}{4}n^2(n+1)^2 + \frac{30}{6}n(n+1)(2n+1) + \frac{10}{2}n(n+1) \quad |M1A2$$

$$= n(n+1)\left(5n(n+1) + 5(2n+1) + 5\right) \quad |A1$$

$$= n(n+1)\left(5n^2 + 15n + 10\right) \quad |A1$$

$$= n(n+1)^2(5n+10) \quad |A1$$

A113 - ID: 591 *[5 marks, 6 minutes]*

(a) $\frac{5x+8}{x+3} = \frac{5(x+3)-7}{x+3} = 5 - \frac{7}{x+3}$ $|B1$

(b) $u_1 = 4.2 \Rightarrow u_1 > 4$ $|B1$

assume true for $n = k \Rightarrow u_k > 4$

$\Rightarrow u_{k+1} = \frac{5u_k + 8}{u_k + 3} = 5 - \frac{7}{u_k + 3}$

$u_k > 4 \Rightarrow \frac{7}{u_k + 3} < 1$

$\Rightarrow u_{k+1} > 4$ $|M1A1$

$\Rightarrow$ hence result true for $n = k + 1$ $|A1$

A114 - ID: 767 *[5 marks, 6 minutes]*

$$A^1 = \begin{pmatrix} 1 & 1 & \frac{1}{2}1(4) \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = A$$

$\Rightarrow$ hence true for $n = 1$ |B1

$$A^{k+1} = A^k.A = \begin{pmatrix} 1 & k & \frac{1}{2}k(k+3) \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$ |M1

$$= \begin{pmatrix} 1 & k+1 & 2+k+\frac{1}{2}k(k+3) \\ 0 & 1 & k+1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & k+1 & \frac{1}{2}(k^2+5k+4) \\ 0 & 1 & k+1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & k+1 & \frac{1}{2}(k+1)(k+4) \\ 0 & 1 & k+1 \\ 0 & 0 & 1 \end{pmatrix}$$ |M1A1

$\Rightarrow$ hence if true for k then true for $k + 1$

$\Rightarrow$ hence true for all n |A1

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A115 - ID: 5102 *[7 marks, 8 minutes]*

$n = 1 \Rightarrow LHS = \sum_{r=1}^{1} (r^2 + 1)(r!) = 2$

$\Rightarrow RHS = 1 \times 2! = 2$ |B1

assume true for $n = k \Rightarrow \sum_{r=1}^{k} (r^2 + 1)(r!) = k(k + 1)!$

$\Rightarrow \sum_{r=1}^{k+1} (r^2 + 1)(r!) = k(k + 1)! + ((k + 1)^2 + 1)((k + 1)!)$ |M1A1

$\Rightarrow \sum_{r=1}^{k+1} (r^2 + 1)(r!) = (k + 1)!(k + ((k + 1)^2 + 1)$ |M1

$\Rightarrow \sum_{r=1}^{k+1} (r^2 + 1)(r!) = (k + 1)!(k^2 + 3k + 2))$

$\Rightarrow \sum_{r=1}^{k+1} (r^2 + 1)(r!) = (k + 1)!(k + 1)(k + 2) = (k + 1)(k + 2)!$ |A2

$\Rightarrow$ hence result true for $n = k + 1$

$\Rightarrow$ hence result true for all n |A1

A116 - ID: 5956 *[8 marks, 10 minutes]*

$n = 1 \Rightarrow LHS = \sum_{r=1}^{1} r^2(r + 1) = 2$

$\Rightarrow RHS = \frac{1}{12}(1)(2)(3)(4) = 2$ |B1

assume true for $n = k \Rightarrow \sum_{r=1}^{k} r^2(r + 1) = \frac{1}{12}k(k + 1)(k + 2)(3k + 1)$

$\Rightarrow \sum_{r=1}^{k+1} r^2(r + 1) = \frac{1}{12}k(k + 1)(k + 2)(3k + 1) + (k + 1)^2(k + 2)$ |M1

$= \frac{1}{12}(k + 1)(k + 2)[k(3k + 1) + 12(k + 1)]$ |M1A1

$= \frac{1}{12}(k + 1)(k + 2)[3k^2 + 13k + 12]$

$= \frac{1}{12}(k + 1)(k + 2)(k + 3)(3k + 4]$ |M1A1

$= \frac{1}{12}(k + 1)((k + 1) + 1)((k + 1) + 2)(3(k + 1) + 1]$ |A1

$\Rightarrow$ hence result true for $n = k + 1$

$\Rightarrow$ hence result true for all n |A1

A117 - ID: 5961 *[8 marks, 10 minutes]*

(a) $a_2 = 11a_1 - 5 = 116$

$a_3 = 11a_2 - 5 = 1271$ | *M*1*A*1

(b) $a_1 = \frac{(21 \times 11^0) + 1}{2} = 11 \Rightarrow$ true for $n = 1$ | *B*1

assume true for $n = k \Rightarrow a_k = \frac{(21 \times 11^{k-1}) + 1}{2}$ | *M*1

$\Rightarrow a_{k+1} = 11a_k - 5$

$\Rightarrow a_{k+1} = 11(\frac{(21 \times 11^{k-1}) + 1}{2}) - 5$

$\Rightarrow a_{k+1} = \frac{(21 \times 11^k) + 11 - 10}{2}$

$\Rightarrow a_{k+1} = \frac{(21 \times 11^k) + 1}{2}$ | *M*1*A*1

$\Rightarrow$ result true for $n = k + 1$

$\Rightarrow$ result true for all n | *B*2

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A118 - ID: 5978 *[9 marks, 11 minutes]*

$n = 1 \Rightarrow f'(x) = x \cdot 8e^{8x} + e^{8x} \cdot 1 = 8xe^{8x} + e^{8x}$
$\Rightarrow f^{(1)}(x) = (8x + 8^0)e^{8x} = 8xe^{8x} + e^{8x}$
$\Rightarrow$ true for $n = 1$ |M1A1
assume true for $n = k \Rightarrow f^{(k)}(x) = (8^k x + k8^{k-1})e^{8x}$ |M1A1
$\Rightarrow f^{(k+1)}(x) = (8^k x + k8^{k-1}) \cdot 8e^{8x} + e^{8x} \cdot (8^k$ |M1A1
$\Rightarrow f^{(k+1)}(x) = (8^{k+1}x + k8^k)e^{8x} + 8^k e^{8x}$
$\Rightarrow f^{(k+1)}(x) = (8^{k+1}x + (k+1)8^k)e^{8x}$ |A2
$\Rightarrow$ result true for $n = k + 1$
$\Rightarrow$ result true for all n |B1

A119 - ID: 5990 *[10 marks, 12 minutes]*

(a) $S_\infty = \frac{a}{1-r} = \frac{54}{1+\frac{1}{3}} = 40.5$ |M1A2

(b) $n = 1 \Rightarrow LHS = a, RHS = \frac{a(1-r)}{1-r} = a$ |M1A1

assume true for $n = k \Rightarrow a + ar + \ldots + ar^{k-1} = \frac{a(1-r^k)}{1-r}$ |M1

add $ar^k \Rightarrow a + ar + \ldots + ar^k = \frac{a(1-r^k)}{1-r} + ar^k$ |M1

$\Rightarrow a + ar + \ldots + ar^k = \frac{a(1-r^k) + ar^k(1-r)}{1-r}$ |A1

$\Rightarrow a + ar + \ldots + ar^k = \frac{a(1-r^{k+1})}{1-r}$ |A1

$\Rightarrow$ so true for $n = k + 1$
$\Rightarrow$ so true for all n |A1

A120 - ID: 5034 *[5 marks, 6 minutes]*

$u_1 = 8 \times 9^{1-1} + 1 = 9 \Rightarrow u_n = 8 \times 9^{n-1} + 1$ for $n = 1$ |B1
assume true for $n = k \Rightarrow u_k = 8 \times 9^{k-1} + 1$ |M1
$\Rightarrow u_{k+1} = 9u_k - 8$
$\Rightarrow u_{k+1} = 9(8 \times 9^{k-1} + 1) - 8$
$\Rightarrow u_{k+1} = 8 \times 9^k + 9 - 8$
$\Rightarrow u_{k+1} = 8 \times 9^k + 1$ |M1A1
$\Rightarrow$ result true for $n = k + 1$
$\Rightarrow$ result true for all n |M1

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A121 - ID: 5686 *[5 marks, 6 minutes]*

$n = 1 \Rightarrow LHS = \frac{1}{1\times 2} = \frac{1}{2}$

$\Rightarrow RHS = \frac{1}{1+1} = \frac{1}{2}$ | B1

assume true for $n = k \Rightarrow \sum_{r=1}^{k} \frac{1}{r(r+1)} = \frac{k}{k+1}$

$\Rightarrow \sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$ | M1

$\Rightarrow \sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \frac{k(k+2)+1}{(k+1)(k+2)}$

$= \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)}$

$= \frac{k+1}{k+2}$ | M1A1

$\Rightarrow$ hence result true for $n = k + 1$

$\Rightarrow$ hence result true for all n | A1

A122 - ID: 5727 *[14 marks, 17 minutes]*

(a) $n = 1 \Rightarrow f(n) = 16$

$\Rightarrow$ divisible by 4 | M1A1

assume true for $n = k \Rightarrow 5^k + 5k + 6 = 4m$

$5^{k+1} + 5(k+1) + 6 = 5(5^k + 5k + 6) - 20k - 24$ | M1A1

$= 5(4m) - 4(5k + 6)$

$\Rightarrow$ divisible by 4 | M1A1

$\Rightarrow$ result true for $n = k + 1$

$\Rightarrow$ result true for all n | M1

(b) $n = 1 \Rightarrow \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = LHS$ | M1A1

assume true for $n = k \Rightarrow \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^k = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}$

$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}$ | M1A1

$= \begin{pmatrix} 3(2k+1)-4k & -6k-2(1-2k) \\ 2(2k+1)-2k & -4k-(1-2k) \end{pmatrix}$

$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$ | M1A1

$\Rightarrow$ result true for $n = k + 1$

$\Rightarrow$ result true for all n | M1

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A123 - ID: 5858 *[7 marks, 8 minutes]*

$n = 1 \Rightarrow LHS = 14, RHS = 14 \times 1 = 14$ |B1

assume true for $n = k \Rightarrow 14 + \ldots + 7 \times 2^k = 14(2^k - 1)$ |A1

add $7 \times 2^{k+1} \Rightarrow$

$14 + \ldots + 7 \times 2^{k+1} = 14(2^k - 1) + 7 \times 2^{k+1}$ |M1

$= 14(2^k - 1) + 14 \times \frac{7}{14} \times 2^{k+1}$

$= 14\left(2^k - 1 + \frac{1}{2} \times 2^{k+1}\right)$ |M1

$= 14\left(2^k + 1 \cdot 2^k - 1\right)$

$= 14\left(2 \cdot 2^k - 1\right)$

$= 14\left(2^{k+1} - 1\right)$ |A1

$\Rightarrow$ result true for $n = k + 1$ |B1

$\Rightarrow$ result true for all n |B1

A124 - ID: 5870 *[7 marks, 8 minutes]*

(a) $u_n + u_{n+1} = 13^n + 6^{n-1} + 13^{n+1} + 6^n$ |B1

$= 13^n(1 + 13) + 6^{n-1}(1 + 6)$

$= 14 \times 13^n + 7 \times 6^{n-1}$ |M1A1

(b) $n = 1 \Rightarrow u_1 = 13^1 + 6^0 = 14$

$\Rightarrow u_1$ = a multiple of 7 |B1

assume $u_k = 7n \Rightarrow u_{k+1} = 14 \times 13^k + 7 \times 6^{k-1} - 7n$

$\Rightarrow u_{k+1} = 7(2 \times 13^k + 6^{k-1} - n)$

$\Rightarrow u_{k+1}$ = a multiple of 7 |B1

$\Rightarrow$ result true for $n = k + 1$ |B1

$\Rightarrow$ result true for all n |B1