FP2 Exam Paper full

Questions: **56** Time: **9 hours 9 minutes** Total Marks: **459**

Q1 - ID: 778 *[7 marks, 8 minutes]*

Find the set of values of x for which

$$\frac{x}{x-6} > \frac{6}{x-1}$$

Q2 - ID: 368 *[5 marks, 6 minutes]*

Find the set of values for which

$$|x-1| > 8x-1.$$

Q3 - ID: 899 *[7 marks, 8 minutes]*

Using algebra, find the set of values of x for which

$$5x-8 > \frac{4}{x}$$

Q4 - ID: 592 *[3 marks, 4 minutes]*

Solve the inequality

$$|5x-4| < 6x-3.$$

Q5 - ID: 17 *[7 marks, 8 minutes]*

(a) Use algebra to find the exact solutions of the equation

$$|3x^2 + x - 4| = 4 - 3x.$$

(b) On the same diagram, sketch the curve with equation $y = |3x^2 + x - 4|$ and the line with equation $y = 4 - 3x$

(c) Find the set of values of x for which

$$|3x^2 + x - 4| > 4 - 3x.$$

Q6 - ID: 369 *[6 marks, 7 minutes]*

Find the set of values of x for which

$$\frac{x^2}{x-7} > 6x$$

Q7 - ID: 850 *[7 marks, 8 minutes]*

(a) Use algebra to find the exact solutions of the equation

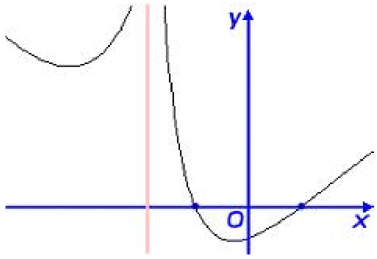
$|3x^2 + x - 4| = 4 - 3x.$

(b) On the same diagram, sketch the curve with equation $y = |3x^2 + x - 4|$ and the line with equation $y = 4 - 3x$

(c) Find the set of values of x for which

$|3x^2 + x - 4| > 4 - 3x.$

Q8 - ID: 658 *[9 marks, 11 minutes]*



The diagram shows a sketch of the curve with equation

$$y = \frac{x^2 - 4}{|x + 4|}, x \neq -4$$

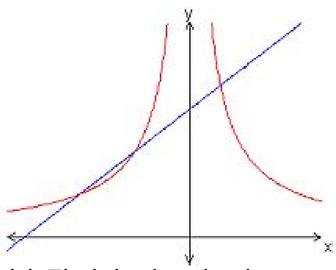
The curve crosses the x-axis at $x = 2$ and $x = -2$ and the line $x = -4$ is an asymptote of the curve.

(a) Use algebra to solve the equation $\dfrac{x^2 - 4}{|x + 4|} = 6(2 - x)$

(b) Hence, or otherwise, find the set of values of x for which

$$\frac{x^2 - 4}{|x + 4|} < 6(2 - x)$$

Q9 - ID: 4239 *[7 marks, 8 minutes]*



(a) Find, in the simplest surd form where appropriate, the exact values of x for which

$$\frac{x}{4} + 4 = \left|\frac{7}{x}\right|, x \neq 0$$

The graph shows the line with equation $\dfrac{x}{4} + 4$ and the graph of $y = \left|\dfrac{7}{x}\right|, x \neq 0$

(c) Find the set of values of x for which $\dfrac{x}{4} + 4 > \left|\dfrac{7}{x}\right|$.

Q10 - ID: 5057 *[6 marks, 7 minutes]*

(a) On the same diagram, sketch the curve with equation $y = |5x–2|$, and the line with equation $y = 3x + 5$.

(b) Show the coordinates of the points at which the graphs meet the x-axis.

(c) Solve the inequality $|5x–2| < 3x + 5$.

Q11 - ID: 828 *[12 marks, 14 minutes]*

(a) On the same diagram, sketch the graphs of $y = |x^2 - 14|$ and $y = |1x - 2|$, showing the coordinates of the points where the graphs meet the axes.
(b) Solve $|x^2 - 14| = |1x - 2|$, giving your answers in surd form where appropriate.
(c) Hence, or otherwise, find the set of values of x for which
$|x^2 - 14| > |1x - 2|$.

Q12 - ID: 924 *[5 marks, 6 minutes]*

(a) By expressing $\dfrac{4}{4r^2 - 1}$ in partial fractions, or otherwise, prove that

$$\sum_{r=1}^{n} \frac{4}{4r^2 - 1} = 2 - \frac{2}{2n + 1}$$

(b) Hence find the exact value of $\displaystyle\sum_{r=11}^{20} \frac{4}{4r^2 - 1}$.

Q13 - ID: 5957 *[7 marks, 8 minutes]*

(a) Show that $\dfrac{1}{6r - 1} - \dfrac{1}{6r + 5} = \dfrac{6}{(6r - 1)(6r + 5)}$ for all integers r.

(b) Hence use the method of differences to find $\displaystyle\sum_{r=1}^{n} \frac{1}{(6r - 1)(6r + 5)}$.

Q14 - ID: 754 *[7 marks, 8 minutes]*

(a) Express $\dfrac{4}{(r + 1)(r + 3)}$ in partial fractions.

(b) Hence prove that $\displaystyle\sum_{r=1}^{n} \frac{4}{(r + 1)(r + 3)} = \frac{10n^2 + 26n}{6(n + 2)(n + 3)}$

Q15 - ID: 879 *[10 marks, 12 minutes]*

Given that for all real values of r,
$(2r + 1)^3 - (2r - 1)^3 = Ar^2 + B,$
where A and B are constants,
(a) find the value of A and the value of B.

(b) Hence, or otherwise, prove that $\displaystyle\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n + 1)(2n + 1).$

(c) Calculate $\displaystyle\sum_{r=1}^{50} (5r - 1)^2$.

Q16 - ID: 5991 *[12 marks, 14 minutes]*

Let $\omega = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$

(a) Show that ω is a root of the equation $z^5 - 1 = 0$.

(b) Show that $(\omega-1)(\omega^4 + \omega^3 + \omega^2 + \omega + 1) = \omega^5 - 1$.

(c) Deduce that $\omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0$.

(d) Hence show that $\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$.

Q17 - ID: 5954 *[3 marks, 4 minutes]*

Indicate on a single Argand diagram the set of points for which
$\arg(z - (2 + 3i)) = \pi$

Q18 - ID: 5963 *[3 marks, 4 minutes]*

On an Argand diagram, sketch the set of points for which
$\arg(z - -5) = \frac{1}{4}\pi$

Q19 - ID: 600 *[6 marks, 7 minutes]*

Find the general solution of the differential equation

$$\frac{dy}{dx} - \frac{y}{x} = 6x^2 + 6x$$

giving y in terms of x in your answer.

Q20 - ID: 562 *[7 marks, 8 minutes]*

Given that $y = 3$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} - y \tan x = 4 \sec^2 x$$

Q21 - ID: 489 *[7 marks, 8 minutes]*

Find the general solution of the differential equation

$$(x + 3)\frac{dy}{dx} + 2y = \frac{1}{x}$$

giving your answer in the form $y = f(x)$.

Q22 - ID: 578 *[8 marks, 10 minutes]*

Obtain the general solution of the differential equation

$$x\frac{dy}{dx} + 11y = \frac{\cos x}{x^9}, \quad x > 0$$

giving your answer in the form $y = f(x)$.

Q23 - ID: 3477 *[5 marks, 6 minutes]*

Solve the differential equation

$$\frac{dy}{dx} - 9y = x$$

to obtain y as a function of x

Q24 - ID: 4845 *[10 marks, 12 minutes]*

A raindrop falls from rest through mist. Its velocity $v\, ms^{-1}$ vertically downward, at time t seconds after it starts to fall is modelled by the differential equation

$$(1+t)\frac{dv}{dt} + 5v = (1+t)g - 35$$

Solve the differential equation to show that

$$v = \frac{g}{6}(1+t) - 7 + (7 - \frac{g}{6})(1+t)^{-5}$$

Q25 - ID: 5058 *[9 marks, 11 minutes]*

A population P is growing at a rate which is modelled by the differential equation

$$\frac{dP}{dt} - 0.1P = 0.19t$$

where t years is the time that has elapsed from the start of observations.
It is given that the population is 8000 at the start of the observations.
(a) Solve the differential equation to obtain an expression for P in terms of t.
(b) Show that the population doubles between the 6th and 7th year after the observations began.

Q26 - ID: 5099 *[9 marks, 11 minutes]*

By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + \frac{4x}{x^2+5}y = x$$

given that $y = 2$ when $x = 2$. Give your answer in the form $y = f(x)$.

Q27 - ID: 742 *[10 marks, 12 minutes]*

(a) Use the substitution y = vx to transform the equation

$$\frac{dy}{dx} = \frac{(4x+y)(x+y)}{x^2}, x > 0 \qquad \text{(I)}$$

into the equation

$$x\frac{dv}{dx} = (2+v)^2, x > 0 \qquad \text{(II)}$$

(b) Solve the differential equation II to find v as a function of x.
(c) Hence show that

$y = -2x - \dfrac{x}{\ln x + c}$, where c is an arbitary constant,

is a general solution of the differential equation I.

Q28 - ID: 2329 *[6 marks, 7 minutes]*

Solve the differential equation $x\dfrac{dy}{dx} - y^2 = 1$
given that $y = 0$ when $x = 4$.
Give your answer in the form $y = f(x)$.

Q29 - ID: 5809 *[11 marks, 13 minutes]*

A liquid is being heated in an oven maintained at a constant temperature of 168°C. It may be assumed that the rate of increase of the temperature of the liquid at any particular time t minutes is proportional to $168 - \theta$, where θ°C is the temperature of the liquid at that time.

(a) Write down a differential equation connecting θ and t.

When the liquid was placed in the oven, its temperature was 18°C and 5 minutes later its temperature had risen to 65°C.

(b) Find the temperature of the liquid, correct to the nearest degree, after another 5 minutes.

Q30 - ID: 5424 *[7 marks, 8 minutes]*

Use the substitution $z = x + y$ to show that the differential equation

$$\frac{dy}{dx} = \frac{x + y + 8}{x + y - 2} \qquad (A)$$

may be written in the form $\dfrac{dz}{dx} = \dfrac{2(z + 3)}{z - 2}$.

(b) Hence find the general solution of the differential equation (A).

Q31 - ID: 470 *[14 marks, 17 minutes]*

(a) Find the general solution of the differential equation

$$2\frac{d^2y}{dt^2} + 13\frac{dy}{dt} + 20y = 4t^2 + 10t$$

(b) Find the particular solution of this equation for which $y = 1$ and $\dfrac{dy}{dt} = 1$ when $t = 0$.

(c) For this particular solution, find the value of y when $t = 1$.

Q32 - ID: 679 *[15 marks, 18 minutes]*

Given that $y = 2$ at $x = 0$ and $\dfrac{dy}{dx} = -6$ when $x = 0$, find y in terms of given further that

$$5\frac{d^2y}{dx^2} + 25\frac{dy}{dx} = 5x + 10$$

Q33 - ID: 632 *[12 marks, 14 minutes]*

(a) Find the value of λ for which $\lambda x \cos 5x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 25y = -40\sin 5x$$

(b) Hence find the general solution of this differential equation.
The particular solution of the differential equation for which $y = 1$ and $\frac{dy}{dx} = 4$ at $x = 0$ is $y = g(x)$.
(c) Find $g(x)$.

Q34 - ID: 513 *[8 marks, 10 minutes]*

Given that $6x\sin 4x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 16y = k\cos 4x$$

where k is a constant,
(a) calculate the value of k,
(b) find the particular solution of the differential equation for which at $x = 0$, $y = 3$, and for which at $x = \frac{\pi}{8}$, $y = \frac{\pi}{4}$.

Q35 - ID: 817 *[9 marks, 11 minutes]*

(a) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0$$

(b) Given that $x = 1$ and $\frac{dx}{dt} = 1$ at $t = 0$, find the particular solution of the differential equation, giving your answer in the form $x = f(t)$.

Q36 - ID: 920 *[12 marks, 14 minutes]*

For the differential equation

$$\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 10y = 5x(x + 7)$$

find the solution for which at $x = 0$, $\frac{dy}{dx} = 1$ and $y = 1$.

Q37 - ID: 4238 *[9 marks, 11 minutes]*

(a) Find, in terms of k, the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = kt + 5$$

For large values of t, this general solution may be approximated by a linear function.
(b) Given that $k = 18$, find the equation of this linear function.

Q38 - ID: 4833 *[10 marks, 12 minutes]*

The differential equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)$ is to be solved for $t \geq 0$ subject to the conditions that $\frac{dy}{dt} = 0, y = 0$ when $t = 0$.

When $f(t) = 8$ find the solution for y in terms of t.

Q39 - ID: 5095 *[10 marks, 12 minutes]*

(a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 13$$

(b) Hence express y in terms of x, given that $y = 3$ and $\frac{dy}{dx} = 5$ when $x = 0$.

Q40 - ID: 5100 *[11 marks, 13 minutes]*

Given that $x = e^t$ and that y is a function of x, show that:

(a) $x\frac{dy}{dx} = \frac{dy}{dt}$

(b) $x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}$

(c) Hence find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2} - 7x\frac{dy}{dx} + 7y = 0$$

Q41 - ID: 5690 *[14 marks, 17 minutes]*

(a) Find the general solution of the differential equation

$$4\frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = x^2$$

(b) Find the particular solution for which, at $x = 0, y = 5$ and $\frac{dy}{dx} = 5$.

Q42 - ID: 7520 *[11 marks, 13 minutes]*

The function f is defined by $f(x) = (1 + 6x)^{\frac{1}{2}}$.

(a) Find $f'''(x)$.

(b) Using Maclaurin's theorem, show that, for small values of x,

$(1 + 6x)^{\frac{1}{2}} \approx 1 + 3x - \frac{9}{2}x^2 + \frac{27}{2}x^3$

(c) Use the expansion of e^x together with the result in part (b) to show that, for small values of x , $e^x(1 + 6x)^{\frac{1}{2}} \approx$ where k is a rational number to be found.

Q43 - ID: 4393 *[6 marks, 7 minutes]*

It is given that $f(x) = \ln(4 + \cos x)$

(a) Find the exact values of $f(0), f'(0), f''(0)$.

(b) Hence find the first two non-zero terms in the Maclaurin series for $f(x)$.

Q44 - ID: 4401 *[8 marks, 10 minutes]*

Given that $f(x) = arc\tan(\sqrt{3} + x)$
(a) find $f'(x), f''(x)$.
(b) Hence find the Maclaurin series for $f(x)$ as far as the term in x^2.

Q45 - ID: 5055 *[7 marks, 8 minutes]*

(a) Given that $y = \ln(1 + 9x), |x| < \frac{1}{9}$, find $\frac{d^3y}{dx^3}$.
(b) Hence find the Maclaurin series for $y = \ln(1 + 9x), |x| < \frac{1}{9}$ as far as the term in x^3.

Q46 - ID: 7512 *[9 marks, 11 minutes]*

(a) Given that $y = \ln(3\cos x)$, find $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$.
(b) Find the value of $\frac{d^4y}{dx^4}$ when $x = 0$.
(c) Hence, by using Maclaurin's theorem, show that the first three non-zero terms in the expansion, in ascending powers of x, of $\ln\cos x$ are $\ln(3) - \frac{x^2}{2} - \frac{x^4}{12}$.

Q47 - ID: 7513 *[8 marks, 10 minutes]*

(a) Write down the expansion of $\sin 5x$ in ascending powers of x up to and including the term in x^3.
(b) Given that $y = \sqrt{8 + e^x}$, find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x = 0$.
(c) Using Maclaurin's theorem, show that, for small values of x, $\sqrt{8 + e^x} \approx 3 + \frac{1}{6}x + \frac{17}{216}x^2$.

Q48 - ID: 7517 *[6 marks, 7 minutes]*

(a) Given that $y = \ln(1 + 2\sin x)$, find $\frac{d^2y}{dx^2}$.
(b) By using Maclaurin's theorem, show that, for small values of x,
$\ln(1 + 2\sin x) \approx 2x - 2x^2$

Q49 - ID: 826 *[8 marks, 10 minutes]*

(a) Find the Taylor expansion of $\cos 3x$ in ascending powers of $\left(x - \frac{\pi}{6}\right)$ up to and including the term in $\left(x - \frac{\pi}{6}\right)^5$
(b) Use your answer to (a) to obtain an estimate of $\cos 3$, giving your answer to 6 decimal places.

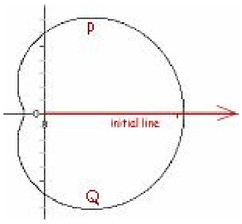
Q50 - ID: 5056 *[8 marks, 10 minutes]*

Use the Taylor Series method to find the series solution, ascending up to and including the term in x^3 , of the differential equation

$$\frac{d^2y}{dx^2} + y\frac{dy}{dx} + y^2 = 5x + 11$$

given that $\frac{dy}{dx} = y = 2$ at $x = 0$.

Q51 - ID: 595 *[14 marks, 17 minutes]*

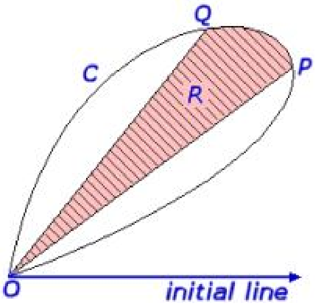


The curve C shown above has polar equation

$r = a(11 + \sqrt{6}\cos\theta),\ for\ -\pi \leq \theta < \pi.$

(a) Find the polar coordinates of the point P and Q where the tangents to C are parallel to the initial line.
The curve C represents the perimeter of the surface of a swimming pool. The direct distance from P to Q is 28 m.
(b) Calculate the value of a.
(c) Find the area of the surface of the pool.

Q52 - ID: 353 *[14 marks, 17 minutes]*



The diagram shows a sketch of the curve C with polar equation

$r = 16\sin\theta\cos^2\theta,\ for 0 \leq \theta < \frac{\pi}{2}.$

The tangent to C at the point P is perpendicular to the initial line.
(a) Find the polar coordinates of P.
The point Q on C has polar coordinates $\frac{\pi}{4}$.
The shaded region R is bounded by OP, OQ and C, as shown in the diagram.
(b) Show that the area of R is given by $16\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\left(\cos 2\theta\sin^2 2\theta + \frac{1-\cos 4\theta}{2}\right)d\theta$
(c) Hence, or otherwise, find the area of R, giving your answer in the form $a + b\pi$, where a and b are rational numbers.

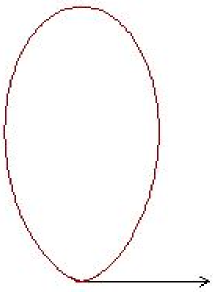
Q53 - ID: 4396 *[5 marks, 6 minutes]*

The equation of a graph in polar coordinates is

$r = 1 + 6\sec\theta,\ for\ -\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$

Find the exact area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{\pi}{6}$.

Q54 - ID: 4400 *[5 marks, 6 minutes]*



The diagram shows the curve with polar equation

$r = a(1 - \cos 2\theta),\ for 0 \leq \theta < \pi$

Find the area of the region enclosed by the curve.

Q55 - ID: 699 *[5 marks, 6 minutes]*

The equation of a curve in cartesian coordinates is given as

$(x^2 + y^2)^2 = 10(x^2 - y^2)$

Show that the equation may be expressed, in polar coordinates, in the form

$r^2 = 10\cos 2\theta$

Q56 - ID: 4395 *[3 marks, 4 minutes]*

The equation of a curve in polar coordinates is

$r = 9 + 5\sec\theta$

Show that a cartesian equation of the curve is

$(x - 5)\sqrt{x^2 + y^2} = 9x$

A1 - ID: 778 *[7 marks, 8 minutes]*

$$\frac{x}{x-6} > \frac{6}{x-1} \Rightarrow \frac{x}{x-6} - \frac{6}{x-1} > 0$$

$$\Rightarrow \frac{x(x-1)-6(x-6)}{(x-6)(x-1)} > 0 \qquad |M1$$

$$\Rightarrow \frac{x^2-7x+36}{(x-6)(x-1)} > 0 \qquad |A1$$

$\Rightarrow$ Numerator always positive $\qquad |B1$

$$\Rightarrow (x-6)(x-1) > 0 \qquad |B1$$

$$\Rightarrow x > 6 \text{ or } x < 1 \qquad |M1A2$$

A2 - ID: 368 *[5 marks, 6 minutes]*

$$x > 1 \Rightarrow x - 1 > 8x - 1$$

$$\Rightarrow 0 > 7x$$

$$\Rightarrow x < 0 \quad \Rightarrow \text{ no values}$$

$$x < 1 \Rightarrow -x + 1 > (8x - 1)$$

$$\Rightarrow -9x > -2$$

$$\Rightarrow 9x < 2 \quad \Rightarrow x < \tfrac{2}{9}$$

A3 - ID: 899 *[7 marks, 8 minutes]*

$$x > 0 \Rightarrow 5x^2 - 8x > 4 \qquad |M1$$

$$\Rightarrow 5x^2 - 8x - 4 > 0$$

$$\Rightarrow (5x+2)(x-2) > 0$$

$$\Rightarrow x < -\tfrac{2}{5} \text{ or } x > 2 \qquad |A2$$

$$\Rightarrow x > 2 \qquad |A1$$

$$x < 0 \Rightarrow (5x+2)(x-2) < 0 \qquad |M1$$

$$\Rightarrow -\tfrac{2}{5} < x < 2$$

$$\Rightarrow -\tfrac{2}{5} < x < 0 \qquad |M1A1$$

A4 - ID: 592 *[3 marks, 4 minutes]*

$$x > \tfrac{4}{5} \Rightarrow 5x - 4 < 6x - 3 \quad \Rightarrow -1 < 1x$$

$$\Rightarrow x > \tfrac{-1}{1} \qquad |M1$$

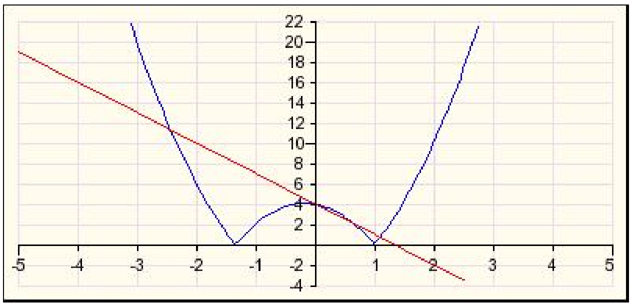
$$x < \tfrac{4}{5} \Rightarrow -5x + 4 < 6x - 3 \quad \Rightarrow 7 < 11x$$

$$\Rightarrow x > \tfrac{7}{11} \qquad |M1$$

answer $\Rightarrow x > \tfrac{7}{11} \qquad |A1$

A5 - ID: 17

[7 marks, 8 minutes]



(a) $3x^2 + x - 4 = 4 - 3x \Rightarrow 3x^2 + 4x - 8 = 0$ $|M1$

$\Rightarrow x = \frac{-4 \pm \sqrt{4^2+96}}{6} = \frac{-4 \pm \sqrt{112}}{6}$ $|M1A1$

$3x^2 + x - 4 = -4 + 3x \Rightarrow 3x^2 - 2x = 0$ $|M1$

$\Rightarrow x = 0, \frac{2}{3}$

(b) see graph below

(c) $|3x^2 + x - 4| > 4 - 3x \Rightarrow 0 < x < \frac{2}{3}$ $|A1$

and $x > \frac{-4+\sqrt{112}}{6} = 1.1$

and $x < \frac{-4-\sqrt{112}}{6} = -2.43$ $|M1A1$

A6 - ID: 369

[6 marks, 7 minutes]

$x > 7 \Rightarrow x^2 > 6x(x-7)$ $|B1$

$\Rightarrow x^2 > 6x^2 - 42x \Rightarrow 5x^2 - 42x < 0$

$\Rightarrow x(5x - 42) < 0$

$\Rightarrow 0 < x < 8.4 \Rightarrow 7 < x < 8.4$ $|M1A1$

$x < 7 \Rightarrow x^2 < 6x(x-7)$ $|B1$

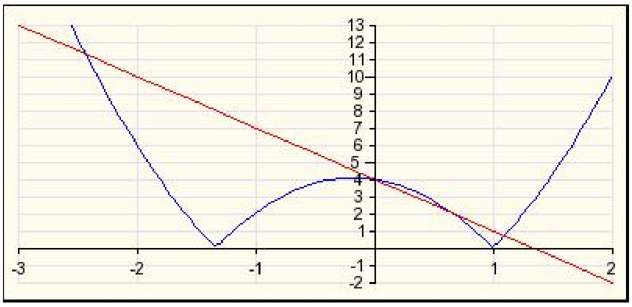
$\Rightarrow x(5x - 42) > 0$

$\Rightarrow x < 0$ or $x > 8.4$

$\Rightarrow x < 0$ $|M1A1$

A7 - ID: 850

[7 marks, 8 minutes]



(a) $3x^2 + x - 4 = 4 - 3x \Rightarrow 3x^2 + 4x - 8 = 0$ $|M1$

$\Rightarrow x = \frac{-4 \pm \sqrt{4^2+96}}{6} = \frac{-4 \pm \sqrt{112}}{6}$ $|M1A1$

$3x^2 + x - 4 = -4 + 3x \Rightarrow 3x^2 - 2x = 0$ $|M1$

$\Rightarrow x = 0, \frac{2}{3}$

(b) see graph below

(c) $|3x^2 + x - 4| > 4 - 3x \Rightarrow 0 < x < \frac{2}{3}$ $|A1$

and $x > \frac{-4+\sqrt{112}}{6}$

and $x < \frac{-4-\sqrt{112}}{6}$ $|M1A1$

A8 - ID: 658 [9 marks, 11 minutes]

(a) $x > -4 \Rightarrow x^2 - 4 = 6(2 - x)(x + 4)$ | $M1$

$\Rightarrow (x - 2)[(x + 2) + 6(x + 4)] = 0$

$\Rightarrow (x - 2)[7x + 26] = 0 \Rightarrow x = 2 \text{ or } -\frac{26}{7}$ | $M1A1$

$x < -4 \Rightarrow x^2 - 4 = -6(2 - x)(x + 4)$ | $M1$

$\Rightarrow (x - 2)[(x + 2) - 6(x + 4)] = 0$

$\Rightarrow -(x - 2)[5x + 22] = 0 \Rightarrow x = 2 \text{ or } -\frac{22}{5}$ | $M1A1$

(b) $\frac{x^2 - 4}{|x + 4|} < 6(2 - x) \Rightarrow -\frac{26}{7} < x < 1$ | $M1A1$

or $x < -\frac{22}{5}$ | $B1$

A9 - ID: 4239 [7 marks, 8 minutes]

(a) $x > 0 \Rightarrow \frac{x}{4} + 4 = \frac{7}{x} \Rightarrow x^2 + 16x - 28 = 0$

$\Rightarrow x = \frac{-16 \pm \sqrt{368}}{2}$ | $M1A1$

$x < 0 \Rightarrow \frac{x}{4} + 4 = -\frac{7}{x} \Rightarrow x^2 + 16x + 28 = 0$

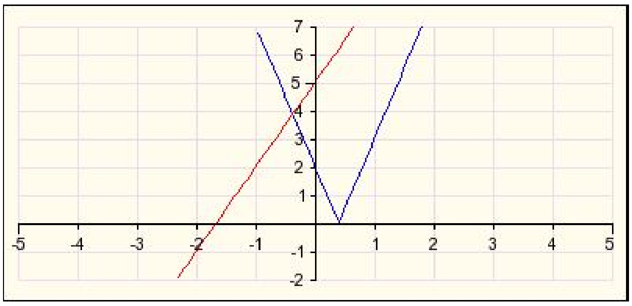
$\Rightarrow (x + 2)(x + 14) = 0$

$\Rightarrow x = -2, -14$ | $M1A1$

$\Rightarrow x = -2, -14, \frac{-16 + \sqrt{368}}{2}$ | $A1$

(b) $\frac{x}{4} + 4 > \left|\frac{7}{x}\right| \Rightarrow -14 < x < -2 \text{ or } x > \frac{-16 + \sqrt{368}}{2}$ | $B2$

A10 - ID: 5057 [6 marks, 7 minutes]



(a) see graph below: line | $B1$

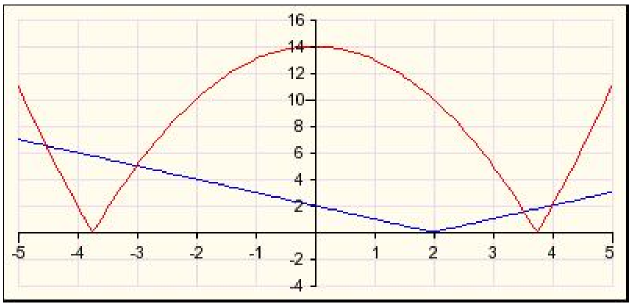
: V shape | $B1$

(b) points : $-\frac{5}{3}, \frac{2}{5}$ | $B1$

(c) $2 - 5x = 3x + 5 \Rightarrow x = -\frac{3}{8}$ | $M1A1$

$\Rightarrow x > -\frac{3}{8}$ | $A1$

A11 - ID: 828 [12 marks, 14 minutes]



(a) graph : see below | $B4$

(b) $x^2 - 14 = 1x - 2 \Rightarrow x^2 - 1x - 12 = 0$ | $M1$

$\Rightarrow (x-4)(x+3) = 0 \Rightarrow x = 4, -3$ | $M1A1$

$x^2 - 14 = -1x + 2 \Rightarrow x^2 + 1x - 16 = 0$

$\Rightarrow x = \dfrac{-1 \pm \sqrt{65}}{2} = -0.5 \pm \dfrac{\sqrt{65}}{2}$ | $M1A1$

(c) $|x^2 - 14| > |1x - 2| \Rightarrow x < -0.5 - \dfrac{\sqrt{65}}{2}$ | $A1$

or $-3 < x < -0.5 + \dfrac{\sqrt{65}}{2}$ | $A1$

or $x > 4$ | $A1$

A12 - ID: 924 [5 marks, 6 minutes]

(a) $\dfrac{4}{4r^2-1} = \dfrac{A}{2r-1} + \dfrac{B}{2r+1} = \dfrac{2}{2r-1} + \dfrac{-2}{2r+1}$ |$B1$

$\displaystyle\sum_{r=1}^{n} \frac{4}{4r^2-1} = \sum_{r=1}^{n} \frac{2}{2r-1} - \frac{2}{2r+1}$

$= \dfrac{2}{1} - \dfrac{2}{3} + \dfrac{2}{3} - \dfrac{2}{5} + \dfrac{2}{5} - \dfrac{2}{7} + \ldots$

$+ \dfrac{2}{2n-1} - \dfrac{2}{2n+1}$ |$M1$

$= 2 - \dfrac{2}{2n+1}$ |$A1$

(b) $\displaystyle\sum_{r=11}^{20} \frac{4}{4r^2-1} = \sum_{r=1}^{20} \frac{4}{4r^2-1} - \sum_{r=1}^{10} \frac{4}{4r^2-1}$ |$M1$

$= -\dfrac{2}{41} + \dfrac{2}{21} = \dfrac{40}{861}$ |$A1$

A13 - ID: 5957 [7 marks, 8 minutes]

(a) $\dfrac{1}{6r-1} - \dfrac{1}{6r+5} = \dfrac{(6r+5)-(6r-1)}{(6r-1)(6r+5)} = \dfrac{6}{(6r-1)(6r+5)}$ |$M1A1$

(b) $\displaystyle\sum_{r=1}^{n} \frac{1}{(6r-1)(6r+5)} = \frac{1}{6}\sum_{r=1}^{n}\left[\frac{1}{6r-1} - \frac{1}{6r+5}\right]$ |$M1$

$= \dfrac{1}{6}\left[\left(\dfrac{1}{5} - \dfrac{1}{11}\right) + \left(\dfrac{1}{11} - \dfrac{1}{17}\right) + \ldots + \left(\dfrac{1}{6n-1} - \dfrac{1}{6n+5}\right)\right]$ |$M1A1$

$= \dfrac{1}{6}\left[\dfrac{1}{5} - \dfrac{1}{6n+5}\right]$ |$A2$

A14 - ID: 754 [7 marks, 8 minutes]

(a) $\dfrac{4}{(r+1)(r+3)} = \dfrac{A}{r+1} + \dfrac{B}{r+3}$

$4 = A(r+3) + B(r+1)$

$r = -3 \Rightarrow 4 = -2B \Rightarrow B = -2$ |$B1$

$r = -1 \Rightarrow 4 = 2A \Rightarrow A = 2$ |$B1$

(b) $\displaystyle\sum_{r=1}^{n} \frac{4}{(r+1)(r+3)} = \sum_{r=1}^{n} \frac{2}{r+1} + \frac{-2}{r+3}$

$= \dfrac{2}{2} + \dfrac{-2}{4}$

$+ \dfrac{2}{3} + \dfrac{-2}{5}$

$+ \dfrac{2}{4} + \dfrac{-2}{6} + \ldots$

$+ \dfrac{2}{n+1} + \dfrac{-2}{n+3}$ |$M1$

$= \dfrac{2}{2} + \dfrac{2}{3} + \dfrac{-2}{n+2} + \dfrac{-2}{n+3}$ |$A2$

$= \dfrac{10}{6} - \dfrac{2(2n+5)}{(n+2)(n+3)}$

$= \dfrac{10(n+2)(n+3) - 12(2n+5)}{6(n+2)(n+3)} = \dfrac{10n^2 + 26n + 0}{6(n+2)(n+3)}$ |$M1A1$

A15 - ID: 879 [10 marks, 12 minutes]

(a) $(2r+1)^3 - (2r-1)^3 = 8r^3 + 12r^2 + 6r + 1 - (8r^3 - 12r^2 + 6r - 1) = 24r^2 + 2$

$\Rightarrow A = 24, B = 2$ |$M1A1$

(b)
$$\sum_{r=1}^{n} r^2 = \sum_{r=1}^{n} \frac{1}{24}((2r+1)^3 - (2r-1)^3 - 2)$$
$$= \frac{1}{24}\sum_{r=1}^{n}((2r+1)^3 - (2r-1)^3) - \frac{2n}{24}$$
$$= \frac{1}{24}(3^3 - 1^3)$$
$$+ \frac{1}{24}(5^3 - 3^3)$$
$$+ \frac{1}{24}(7^3 - 5^3) + \ldots$$
$$+ \frac{1}{24}((2n+1)^3 - (2n-1)^3) - \frac{2n}{24}$$
$$= \frac{1}{24}[(2n+1)^3 - 1^3] - \frac{2n}{24} = \frac{1}{24}[(2n+1)^3 - 1^3 - 2n]$$ |$M1A2$
$$= \frac{1}{24}[8n^3 + 12n^2 + 4n] = \frac{1}{24}n[8n^2 + 12n + 4]$$
$$= \frac{1}{6}n(n+1)(2n+1)$$ |$M1A1$

(b)
$$\sum_{r=1}^{50}(5r-1)^2 = \sum_{r=1}^{50}(25r^2 - 10r + 1) = 1060425$$ |$M2A1$

A16 - ID: 5991 [12 marks, 14 minutes]

(a)
$$\omega^5 = \left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)^5$$ |$M1A1$
$$= \cos 2\pi + i\sin 2\pi = 1$$ |$A1$

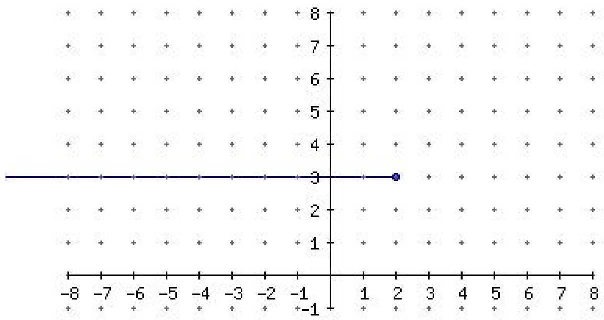
(b)
$$LHS = (\omega^5 + \omega^4 + \omega^3 + \omega^2 + \omega) - (\omega^4 + \omega^3 + \omega^2 + \omega + 1)$$
$$= \omega^5 - 1$$ |$M1A1$

(c)
$$\omega^5 - 1 = 0 \Rightarrow (\omega - 1)(\omega^4 + \omega^3 + \omega^2 + \omega + 1) = 0$$
$$\omega \neq 1 \Rightarrow \omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0$$ |$A1$

(d)
$$\omega^4 + \omega^3 + \omega^2 + \omega + 1 = \left(cis\frac{2\pi}{5}\right)^4 + \left(cis\frac{2\pi}{5}\right)^3 + \left(cis\frac{2\pi}{5}\right)^2 + \left(cis\frac{2\pi}{5}\right) + 1$$ |$M1$
$$= \left(cis\frac{8\pi}{5}\right) + \left(cis\frac{6\pi}{5}\right) + \left(cis\frac{4\pi}{5}\right) + \left(cis\frac{2\pi}{5}\right) + 1$$ |$A1$
$$= \cos\frac{2\pi}{5} - \sin\frac{2\pi}{5} + \cos\frac{4\pi}{5} - \sin\frac{4\pi}{5} + cis\frac{4\pi}{5} + (cis\frac{2\pi}{5} + 1$$ |$A1$
$$= 2\cos\frac{2\pi}{5} + 2\cos\frac{4\pi}{5} + 1$$ |$A2$
$$\omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0 \Rightarrow 2\cos\frac{2\pi}{5} + 2\cos\frac{4\pi}{5} + 1 = 0 \Rightarrow \cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$$ |$A1$

A17 - ID: 5954

[3 marks, 4 minutes]



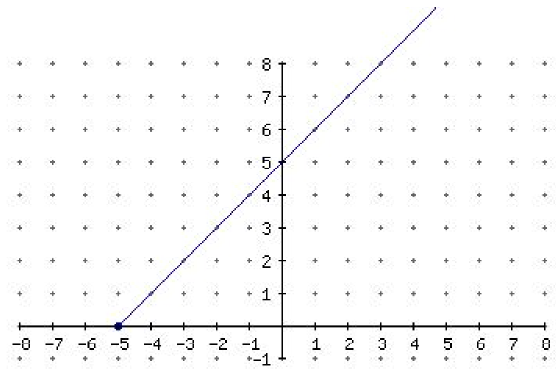
locus = half line |B1

= through (2, 3) |B1

direction = parallel to real axis |B1

A18 - ID: 5963

[3 marks, 4 minutes]



locus = half line |B1

= through $(-5, 0)$ |B1

direction = at $\frac{\pi}{4}$ to real axis |B1

A19 - ID: 600 *[6 marks, 7 minutes]*

integrating factor $= e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$ |$M1A1$

$\Rightarrow \frac{d}{dx}\left(\frac{y}{x}\right) = \frac{1}{x}(6x^2 + 6x)$ |$M1$

$\Rightarrow \frac{d}{dx}\left(\frac{y}{x}\right) = 6x + 6$

$\Rightarrow \frac{y}{x} = \int 6x + 6\, dx$

$\Rightarrow \frac{y}{x} = 3x^2 + 6x + c$ |$M1A1$

$\Rightarrow y = 3x^3 + 6x^2 + cx$ |$A1$

A20 - ID: 562 *[7 marks, 8 minutes]*

int. factor $= e^{-\int \tan x\, dx}$ |$M1$

$= e^{\ln \cos x}$ |$A1$

$= \cos x$ |$A1$

$\Rightarrow \cos x \frac{dy}{dx} - y \sin x = 4 \sec x$

$\Rightarrow y \cos x = \int 4 \sec x\, dx$ |$M1$

$\Rightarrow y \cos x = 4 \ln(\sec x + \tan x) + c$ |$A1$

$y = 3, x = 0 \Rightarrow 3 = c$ |$B1$

$\Rightarrow y \cos x = 4 \ln(\sec x + \tan x) + 3$

$\Rightarrow y = \sec x(4 \ln(\sec x + \tan x) + 3)$ |$A1$

A21 - ID: 489 *[7 marks, 8 minutes]*

$(x + 3)\frac{dy}{dx} + 2y = \frac{1}{x} \Rightarrow \frac{dy}{dx} + \frac{2}{x+3}y = \frac{1}{x(x+3)}$ |$M1$

integrating factor $= e^{\int \frac{2}{x+3}dx} = e^{2\ln(x+3)} = (x + 3)^2$ |$M1A1$

$\Rightarrow y(x + 3)^2 = \int \frac{(x+3)}{x}\, dx$ |$M1$

$\Rightarrow y(x + 3)^2 = x + 3 \ln x + c$ |$M1A1$

$\Rightarrow y = \frac{x + 3 \ln x + c}{(x+3)^2}$ |$A1$

A22 - ID: 578 *[8 marks, 10 minutes]*

$$x\frac{dy}{dx} + 11y = \frac{\cos x}{x^9} \Rightarrow \frac{dy}{dx} + \frac{11}{x}y = \frac{\cos x}{x^{10}}$$ M1

integrating factor $= e^{\int \frac{11}{x}dx} = e^{11\ln(x)} = x^{11}$ M1A1

$$\Rightarrow x^{11}\frac{dy}{dx} + 11x^{10}y = x^{11}\frac{\cos x}{x^{10}}$$

$$\Rightarrow x^{11}y = \int x\cos x\,dx$$ M1

$$\Rightarrow x^{11}y = x.\sin x - \int \sin x\,dx$$ M1

$$\Rightarrow x^{11}y = x.\sin x + \cos x + c$$ A2

$$\Rightarrow y = \frac{x.\sin x + \cos x + c}{x^{11}}$$ A1

A23 - ID: 3477 *[5 marks, 6 minutes]*

integrating factor $= e^{\int -9\,dx} = e^{-9x}$ |$M1A1$

$\Rightarrow e^{-9x}\dfrac{dy}{dx} - e^{-9x}9y = e^{-9x}x$

$\Rightarrow ye^{-9x} = \int e^{-9x}x\,dx$ |$A1$

$\Rightarrow ye^{-9x} = -\dfrac{x}{9}e^{-9x} + \int \dfrac{1}{9}e^{-9x}\,dx$

$\Rightarrow ye^{-9x} = -\dfrac{x}{9}e^{-9x} - \dfrac{1}{81}e^{-9x} + c$

$\Rightarrow y = -\dfrac{x}{9} - \dfrac{1}{81} + ce^{9x}$ |$M1A1$

A24 - ID: 4845 *[10 marks, 12 minutes]*

equation $\Rightarrow \dfrac{dv}{dt} + \dfrac{5}{1+t}v = g - \dfrac{35}{1+t}$ |$M1$

I.F. $= \exp\int \dfrac{5}{1+t}\,dt$

$= \exp 5\ln(1+t) = (1+t)^5$ |$M1A2$

$\Rightarrow (1+t)^5\dfrac{dv}{dt} + 5(1+t)^4v = (1+t)^5g - 35(1+t)^4$ |$B1$

integrate $\Rightarrow (1+t)^5v = \dfrac{g}{6}(1+t)^6 - \dfrac{35}{5}(1+t)^5 + c$ |$M1A1$

$\Rightarrow v = \dfrac{g}{6}(1+t) - 7 + c(1+t)^{-5}$ |$B1$

$t = 0, v = 0 \Rightarrow c = 7 - \dfrac{g}{6}$ |$M1$

$\Rightarrow v = \dfrac{g}{6}(1+t) - 7 + (7 - \dfrac{g}{6})(1+t)^{-5}$ |$A1$

A25 - ID: 5058 *[9 marks, 11 minutes]*

(a) integrating factor $= e^{\int -0.1\,dt} = e^{-0.1t}$ |$M1A1$

$\Rightarrow e^{-0.1t}\dfrac{dP}{dt} - e^{-0.1t}0.1P = e^{-0.1t}0.19t$

$\Rightarrow Pe^{-0.1t} = \int e^{-0.1t}0.19t\,dx$ |$A1$

$\Rightarrow Pe^{-0.1t} = -1.9te^{-0.1t} + \int 1.9e^{-0.1t}\,dx$

$\Rightarrow Pe^{-0.1t} = -1.9te^{-0.1t} - 19e^{-0.1t} + c$

$\Rightarrow P = -1.9t - 19 + ce^{0.1t}$ |$M1A1$

$t = 0, P = 8000 \Rightarrow 8000 = -19 + c \Rightarrow c = 8019$

$\Rightarrow P = -1.9t - 19 + 8019e^{0.1t}$ |$M1A1$

(b) $t = 6 \Rightarrow P = 14581$

$t = 7 \Rightarrow P = 16116$

$\Rightarrow P$ reaches 16000 between year 6 and 7 |$M1A1$

A26 - ID: 5099

[9 marks, 11 minutes]

$$\text{integrating factor} = e^{\int \frac{4x}{x^2+5}} = e^{2\ln(x^2+5)} = (x^2+5)^2 \quad |M1A2$$

$$\Rightarrow \frac{d}{dx}\left(y(x^2+5)^2\right) = x(x^2+5)^2 \quad |M1A1$$

$$\Rightarrow y(x^2+5)^2 = \int x(x^2+5)^2\,dx$$

$$\Rightarrow y(x^2+5)^2 = \frac{1}{6}(x^2+5)^3 + c \quad |M1A1$$

$$y = 2, x = 2 \Rightarrow 162 = \frac{729}{6} + c \Rightarrow c = \frac{243}{6} \quad |M1$$

$$\Rightarrow y = \frac{1}{6}(x^2+5) + \frac{243}{6}(x^2+5)^{-2} \quad |A1$$

A27 - ID: 742

[10 marks, 12 minutes]

(a)
$$\frac{dy}{dx} = v + x\frac{dv}{dx} = \frac{(4x+vx)(x+vx)}{x^2} \quad |M2$$

$$\Rightarrow v + x\frac{dv}{dx} = (4+v)(1+v)$$

$$\Rightarrow x\frac{dv}{dx} = 4 + 5v + v^2 - v = 4 + 4v + v^2 = (2+v)^2 \quad |M1A1$$

(b)
$$x\frac{dv}{dx} = (2+v)^2 \Rightarrow \frac{dv}{(2+v)^2} = \frac{dx}{x} \Rightarrow \int \frac{dv}{(2+v)^2} = \int \frac{dx}{x} \quad |M1$$

$$\Rightarrow -(2+v)^{-1} = \ln x + c \quad |M1A1$$

$$\Rightarrow -(2+v) = \frac{1}{\ln x + c} \Rightarrow v = -2 - \frac{1}{\ln x + c} \quad |M1A1$$

(c)
$$v = \frac{y}{x} \Rightarrow y = -2x - \frac{x}{\ln x + c} \quad |M1$$

A28 - ID: 2329

[6 marks, 7 minutes]

$$x\frac{dy}{dx} - y^2 = 1 \Rightarrow x\frac{dy}{dx} = y^2 + 1 \Rightarrow \frac{dy}{y^2+1} = \frac{dx}{x} \quad |M1A1$$

$$\Rightarrow \frac{1}{1}\tan^{-1}\frac{y}{1} = \ln x + c \quad |M1A1$$

$$y = 0, x = 4 \Rightarrow \frac{1}{1}\tan^{-1}\frac{0}{1} = \ln 4 + c$$

$$\Rightarrow c = -\ln 4 \quad |A1$$

$$\Rightarrow \frac{1}{1}\tan^{-1}\frac{y}{1} = \ln x - \ln 4$$

$$\Rightarrow \frac{1}{1}\tan^{-1}\frac{y}{1} = \ln\frac{x}{4}$$

$$\Rightarrow \tan^{-1}\frac{y}{1} = 1\ln\frac{x}{4}$$

$$\Rightarrow \frac{y}{1} = \tan\left(1\ln\frac{x}{4}\right)$$

$$\Rightarrow y = 1\tan\left(1\ln\frac{x}{4}\right) \quad |A1$$

A29 - ID: 5809

[11 marks, 13 minutes]

(a) $\frac{d\theta}{dt} = k(168 - \theta)$ |B2

(b) $\frac{d\theta}{dt} = k(168 - \theta) \Rightarrow \int \frac{d\theta}{168 - \theta} = \int k\,dt$ |M1

$\Rightarrow -\ln(168 - \theta) = kt + c$ |A2

$t = 0, \theta = 18 \Rightarrow c = -\ln 150)$ |M1A1

$t = 5, \theta = 65 \Rightarrow -\ln 103 = 5k + -\ln 150)$

$\Rightarrow k = \frac{1}{5}\ln\frac{150}{103}$ |M1A1

$t = 10 \Rightarrow -\ln(168 - \theta) = 10k + c$

$\Rightarrow 168 - \theta = e^{-10k-c}$

$\Rightarrow \theta = 168 - e^{-10k-c} = 97.27°$ |M1A1

A30 - ID: 5424

[7 marks, 8 minutes]

(a) $z = x + y \Rightarrow \frac{dz}{dx} = 1 + \frac{dy}{dx}$ |M1

$(A) \Rightarrow \frac{dz}{dx} - 1 = \frac{z + 8}{z - 2}$

$\Rightarrow \frac{dz}{dx} = \frac{z + 8 + z - 2}{z - 2} = \frac{2(z + 3)}{z - 2}$ |M1A1

(b) $\frac{dz}{dx} = \frac{2(z + 3)}{z - 2} \Rightarrow \int \frac{z - 2}{z + 3}\,dz = \int 2dx$ |B1

$\Rightarrow \int \left(1 - \frac{5}{z + 3}\right) dz = 2x + c$ |M1

$\Rightarrow z - 5\ln(z + 3) = 2x + c$

$\Rightarrow x + y - 5\ln(x + y + 3) = 2x + c$

$\Rightarrow -5\ln(x + y + 3) = x - y + c$ |M1A1

A31 - ID: 470 *[14 marks, 17 minutes]*

(a) $CF \Rightarrow 2m^2 + 13m + 20 = 0 \Rightarrow (2m+5)(m+4) = 0$

$\Rightarrow m = -\frac{5}{2}, -4 \Rightarrow$ CF is $y = Ae^{-\frac{5}{2}t} + Be^{-4t}$ | $M1A1$

$PI \Rightarrow y = at^2 + bt + c \Rightarrow y' = 2at + b,\ y'' = 2a$ | $B1$

$\Rightarrow 2(2a) + 13(2at + b) + 20(at^2 + bt + c) = 4t^2 + 10t$ | $M1$

$\Rightarrow 20a = 4 \Rightarrow a = 0.2$

$\Rightarrow 26a + 20b = 10 \Rightarrow b = 0.24$ | $A1$

$\Rightarrow 4a + 13b + 20c = 0 \Rightarrow c = -0.196$ | $M1A1$

$GS \Rightarrow y = Ae^{-\frac{5}{2}t} + Be^{-4t} + 0.2t^2 + 0.24t + -0.196$ | $A1$

(b) $t = 0 \Rightarrow y' = -\frac{5}{2}Ae^{-\frac{5}{2}t} - 4Be^{-4t} + 0.4t + 0.24$ | $M1$

$\Rightarrow 1 = -\frac{5}{2}A - 4B + 0.24$

$\Rightarrow 1 = A + B + -0.196$ | $M1A1$

$\Rightarrow 1.196 = A + B,\ 1.52 = -5A - 8B$

$\Rightarrow 1.52 = -5A - 8(1.196 - A) \Rightarrow A = 3.696,\ B = -2.5$ | $M1$

$\Rightarrow y = 3.696(e^{-\frac{5}{2}t} - e^{-4t}) + 0.2t^2 + 0.24t + -0.196$ | $A1$

(c) $t = 1 \Rightarrow y = 3.696(e^{-\frac{5}{2}} - e^{-4}) + 0.244 = 0.48$ | $A1$

A32 - ID: 679 *[15 marks, 18 minutes]*

$CF \Rightarrow 5m^2 + 25m = 0 \Rightarrow 5m(m+5) = 0$

$\Rightarrow m = 0, -5$ | $M1A1$

$\Rightarrow y = A + Be^{-5x}$ | $M1A1$

$PI \Rightarrow y = ax^2 + bx + c \Rightarrow y' = 2ax + b,\ y'' = 2a$ | $M1B1$

$\Rightarrow 5(2a) + 25(2ax + b) = 5x + 10$ | $M1$

$\Rightarrow 50a = 5 \Rightarrow a = 0.1$ | $A1$

$\Rightarrow 10a + 25b = 10 \Rightarrow b = 0.36$ | $A1$

$GS \Rightarrow y = A + Be^{-5x} + 0.1x^2 + 0.36x$ | $A1$

$x = 0 \Rightarrow 2 = A + B$ | $B1$

$\Rightarrow y' = -5Be^{-5x} + 0.2x + 0.36$ | $M1$

$\Rightarrow -6 = -5B + 0.36$ | $A1$

$\Rightarrow B = 1.272,\ A = 0.728$ | $A1$

$PS \Rightarrow y = 0.728 + 1.272e^{-5x} + 0.1x^2 + 0.36x$ | $A1$

A33 - ID: 632 [12 marks, 14 minutes]

(a) $\frac{dy}{dx} = -5\lambda x\sin 5x + \lambda\cos 5x$ |$M1A1$

$\Rightarrow \frac{d^2y}{dx^2} = -25\lambda x\cos 5x - 5\lambda\sin 5x - 5\lambda\sin 5x$ |$A1$

$\Rightarrow -25\lambda x\cos 5x - 5\lambda\sin 5x - 5\lambda\sin 5x + 25\lambda x\cos 5x = -40\sin 5x$

$\Rightarrow -5\lambda\sin 5x - 5\lambda\sin 5x = -40\sin 5x$

$\Rightarrow -10\lambda = -40 \Rightarrow \lambda = 4$ |$A1$

(b) $m^2 + 25 = 0$ |$M1$

$\Rightarrow m = \pm 5i$ |$A1$

$\Rightarrow GS : y = A\cos 5x + B\sin 5x + 4x\cos 5x$ |$M1A1$

(c) $x = 0 \Rightarrow 1 = A$ |$B1$

$\frac{dy}{dx} = -5\sin 5x + 5B\cos 5x + -20x\sin 5x + 4\cos 5x$ |$M1A1$

$x = 0 \Rightarrow 4 = 5B + 4 \Rightarrow B = 0$

$\Rightarrow g(x) = \cos 5x + 4x\cos 5x$ |$A1$

A34 - ID: 513 [8 marks, 10 minutes]

(a) $\frac{dy}{dx} = 24x\cos 4x + 6\sin 4x$

$\Rightarrow \frac{d^2y}{dx^2} = -96x\sin 4x + 24\cos 4x + 24\cos 4x$ |$M1A1$

$\Rightarrow k\cos 4x = -96x\sin 4x + 48\cos 4x + 16(6x\sin 4x)$

$\Rightarrow k = 48$ |$M1A1$

(b) $GS : y = A\cos 4x + B\sin 4x + 6x\sin 4x$ |$B1$

$x = 0, y = 3 \Rightarrow 3 = A$ |$B1$

$x = \frac{\pi}{8}, y = \frac{\pi}{4} \Rightarrow \frac{\pi}{4} = B + 6\frac{\pi}{8} \Rightarrow B = \frac{-16\pi}{32}$ |$B1$

$\Rightarrow y = 3\cos 4x + \frac{-16\pi}{32}\sin 4x + 6x\sin 4x$ |$A1$

A35 - ID: 817 [9 marks, 11 minutes]

(a) $m^2 + 2m + 5 = 0 \Rightarrow m = -1 \pm 2i$ |$M1A1$

$\Rightarrow x = e^{-1t}(A\cos 2t + B\sin 2t)$ |$M1A1$

(b) $x = 1, t = 0 \Rightarrow 1 = A$ |$B1$

$\frac{dx}{dt} = -1e^{-1t}(A\cos 2t + B\sin 2t) +$

$+ e^{-1t}2(-A\sin 2t + B\cos 2t)$ |$M1$

$\frac{dx}{dt} = 1, t = 0 \Rightarrow 1 = -1 + 2B \Rightarrow B = 1$ |$M1$

$\Rightarrow x = e^{-1t}(\cos 2t + 1\sin 2t)$ |$M1A1$

A36 - ID: 920 [12 marks, 14 minutes]

$CF \Rightarrow m^2 + 7m + 10 = 0 \Rightarrow (m+2)(m+5) = 0$
$\Rightarrow m = -2, -5 \Rightarrow$ CF is $y = Ae^{-2x} + Be^{-5x}$ |$M1A1$
$PI \Rightarrow y = ax^2 + bx + c \Rightarrow y' = 2ax + b, y'' = 2a$ |$B1$
$\Rightarrow 2a + 7(2ax + b) + 10(ax^2 + bx + c) = 5x(x + 7)$ |$M1$
$\Rightarrow 10a = 5 \Rightarrow a = 0.5$
$\Rightarrow 14a + 10b = 35 \Rightarrow b = 2.8$ |$A1$
$\Rightarrow 2a + 7b + 10c = 0 \Rightarrow c = -2.06$ |$A1$
$GS \Rightarrow y = Ae^{-2x} + Be^{-5x} + 0.5x^2 + 2.8x + -2.06$ |$A1$
$\Rightarrow y' = -2Ae^{-2x} - 5Be^{-5x} + 1x + 2.8$ |$M1$
$x = 0 \Rightarrow 1 = A + B + -2.06$
$\Rightarrow 1 = -2A - 5B + 2.8$ |$M1A1$
$\Rightarrow 3.06 = A + B, -1.8 = -2A - 5B$
$\Rightarrow -1.8 = -2A - 5(3.06 - A) \Rightarrow A = 6.75, B = -3.69$ |$M1$
$\Rightarrow y = 6.75e^{-2x} + -3.69e^{-5x} + 0.5x^2 + 2.8x + -2.06$ |$A1$

A37 - ID: 4238 [9 marks, 11 minutes]

(a) $CF \Rightarrow m^2 + 5m + 6 = 0 \Rightarrow (m+3)(m+2) = 0$
$\Rightarrow m = -3, -2$ |$M1A1$
$\Rightarrow$ CF is $x = Ae^{-3t} + Be^{-2t}$ |$A1$
$PI \Rightarrow x = at + b \Rightarrow x' = a$ |$B1$
$\Rightarrow 5a + 6(at + b) = kt + 5$
$\Rightarrow 6a = k, 5a + 6b = 5$ |$M1$
$\Rightarrow a = \frac{k}{6}, b = \frac{5}{6} - \frac{5k}{36}$ |$A1$
$GS \Rightarrow x = Ae^{-3t} + Be^{-2t} + \frac{k}{6}t + \frac{5}{6} - \frac{5k}{36}$ |$A1$

(b) $k = 18 \Rightarrow x = 3t - \frac{10}{6}$ |$M1A1$

A38 - ID: 4833 *[10 marks, 12 minutes]*

$CF \Rightarrow m^2 + 4m + 4 = 0 \Rightarrow (m+2)^2 = 0$
$\Rightarrow m = -2$ |$M1A1$
$\Rightarrow$ CF is $y = (A + Bt)e^{-2t}$ |$A1$
$PI : y = a \Rightarrow y' = 0 \Rightarrow 4(a) = 8$ |$B1$
$\Rightarrow a = 2$ |$A1$
$GS \Rightarrow y = (A + Bt)e^{-2t} + 2$ |$A1$
$t = 0, y = 0 \Rightarrow 0 = (A)e^0 + 2$
$\Rightarrow A = -2$ |$M1$
$y = (A + Bt)e^{-2t} + 2 \Rightarrow y' = (A + Bt)e^{-2t} \cdot -2 + e^{-2t} \cdot B$ |$M1$
$t = 0, y' = 0 \Rightarrow 0 = -2A + B$
$\Rightarrow B = -4$ |$M1$
$\Rightarrow y = -(2 + 4t)e^{-2t} + 2$ |$A1$

A39 - ID: 5095 *[10 marks, 12 minutes]*

(a) $m^2 + 6m + 13 = 0 \Rightarrow m = -3 \pm 2i$ |$M1A1$
$\Rightarrow y = e^{-3x}(A\cos 2x + B\sin 2x)$ |$M1A1$
$y = p \Rightarrow 13p = 13 \Rightarrow p = 1$ |$B1$
$\Rightarrow y = e^{-3x}(A\cos 2x + B\sin 2x) + 1$ |$B1$
(b) $x = 0, y = 3 \Rightarrow 3 = A + 1 \Rightarrow A = 2$ |$B1$
$\frac{dy}{dx} = -3e^{-3x}(A\cos 2x + B\sin 2x) +$
$+ e^{-3x}2(-A\sin 2x + B\cos 2x)$ |$M1A1$
$\frac{dy}{dx} = 5, x = 0 \Rightarrow 5 = -3A + 2B \Rightarrow B = 5.5$
$\Rightarrow y = e^{-3x}(2\cos 2x + 5.5\sin 2x) + 1$ |$A1$

A40 - ID: 5100 [11 marks, 13 minutes]

(a) $\frac{dx}{dt} = e^t = x$ |B1

$\Rightarrow x\frac{dy}{dx} = x\frac{dy}{dt}\times\frac{dt}{dx} = \frac{dy}{dt}$ |M1A1

(b) $\frac{d^2y}{dt^2} = \frac{d}{dt}\left(x\frac{dy}{dx}\right) = \frac{dx}{dt}\frac{dy}{dx} + x\frac{d}{dt}\left(\frac{dy}{dx}\right)$

$= \frac{dy}{dt} + x\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{dt}$ |M1

$= \frac{dy}{dt} + x\frac{d^2y}{dx^2}\frac{dx}{dt} = \frac{dy}{dt} + x^2\frac{d^2y}{dx^2}$

$\Rightarrow x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}$ |M1A1

(c) $x^2\frac{d^2y}{dx^2} - 7x\frac{dy}{dx} + 7y = (\Rightarrow \frac{d^2y}{dt^2} - \frac{dy}{dt} - 7\frac{dy}{dt} + 7y = 0$

$\Rightarrow \frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 7y = 0$

$\Rightarrow m^2 - 8m + 7 = 0$

$\Rightarrow (m-7)(m-1) = 0$

$\Rightarrow m = 7, 1$ |M3

$\Rightarrow y = Ae^{7t} + Be^t = Ax^7 + Bx$ |A2

A41 - ID: 5690 [14 marks, 17 minutes]

(a) $4m^2 - m - 3 = 0 \Rightarrow (4m+3)(m-1)$

$\Rightarrow m = -\frac{3}{4}, 1$ |M1A1

$\Rightarrow CF = Ae^{-\frac{3}{4}x} + Be^x$ |A1

$y = px^2 + qx + r \Rightarrow y' = 2px + q, \quad y'' = 2p$

$\Rightarrow 8p - 2px - q - 3(px^2 + qx + r) = x^2$ |M1

$\Rightarrow p = -\frac{1}{3}, \quad q = \frac{2}{9}, \quad r = -\frac{26}{27}$ |A3

$\Rightarrow y = Ae^{-\frac{3}{4}x} + Be^x - \frac{1}{3}x^2 + \frac{2}{9}x - \frac{26}{27}$ |A1

(b) $x = 0, y = 5 \Rightarrow 5 = A + B - \frac{26}{27}$ |M1A1

$y' = -\frac{3}{4}Ae^{-\frac{3}{4}x} + Be^x - \frac{2}{3}x + \frac{2}{9}$

$x = 0, \frac{dy}{dx} = 5 \Rightarrow 5 = -\frac{3}{4}A + B + \frac{2}{9}$ |M1A1

$\Rightarrow A = 0.677, B = 5.286$ |M1A1

$\Rightarrow y = 0.677e^{-\frac{3}{4}x} + 5.286e^x - \frac{1}{3}x^2 + \frac{2}{9}x - \frac{26}{27}$

A42 - ID: 7520 [11 marks, 13 minutes]

(a) $f'(x) = \frac{6}{2}(1+6x)^{-\frac{1}{2}}$ |$M1A1$

$f''(x) = -\frac{36}{4}(1+6x)^{-\frac{3}{2}} \quad f'''(x) = \frac{648}{8}(1+6x)^{-\frac{5}{2}}$ |$A2$

(b) $f(0) = 1, \quad f'(0) = 3, \quad f''(0) = -9, \quad f'''(0) = 81$ |$M1A1$

$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f''(0) + \ldots$

$= 1 + 3x - \frac{9}{2}x^2 + \frac{27}{2}x^3$ |$M1A1$

(c) $e^x(1+6x)^{\frac{1}{2}} = (1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3)(1 + 3x - \frac{9}{2}x^2 + \frac{27}{2}x^3)$ |$M1$

$= 1 + 4x + -1x^2 + \frac{32}{3}x^3$ |$M1A1$

A43 - ID: 4393 [6 marks, 7 minutes]

(a) $f(0) = \ln(4+\cos 0) = \ln(5)$ |$B1$

$f'(x) = \frac{-\sin x}{4+\cos x}$ |$M1$

$\Rightarrow f'(0) = \frac{-\sin 0}{4+\cos 0} = 0$

$f''(x) = \frac{(4+\cos x)\cdot -\cos x + \sin x \cdot -\sin x}{(4+\cos x)^2}$ |$M1$

$\Rightarrow f''(0) = -\frac{1}{5}$ |$A1$

(b) $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots$

$= \ln(5) - \frac{x^2}{10} + \ldots$ |$M1A1$

A44 - ID: 4401 [8 marks, 10 minutes]

(a) $f'(x) = \frac{1}{1+(\sqrt{3}+x)^2}$ |$M1A1$

$f''(x) = \frac{-2(\sqrt{3}+x)}{(1+(\sqrt{3}+x)^2)^2}$ |$M1A1$

(b) $f(0) = arc\tan(\sqrt{3}) = \frac{\pi}{3}$

$f'(0) = \frac{1}{4}, \quad f''(0) = \frac{-2\sqrt{3}}{16}$ |$B1M1$

$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots$

$= \frac{\pi}{3} + \frac{x}{4} - \frac{\sqrt{3}x^2}{16}$ |$A2$

A45 - ID: 5055 *[7 marks, 8 minutes]*

(a) $\frac{dy}{dx} = \frac{9}{1+9x}$ |$M1A1$

$\frac{d^2y}{dx^2} = -\frac{81}{(1+9x)^2}$ |$A1$

$\frac{d^3y}{dx^3} = \frac{1458}{(1+9x)^3}$ |$A1$

(b) $f(0) = \ln(1), \quad f'(0) = 9$

$f''(0) = -81, \quad f'''(0) = 1458$

$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0)...$

$= 9x - \frac{81}{2}x^2 + \frac{729}{3}x^3$ |$M1A2$

A46 - ID: 7512 *[9 marks, 11 minutes]*

(a) $\frac{dy}{dx} = \frac{-3\sin x}{3\cos x} = -\tan x$ |$M1$

$\frac{d^2y}{dx^2} = -\sec^2 x$ |$A1$

$\frac{d^3y}{dx^3} = -2\sec x\sec x\tan x = -2\sec^2 x\tan x$ |$M1A1$

(b) $\frac{d^4y}{dx^4} = -2\sec^2 x\sec^2 x + \tan x - 4\sec x\sec x\tan x$ |$M1A1$

$x = 0 \Rightarrow \frac{d^4y}{dx^4} = -2$ |$A1$

(c) $f(0) = \ln(3\cos 0) = \ln(3), \quad f'(0) = -\tan 0 = 0$

$f''(0) = -\sec^2 0 = -1, \quad f'''(0) = 2\sec^2 0\tan 0 = 0$

$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{(4)}(0)...$

$= \ln(3) - \frac{x^2}{2!} - \frac{2x^4}{4!} = \ln(3) - \frac{x^2}{2} - \frac{x^4}{12}$ |$M1A1$

A47 - ID: 7513 *[8 marks, 10 minutes]*

(a) $\sin 5x = 5x - \frac{(5x)^3}{3!} = 5x - \frac{125}{6}x^3$ |$B1$

(b) $\frac{dy}{dx} = \frac{1}{2}(8+e^x)^{-\frac{1}{2}}e^x$ |$M1A1$

$\frac{d^2y}{dx^2} = \frac{1}{2}(8+e^x)^{-\frac{1}{2}}e^x - e^x\frac{1}{4}(8+e^x)^{-\frac{3}{2}}e^x$ |$M1A1$

$\Rightarrow f'(0) = \frac{1}{2}(9)^{-\frac{1}{2}} = \frac{1}{6}$

$\Rightarrow f''(0) = \frac{1}{2}(9)^{-\frac{1}{2}} - \frac{1}{4}(9)^{-\frac{3}{2}} = \frac{1}{6} - \frac{1}{108} = \frac{17}{108}$ |$A1$

(c) $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + ...$

$= 3 + \frac{1}{6}x + \frac{17}{216}x^2$ |$M1A1$

A48 - ID: 7517 *[6 marks, 7 minutes]*

(a) $\frac{dy}{dx} = \frac{2\cos x}{1+2\sin x}$ |$M1A1$

$\frac{d^2y}{dx^2} = \frac{(1+2\sin x)\cdot -2\sin x - 2\cos x \cdot 2\cos x}{(1+2\sin x)^2}$

$= \frac{-2\sin x - 4}{(1+2\sin x)^2}$ |$M1A1$

(b) $f(0) = \ln(1+2\sin 0) = 0, \quad f'(0) = \frac{2\cos 0}{1+2\sin 0} = 2$

$f''(0) = \frac{-2\sin 0 - 4}{(1+2\sin 0)^2} = -4$

$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots = 2x - 2x^2$ |$M1A1$

A49 - ID: 826 *[8 marks, 10 minutes]*

(a) $f(x) = \cos 3x, \quad f(\frac{\pi}{6}) = 0$

$f'(x) = -3\sin 3x, \quad f'(\frac{\pi}{6}) = -3$ |$M1$

$f''(x) = -9\cos 3x, \quad f''(\frac{\pi}{6}) = 0$

$f'''(x) = 27\sin 3x, \quad f'''(\frac{\pi}{6}) = 27$ |$A1$

$f^{(4)}(x) = 81\cos 3x, \quad f^{(4)}(\frac{\pi}{6}) = 0$

$f^{(5)}(x) = -243\sin 3x, \quad f^{(5)}(\frac{\pi}{6}) = -243$ |$A1$

$\Rightarrow \cos 3x = f(\frac{\pi}{6}) + f'(\frac{\pi}{6})(x-\frac{\pi}{6}) + \frac{f''(\frac{\pi}{6})}{2!}(x-\frac{\pi}{6})^2 + \ldots$

$\Rightarrow \cos 3x = -3(x-\frac{\pi}{6}) + \frac{9}{2}(x-\frac{\pi}{6})^3 - \frac{81}{40}(x-\frac{\pi}{6})^5$ |$M1A1$

(b) $x = 1 \Rightarrow \cos 3 = -3(1-\frac{\pi}{6}) + \frac{9}{2}(1-\frac{\pi}{6})^3 - \frac{81}{40}(1-\frac{\pi}{6})^5$ |$B1$

$= -0.992342$ |$M1A1$

A50 - ID: 5056 *[8 marks, 10 minutes]*

$x = 0 \Rightarrow \frac{d^2y}{dx^2} + 4 + 4 = 11$

$\Rightarrow \frac{d^2y}{dx^2} = 3$ |$B1$

$\text{differentiate} \Rightarrow \frac{d^3y}{dx^3} + y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx} =$ |$M2A2$

$x = 0 \Rightarrow \frac{d^3y}{dx^3} + 6 + 4 + 8 = 5$

$\Rightarrow \frac{d^3y}{dx^3} = -13$ |$B1$

$\Rightarrow y = 2 + 2x + \frac{3}{2}x^2 - \frac{13}{6}x^3$ |$M1A1$

A51 - ID: 595 [14 marks, 17 minutes]

(a) $y = r\sin\theta = a(11 + \sqrt{6}\cos\theta)\sin\theta = 11a\sin\theta + a\sqrt{6}\cos\theta\sin\theta$

$\Rightarrow \frac{dy}{d\theta} = 11a\cos\theta + a\sqrt{6}(\cos\theta\cos\theta - \sin\theta\sin\theta)$

$\Rightarrow \frac{dy}{d\theta} = 2\sqrt{6}a\cos^2\theta + 11a\cos\theta - \sqrt{6}a$ |M1A1

$\Rightarrow 2\sqrt{6}\cos^2\theta + 11\cos\theta - \sqrt{6} = 0$

$\Rightarrow \cos\theta = \frac{-11\pm\sqrt{121+48}}{4\sqrt{6}} = \frac{2}{4\sqrt{6}}$ |M1A1

$\Rightarrow \theta = \pm 1.365$ |A1

$\Rightarrow r = 11.5a$ |A1

(b) $2r\sin\theta = 28 \Rightarrow a = \frac{28}{23\sin\theta} = 1.244$ |M1A1

(c) Area $= \frac{1}{2}\int_0^{2\pi} r^2\,d\theta = \frac{1}{2}\int_0^{2\pi}(a(11 + \sqrt{6}\cos\theta))^2\,d\theta$

$= \frac{1}{2}\int_0^{2\pi} a^2(121 + 22\sqrt{6}\cos\theta + 6\cos^2\theta)\,d\theta$ |B1

$= \frac{1}{2}\left[a^2(121\theta + 22\sqrt{6}\sin\theta + 6(\frac{\sin 2\theta}{4} + \frac{\theta}{2}))\right]_0^{2\pi}$ |M1A1

$= \frac{1}{2}a^2(242\pi + 6\pi) = 603$ |M1A2

A52 - ID: 353 [14 marks, 17 minutes]

(a) $x = r\cos\theta = 16\sin\theta\cos^3\theta$ |M1

$\frac{dx}{d\theta} = 16\cos^4\theta - 48\sin^2\theta\cos^2\theta = 16\cos^2\theta(\cos^2\theta - 3\sin^2\theta)$ |M1A1

$\frac{dx}{d\theta} = 0 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$ |M1A1

$\Rightarrow r = 16\sin\theta\cos^2\theta = 16\frac{1}{\sqrt{4}}\left(\frac{\sqrt{3}}{\sqrt{4}}\right)^2 = \frac{48}{8}$ |A1

(b) area $= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} r^2\,d\theta = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 256\sin^2\theta\cos^4\theta\,d\theta$

$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 256\cos^2\theta\sin^2\theta\cos^2\theta\,d\theta$

$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 32(\cos 2\theta + 1)\sin^2 2\theta\,d\theta$

$= 16\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\left(\cos 2\theta\sin^2 2\theta + \frac{1 - \cos 4\theta}{2}\right)d\theta$ |M2A1

(c) area $= 16\left[\frac{1}{6}\sin^3 2\theta + \frac{\theta}{2} - \frac{\sin 4\theta}{8}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ |M1A1

$= 16\left[\frac{1}{6} + \frac{\pi}{8}\right] - \left[\frac{\sqrt{3}}{16} + \frac{\pi}{12} - \frac{\sqrt{3}}{16}\right] = 16\left[\frac{1}{6} + \frac{\pi}{24}\right]$ |M1A2

A53 - ID: 4396 [5 marks, 6 minutes]

Area $= \frac{1}{2}\int_0^{\frac{\pi}{6}} r^2\,d\theta = \frac{1}{2}\int_0^{\frac{\pi}{6}}(1 + 6\sec\theta)^2\,d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{6}}(1 + 12\sec\theta + 36\sec^2\theta)\,d\theta$ |M2

$= \frac{1}{2}\left[1\theta + 12\ln(\sec\theta + \tan\theta) + 36\tan\theta\right]_0^{\frac{\pi}{6}}$ |B1M1

$= \frac{1}{2}\left[\frac{1\pi}{6} + 12\ln(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}) + \frac{36}{\sqrt{3}}\right]$

$= \frac{1\pi}{12} + 6\ln(\sqrt{3}) + \frac{18\sqrt{3}}{3}$ |A1

A54 - ID: 4400

[5 marks, 6 minutes]

$$\text{Area} = \frac{1}{2}\int_0^{\frac{\pi}{6}} r^2\, d\theta = \frac{a^2}{2}\int_0^{\pi} (1 - \cos 2\theta)^2\, d\theta$$

$$= \frac{a^2}{2}\int_0^{\pi} (1 - 2\cos 2\theta + \cos^2 2\theta)\ d\theta \qquad |M2$$

$$= \frac{a^2}{2}\int_0^{\pi} (1 - 2\cos 2\theta + \frac{1}{2}(1 + \cos 4\theta)\ d\theta \qquad |B1$$

$$= \frac{a^2}{2}\left[1\theta - 1\sin 2\theta + \frac{1}{2}(\theta + \frac{1}{4}\sin 4\theta)\right]_0^{\pi} \qquad |B3$$

$$= \frac{3a^2\pi}{4} \qquad |A1$$

A55 - ID: 699

[5 marks, 6 minutes]

$$x = r\cos\theta,\ y = r\sin\theta \Rightarrow (x^2 + y^2)^2 = 10(x^2 - y^2)$$

$$\Rightarrow (r^2\cos^2\theta + r^2\sin^2\theta)^2 = 10(r^2\cos^2\theta - r^2\sin^2\theta) \qquad |M1A1$$

$$\Rightarrow r^4(\cos^2\theta + \sin^2\theta)^2 = 10r^2(\cos^2\theta - \sin^2\theta)$$

$$\Rightarrow r^4 = 10r^2(\cos 2\theta) \qquad |M1A1$$

$$\Rightarrow r^2 = 10(\cos 2\theta) \qquad |A1$$

A56 - ID: 4395

[3 marks, 4 minutes]

$$x = r\cos\theta,\ r^2 = x^2 + y^2$$

$$(x - 5)\sqrt{x^2 + y^2} = 9x \Rightarrow (r\cos\theta - 5)r = 9r\cos\theta \qquad |B1M1$$

$$\Rightarrow (r\cos\theta - 5) = 9\cos\theta$$

$$\Rightarrow r\cos\theta = 9\cos\theta + 5$$

$$\Rightarrow r = 9 + 5\sec\theta \qquad |A1$$