

**www.m4ths.com – AS Year 1**  
**Discriminant**

(1) State the conditions for the quadratic equation

$ax^2 + bx + c = 0$  to have:

- (i) 2 distinct real roots.
- (ii) A repeated root (equal).
- (iii) No real roots.

(2) Sketch a graph of the form  $y = ax^2 + bx + c$  given each of the following conditions:

- (i)  $b^2 - 4ac > 0$  and  $a > 0$
- (ii)  $b^2 - 4ac > 0$  and  $a < 0$
- (iii)  $b^2 - 4ac = 0$  and  $a > 0$
- (iv)  $b^2 - 4ac = 0$  and  $a < 0$
- (v)  $b^2 - 4ac < 0$  and  $a > 0$
- (vi)  $b^2 - 4ac < 0$  and  $a < 0$

(3) Calculate the discriminant for the each the questions below and state the number of real roots to each equation:

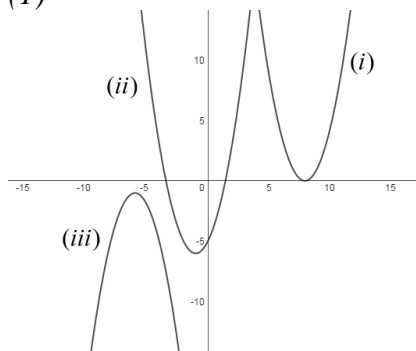
- (a)  $x^2 - 3x + 5 = 0$
- (b)  $2x^2 + 3x - 1 = 0$
- (c)  $-x^2 + 4x - 5 = 0$
- (d)  $0.5x^2 + 2x - 3 = 0$
- (e)  $5x^2 + 3x - 2 = 6$
- (f)  $2(x-1)^2 = x + 4$
- (g)  $x(x-3) = 1 - x$
- (h)  $(x-5)^2 = 0$

(4) In diagrams (1) and (2) the graphs of 6 different quadratic equations in the form

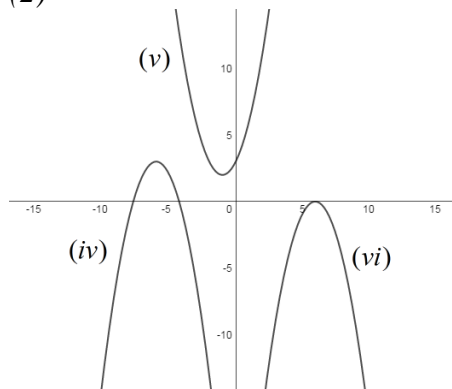
$y = ax^2 + bx + c$  are shown.

- (a) For each graph state whether  $b^2 - 4ac$  is  $0, > 0$  or  $< 0$ .
- (b) For each graph write down whether  $a > 0$  or  $a < 0$ .

(1)



(2)



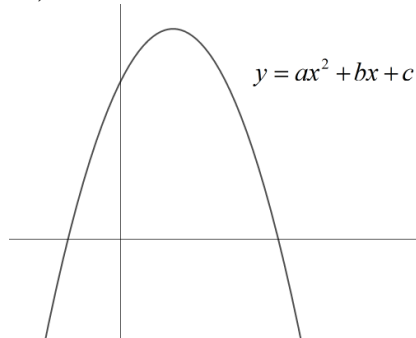
(5) The equation  $kx^2 - 2kx + 1 = 0$ ,  $k > 0$  where  $k$  is a constant has equal roots. Find the value of  $k$ .

(6) The equation  $x^2 - 3px + 1 = 0$  where  $p$  is a constant has no real roots. Find the possible values for the constant  $p$ .

(7) The equation  $x^2 + 2qx + q = -12$  where  $q$  is a constant has two distinct real roots. Find the possible values of the constant  $q$ .

(8) Sketch a graph with equation  $y = ax^2 + bx + c$  where  $a < 0$  and  $b^2 - 4ac > 0$ . (You do not have to state any points of intersection with coordinate axis).

(9) The diagram below shows part of the curve  $y = ax^2 + bx + c$  where  $a, b$  and  $c$  are constants.



- (a) State the range of values of the constant  $a$ .
- (b) State the range of values of  $b^2 - 4ac$ .

(10) The line  $y = mx - 3$  is a tangent to the curve  $y = x^2 + 3x + 1$ .

- (a) Find the possible values of  $m$ .
- (b) Given  $m > 0$  find the point where the tangent touches the curve.

(11) The line  $y = c - 2x$  where  $c$  is a constant is a tangent to the curve  $y = \frac{2}{x}$ ,  $x \neq 0$ .

- (a) Sketch the graph of  $y = \frac{2}{x}$  stating the equations of any asymptotes.
- (b) Find the possible values of the constant  $c$ .
- (c) Given  $c < 0$  sketch the graph of  $y = c - 2x$  on a separate set of axis showing any points of intersection with the coordinate axis.

(12)  $f(x) = -x^2 + 4x - 7$

- (a) Show the equation  $-x^2 + 4x - 7 = 0$  has no real roots.
- (b) Sketch the graph of  $y = f(x)$  stating the coordinates of the maximum point and the coordinates of the point where the curve cross the coordinate axis.
- (c) Find the values of  $k$  such that  $f(x) + k$  has 2 distinct real roots.

(13) Given the  $x$  axis is a tangent to the curve

$y = 2(x-3)^2 + k$

- (a) Write down the value of  $k$ .
- (b) Sketch the curve showing any points where the curve touches or crosses the coordinate axis. The discriminant of the equation  $2(x-3)^2 + p = 0$  is 128.
- (c) Find the value of  $p$ .