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Core Mathematics C4

Advanced

Practice Paper 2

Time: 1 hour 30 minutes

(1) (a) Express $\dfrac{x-10}{(x-2)x^2}$ in partial fractions. *(5 marks)*

(b) Hence or otherwise show that $\displaystyle\int \frac{x-10}{(x-2)x^2}\,dx = \ln\left(\frac{x}{x-2}\right)^2 - \frac{5}{x} + c$ *(5 marks)*

(2) (a) In the binomial expansion of $(1+ax)^{-\frac{1}{4}}, |x| < \dfrac{1}{a}$ where a is a positive constant the coefficient of the term in x^2 is 10. Find the coefficient of the term in x^3. *(5 marks)*

(b) Hence or otherwise find the term in x^3 in the expansion of:

(i) $(1+2ax)^{-\frac{1}{4}}$ (ii) $(1-ax)^{-\frac{1}{4}}$ *(iii)* $\left(1+\dfrac{ax}{2}\right)^{-\frac{1}{4}}$ *(6 marks)*

State the values of x for which each expansion is valid for.

(3) (a) Show that the parametric equations $x = 3\cos\theta - 2$ and $y = 3\sin\theta + 1$ can be written in cartesian form to produce an equation of a circle. *(4 marks)*

(b) Write down the centre and the radius of the circle. *(2 marks)*

(c) Find an equation of the tangent to the circle at the point where $\theta = \dfrac{\pi}{4}$ *(6 marks)*

(4) Use integration by parts to show that $\displaystyle\int e^x \cos(x)\,dx = \frac{e^x\left(\cos(x)+\sin(x)\right)}{2} + c$ *(7 marks)*

(5) Find the gradient of the curve $x^2e^{3y} - y^2 = \ln(x) - 4$ at the point $(1,0)$. *(6 marks)*

(6) The parallelogram $ABCD$ has vertices $A\begin{pmatrix}1\\0\\4\end{pmatrix}, B\begin{pmatrix}5\\3\\7\end{pmatrix}, C\begin{pmatrix}6\\4\\8\end{pmatrix}$ and $D\begin{pmatrix}2\\p\\q\end{pmatrix}$ where A, B, C & D are position vectors.

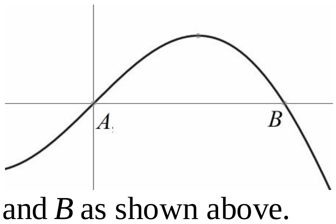
(a) Find the values of p and q. *(2 marks)*

(b) Find the direction vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$. *(2 marks)*

(b) Find the area of the parallelogram $ABCD$ giving your answer to 3 significant figures. *(5 marks)*

(7) Find a general solution to the differential equation $\left(1-\cos^2(x)\right)\dfrac{dy}{dx} = \dfrac{1}{y\left(y^2-3\right)^4}$ *(6 marks)*

(8) Part of the curve $y = x\cos(x)$ is shown below.



The curve crosses the x axis at the points A and B as shown above.

(a) Find the coordinates of A and B giving your answer for the x coordinate of B as a multiple of π. *(3 marks)*

(b) Show that the area trapped between the curve and the x axis from A to B is $\dfrac{\pi}{2} - 1$ *(6 marks)*

(c) The curve $y = x\cos(x)$ has a stationary point P, $A < P < B$. Show the x coordinate of the stationary point P satisfies the equation $x = \cot(x)$ *(5 marks)*

End of Questions