Core Mathematics C3

Advanced

Practice Paper 1

Time: 1 hour 30 minutes

(1) (a) Draw the graphs of $y = 4 - x^2$ and $y = 2x^3$ on the same set of axis. *(3 marks)*

(b) Using your graphs, explain why there is only one solution to the equation $2x^3 + x^2 - 4 = 0$ *(1 mark)*

(c) Show that the solution to the equation $2x^3 + x^2 - 4 = 0$ lies between 1.1 and 1.2 *(2 marks)*

(2) (a) Solve the equation $|2x - 1| = x + 4$ *(3 marks)*

(b) Hence solve the inequality $|2x - 1| > x + 4$ *(2 marks)*

(3) Given A is an acute angle and B is an obtuse angle and $\cos A = \dfrac{3}{5}$ and $\sin B = \dfrac{5}{13}$

(a) Find the value of $\sin 2A$ *(2 marks)*

(b) Find the value of $\tan B$ *(1 marks)*

(c) Find the value of $\sec^2 2A$ *(3 marks)*

(4) The population of a small island is modelled by the equation $P = P_o e^{kt}$ where k is a positive constant and t is the time and is measured in years.

(a) State whether the population is increasing or decreasing giving a reason for your answer. *(1 marks)*

(b) Given the initial population is 4000 and after 5 years the population is 6300 write down the value of P_o and find the value of k to 3 significant figures. *(4 marks)*

(c) Find the rate of change of the population after 9 years giving your answer to 3 significant figures. *(3 marks)*

(d) Draw the graph of $\dfrac{dP}{dt}$ showing any points where the curve meets the coordinate axis. *(3 marks)*

(5) Find the first positive value of x in radians for which $f(x) = 3e^x \sin(2x)$ is stationary. *(7 marks)*

(6) (a) Simplify fully $\dfrac{x^2 + x - 12}{2x^2 - 7x + 3}$ *(3 marks)*

(b) Hence solve the equation $\ln(x^2 + x - 12) = 4 + \ln\left(2x^2 - 7x + 3\right)$ giving your answer in terms of e. *(4 marks)*

(7) $f(x)$ is a linear function is defined for all values of x. $y = f(x)$ passes through the points $A(1, 2)$ and $B(5, 10)$.

(a) Find the inverse function $f^{-1}(x)$ and state its range. *(3 marks)*

(b) Given $g(x) = (2x + 1)^5$ solve the equation $gf(x) = 32$ *(3 marks)*

(c) Find $\dfrac{d}{dx} gf(x)$ *(3 marks)*

(8) (a) Show that $3\cot(x) + 3\tan(x) \equiv p\,\mathrm{cosec}(qx)$ stating the values of p and q. *(4 marks)*

(b) Hence or otherwise solve the equation $\cot(x) + \tan(x) = 4$, $0 \le x \le \pi$ giving your answers in terms of π. *(5 marks)*

(9) (a) $f(x) = e^x$. Draw the graph of $y = f(x)$ showing any points of intersection with the coordinate axis and stating the equations of any asymptotes. *(3 marks)*

(b) Draw the graph of $f(x) + k$, $-1 < k < 0$ showing any points of intersection with the coordinate axis and stating the equations of any asymptotes. *(3 marks)*

(c)The graph of $y = f(x - 1) - 3$ crosses the x axis at the point $A(p, 0)$, Find the exact value of p. *(3 marks)*

(10) Show the curve $y = \dfrac{xe^x}{x + 1}$ has no stationary points. *(7 marks)*

End of Questions