C3 help sheet on Trigonometry

$$\sec(x)=\frac{1}{\cos(x)}$$

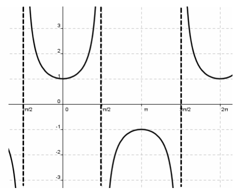
$$Domain: x\in\Re$$

$$x\neq(2n+1)\frac{\pi}{2}$$

$$Range: y\le -1$$

$$and\ \ y\ge 1$$

Where cos(x) = 0 draw an asymptote! 1/0 is undefined



$$\text{cosec}(x)=\frac{1}{\sin(x)}$$

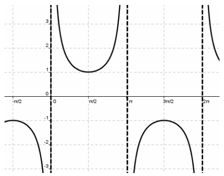
$$Domain: x\in\Re$$

$$x\neq(2n+1)\frac{\pi}{2}$$

$$Range: y\le -1$$

$$and\ \ y\ge 1$$

Where sin(x) = 0 draw an asymptote! 1/0 is undefined



$$\cot(x)=\frac{1}{\tan(x)}$$

or

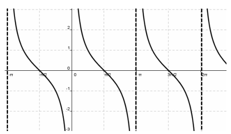
$$\cot(x)=\frac{\cos(x)}{\sin(x)}$$

$$Domain: x\in\Re$$

$$x\neq n\pi$$

$$Range: y\in\Re$$

Where sin(x) and tan(x) = 0 draw an asymptote!



$$\cos^2(x)+\sin^2(x)\equiv 1$$

$$\text{cosec}^2(x)\equiv\cot^2(x)+1$$

$$\sec^2(x)\equiv\tan^2(x)+1$$

These are reciprocal trig identities. You can derive the 2nd two by dividing through by $\sin^2 x$ or by $\cos^2 x$

$a\cos(bx+c)+d$ Graph transformations!

Be careful with multiple transformations in the same direction. Carry out 'a' followed by 'd' with vertical transformations and 'c' followed by 'b' with horizontal!

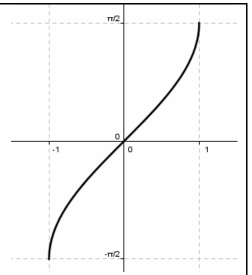
$$y=arc\sin(x)$$

$$Domain: -1\le x\le 1$$

$$Range: \frac{-\pi}{2}\le y\le\frac{\pi}{2}$$

This is the inverse of sin. To have an inverse you must restrict the domain to make it a 1-2-1

Reflect sin(x) in y = x



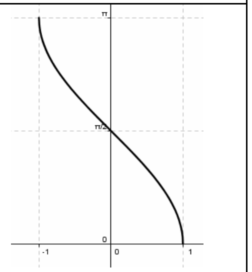
$$y=arc\cos(x)$$

$$Domain: -1\le x\le 1$$

$$Range: 0\le y\le\pi$$

This is the inverse of cos. To have an inverse you must restrict the domain to make it a 1-2-1

Reflect cos(x) in the line y=x

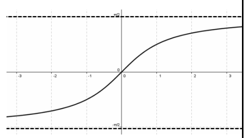


$$y=arc\tan(x)$$

$$Domain: x\in\Re$$

$$Range: \frac{-\pi}{2}< y<\frac{\pi}{2}$$

This is the inverse of tan. Be careful with the asymptotes!



The addition formulae below are given in the exam.

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

$$\sin(A-B)=\sin A\cos B-\cos A\sin B$$

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

The Double angle identity for sin. This can be derived from the addition formulae but are given in the formula book.

$$\cos 2A=\cos^2 A-\sin^2 A$$

$$\cos 2A=2\cos^2 A-1$$

$$\cos 2A=1-2\sin^2 A$$

The double angle for cos. This too can be derived from the addition formulae. The second two versions come from swapping the sin or cosine in the previous one.

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

The double angle identity for tan(x). Look out for equations or identities with cot(2x) in them!

$$\sin A=2\sin\frac{A}{2}\cos\frac{A}{2}$$

This is the half angle for sin. It can be derived from the double angle by swapping A for ½ A

$$\cos A=\cos^2\frac{A}{2}-\sin^2\frac{A}{2}$$

$$\cos A=2\cos^2\frac{A}{2}-1$$

$$\cos A=1-2\sin^2\frac{A}{2}$$

Half angle identity for cos. The same applies for tan(x)

$$\tan A=\frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$$

Remember! If you are solving equations with ½x or 2x you will need to consider the interval you are solving for.

$$\sin P+\sin Q=2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

$$\sin P-\sin Q=2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

$$\cos P+\cos Q=2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

$$\cos P-\cos Q=-2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

These formulae are given in the exam but can be derived from the addition formulae. Examples of equations related to these identities could be an equation such as $\sin 6x+\sin 4x=0$ for example.

The form $R\sin(x\pm\alpha)$ and $R\cos(x\pm\alpha)$

$$R>0,\ \alpha<\frac{\pi}{2}$$

$$R=\sqrt{a^2+b^2}$$

$$\alpha=\tan^{-1}\left(\frac{a}{b}\right)\ \text{or}\ \tan^{-1}\left(\frac{b}{a}\right)$$

R gives the stretch in the y direction

a gives the horizontal shift

Use the addition formulae to expand either form (which will be given) and equate the coefficients.

Equations in the form $a\cos(x)+b\sin(x)$ will be solved using this method.

Often sketching the new graph can help solve problems. The value of R, in general, will give the maximum/minimum value and the phase shift will tell you how far the maximum point has move horizontally. Sketch it!

cos(x) is an even function which means:

$$\cos(-x)=\cos(x)$$

Both sin(x) and tan(x) are odd functions which means:

$$\sin(-x)=-\sin(x)$$

$$\tan(-x)=-\tan(x)$$

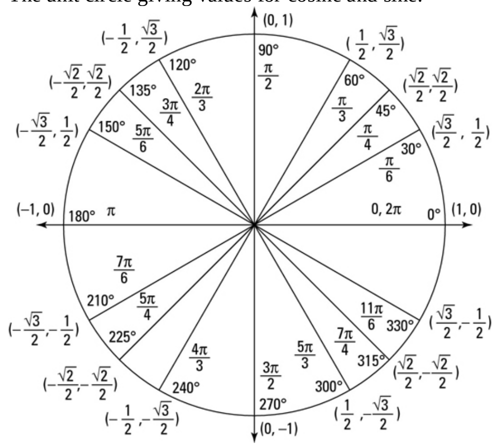
This also holds true for the corresponding reciprocal functions.

Trig Values from 0-90º/π/2
(invert the fraction for reciprocal functions)

x	$\cos(x)$	$\sin(x)$	$\tan(x)$
0	1	0	0
$\frac{\pi}{6}/30^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}/45^\circ$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}/60^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{2}/90^\circ$	0	1	∞^+

The same rules apply for positive and negative values:
All values are positive in the 1st quadrant
sin(x) and cosec(x) are positive in the 2nd
tan(x) and cot(x) are positive in the 3rd
cos(x) and sec(x) are positive in the 4th

The unit circle giving values for cosine and sine.



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I do not own the rights to the unit circle.