

Core Mathematics C4

Advanced

Practice Paper 3

Time: 1 hour 30 minutes

- (1) Find an expression for $\frac{dy}{dx}$ given $\frac{\cos(y)}{x} - \ln(x) = 3y - 1$ (6 marks)
- (2) (a) Find the values of A and B such that $\frac{x^2 + x + 7}{(1-x)(2+x)^2} \equiv \frac{A}{(1-x)} + \frac{B}{(2+x)^2}$ (4 marks)
- (b) Hence or otherwise find the binomial expansion of $\frac{x^2 + x + 7}{(1-x)(2+x)^2}$, up to an including the term in x^2 stating the set of values of x for which expansion is valid. (6 marks)
- (3) (a) Find $\int 3^x dx$ (2 marks)
- (b) Solve the differential equation $3^{-x} \frac{dy}{dx} = \cos^2(y)$ given the point $\left(0, \frac{\pi}{4}\right)$ lies on the curve. (6 marks)
- (4) A curve has parametric equations $x = t^2$ and $y = 2t - 1, t \in \mathfrak{R}$.
- (a) The curve cross the x axis at the point $A(p, 0)$ and touches the y axis at the point $B(0, q)$. Find the values of p and q . (4 marks)
- (b) Show that the cartesian equation of the curve can be written in the form $x = f(y)$. (3 marks)
- (c) Find an equation of the normal to the curve at the point C where $t = 1$. (5 marks)
- (d) The normal intersects the curve again at the point $D(r, s)$. Find the value of r and s . (5 marks)
- (5) (a) The curve $y = x^2\sqrt{x+1}$ crosses the x axis at the points A and B where $B > A$. Find the coordinates of A and B . (3 marks)
- (b) Use the trapezium rule with 3 strips to find an approximate value for the area trapped between the curve $y = x^2\sqrt{x+1}$ and the x axis from A to B giving your answer to 3 decimal places. (4 marks)
- (6) The curve $y = xe^{\frac{1}{2}x}$ is rotated 2π radians about the x axis from the point $x = 0$ to $x = 1$. Show the exact volume of the solid generated is $\pi(e - 2)$. (8 marks)
- (7) Using the substitution $u^2 = 1 + x^5$ find $\int x^4\sqrt{1+x^5} dx$. (6 marks)
- (8) Line l_1 has equation $r_1 = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ and line l_2 has equation $r_2 = \begin{pmatrix} 14 \\ -8 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$.
- (a) Show lines l_1 and l_2 are perpendicular. (2 marks)
- (b) The lines intersect at the point P . Find the coordinates of P . (4 marks)
- (c) Show the point Q with coordinates $(4, -3, 5)$ lies on l_1 and the point $R (11, -7, 8)$ lies on l_2 . (2 marks)
- (d) Show that the area of the triangle $QPR = \sqrt{330}$ (5 marks)

End of Questions