

Core Mathematics C4

Advanced

Practice Paper 1

Time: 1 hour 30 minutes

(1) (a) Express $\frac{2x^2 - 2x + 3}{(1 + 2x)(1 - x)^2}$ in partial fractions. (5 marks)

(b) Hence find the integral of $\frac{4x^2 - 4x + 6}{(1 + 2x)(1 - x)^2}$ (5 marks)

(2) (a) Find an equation of the normal to the curve with parametric equations $x = 3t^2$ and $y = (4t - 1)^3$ at the point where $t = 1$. (7 marks)

(b) Show the equation of the curve can be written in the form $y = f(x)$ (2 marks)

(c) Find where the curve crosses the coordinate axis. (3 marks)

(3) Using the binomial expansion $(1 - 2x)^{0.5}$ with a suitable value of x find an approximation for $\frac{7\sqrt{2}}{10}$ giving your answer to 4 decimal places. (7 marks)

(4) The population of an island is increasing proportionally to the square of the population. Write a differential equation to model this situation. (You do not have to solve the equation). (3 marks)

(5) (a) The lines l_1 with equation $r_1 = \begin{pmatrix} 6 \\ -2 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix}$ and l_2 with equation $r_2 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 5 \\ -8 \end{pmatrix}$ intersect at the point Q . Find the coordinates of point Q . (4 marks)

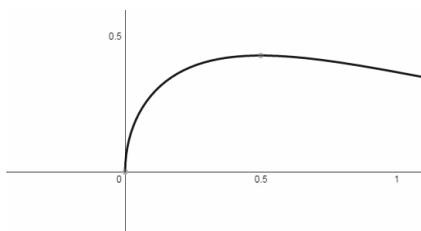
(b) Show the point $P(7, -7, 15)$ lies on l_1 and the point $R(-2, -2, 10)$ lies on l_2 . (2 marks)

(c) Hence find the area of the triangle PQR . (6 marks)

(6) Solve the differential equation $(1 + x^2) \frac{dy}{dx} = 2xe^{3y}$ given the point $(0, 1)$ lies on the curve. (7 marks)

(7) Express $\frac{dy}{dx}$ in terms of x and y given $2xy + \cos(2x + 1) = 3\sin(4y) - 7$ (6 marks)

(8) Part of the curve $y = \frac{\sqrt{x}}{e^x}$ is shown below.



The curve is rotated 360° about the x axis from the point $x = 0$ to the point $x = 1$. Find the exact volume of the solid generated. (7 marks)

(9) (a) Using the trapezium rule with 3 strips find an approximation to the area trapped under the curve $y = \cos^2(x)$ from $x = 0$ to $x = \frac{\pi}{2}$ (4 marks)

(b) Using integration find the exact area trapped under the curve $y = \cos^2(x)$ from $x = 0$ to $x = \frac{\pi}{2}$ (5 marks)

(c) Find the percentage error between the approximation in part (a) and your answer in part (b). (2 marks)

End of Questions