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Core Mathematics C3

Advanced

Practice Paper 3

Time: 1 hour 30 minutes

(1) (a) Simplify $\dfrac{2x^2+x-3}{x^2-1}$ *(3 marks)*

(b) Hence or otherwise solve the equation $\ln(2x^2+x-3)-\ln(x^2-1)=5$. *(4 marks)*

(2) (a) Sketch the graph of $y=\mathrm{cosec}(x)$, $0^\circ \le x \le 360^\circ$ stating the equation of any asymptotes. *(4 marks)*

(b) The graph of the $y=a\,\mathrm{cosec}(x-b)$, where x is measured in degrees, has a minimum point with coordinates $(120^\circ, 2)$ and a maximum point with coordinates $(300^\circ, -2)$. Find the values of a and b. *(4 marks)*

(c) Using your answers from part (a) solve the equation $a\,\mathrm{cosec}(x-b)=4$ for $0^\circ \le x \le 360^\circ$ *(4 marks)*

(3) (a) Show that the equation $x^2e^x-1=0$ can be written as $x=\left(\dfrac{1}{e^x}\right)^{\frac{1}{2}}$. *(2 marks)*

(b) Use the iterative formula $x_{n+1}=\left(\dfrac{1}{e^{x_n}}\right)^{\frac{1}{2}}$, $x_0=0.6$ to find $x_{1,}\, x_2$ and x_3 giving each answer to 3 decimal places.

(3 marks)

(4) (a) $\mathrm{f}(x)=e^{x-2}$. Sketch the graph of $y=\mathrm{f}(x)$ showing the point of intersection with the y axis. *(3 marks)*

(b) State the range of the function $\mathrm{f}(x)$. *(1 marks)*

(c) Find $\mathrm{f}^{-1}(x)$ and state its domain. *(3 marks)*

(d) Sketch the graph of $y=\mathrm{f}^{-1}(x)$ and $y=\mathrm{f}(x)$ on the same set of axis and state the number of solutions to the equation $\mathrm{f}^{-1}(x)=\mathrm{f}(x)$. *(3 marks)*

(5) $\mathrm{f}(x)=3x^2,\ x>0$ and $\mathrm{g}(x)=\sqrt{x},\ x>0$.

(a) Find (i) $\mathrm{f}(5)$ and (ii) $\mathrm{g}(16)$. *(2 marks)*

(b) $\mathrm{gf}^{-1}(48)$. *(3 marks)*

(c) Solve the equation $\mathrm{fg}(x)=12$. *(3 marks)*

(6) (a) Sketch the graph of $y=\left|\dfrac{2}{x}\right|$ and $y=x+1$ on the same set of axis showing any points where the graphs cross the coordinate axis. *(4 marks)*

(b) Find the solution to the equation $\left|\dfrac{2}{x}\right|=x+1$. *(5 marks)*

(c) Hence or otherwise solve the inequality $\left|\dfrac{2}{x}\right|<x+1$. *(2 marks)*

(7) $\mathrm{f}(x)=2x^2\ln(x)$, $x\in\Re, x>0$. Show the coordinates of the stationary point on the curve $y=\mathrm{f}(x)$ can be written as $\dfrac{1}{\sqrt{e}},\dfrac{2}{e}$. *(7 marks)*

(8) (a) Show that $\sin\left(x+\dfrac{\pi}{4}\right)$ can be written as $\dfrac{1}{\sqrt{2}}(\sin(x)+\cos(x))$. *(4 marks)*

(b) Hence or otherwise solve the equation $\dfrac{\sqrt{3}}{2}=\dfrac{1}{\sqrt{2}}(\sin(x)+\cos(x))$, $0\le x\le 2\pi$, giving your answers as multiples of π. *(4 marks)*

(9) Find the equation of the tangent to the curve $x=4\sin(2y)+\dfrac{\pi}{2}$ at the point where $y=\dfrac{\pi}{2}$ in the form $ax+by+c=0$, where a and b are integers and c is a multiple of π. *(7 marks)*

End of Questions