www.m4ths.com

Core Mathematics C3

Advanced

Practice Paper 2

Time: 1 hour 30 minutes

(1) Solve the equation $\tan(2x-15°) = \tan(15°)$ for $-180° \leq x \leq 180°$ *(5 marks)*

(2) (a) Show that $\dfrac{1}{2x+1} + \dfrac{1}{2x-1} - \dfrac{1}{x}$ can be written as $\dfrac{a}{4x^3 - x}$ where a is an integer to be found. *(5 marks)*

(b) Given $f(x) = \dfrac{1}{2x+1} + \dfrac{1}{2x-1} - \dfrac{1}{x}$ using your answer to part (a) or otherwise find $f'(x)$. *(4 marks)*

(3) Given $x = 3\sec^2(y)$ find $\dfrac{dy}{dx}$ in terms of x. *(6 marks)*

(4) $f(x) = x^2 - 6$, $x \geq 0$

(a) Find the inverse function $f^{-1}(x)$ stating its range. *(3 marks)*

(b) Sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axis showing any points of intersection with the coordinate axis. *(4 marks)*

(c) Solve the equation $f(x) = f^{-1}(x)$ *(3 marks)*

(5) Given that $f(x) = 3e^x$ and $g(x) = 2\ln(x)$, $x > 0$

(a) Show that $gf(x) = \ln 9 + 2x$ *(3 marks)*

(b) Find a simplified expression for $fg(x)$. *(3 marks)*

(c) By sketching two different graphs on the same set of axis state the number of solutions to the equation $f(x) = g(x)$ *(3 marks)*

(6) Find an equation of the normal to the curve $y = \dfrac{x^2}{\ln(x)}$ at the point where $x = e$. *(7 marks)*

(7) Express $3\cos(x) + 4\sin(x)$ in the form $R\cos(x-\alpha)$, $R > 0$, $0° < \alpha < 90°$ giving your answer for α to 3 significant figures. *(3 marks)*

(a) Hence find the maximum and minimum vale of the function $f(x) = \dfrac{1}{3\cos(x) + 4\sin(x)} + 3$ *(3 marks)*

(b) A wave machine is modelled by the equation $h = 3\cos\left(2t - 30°\right) + 4\sin\left(2t - 30°\right) - 1$ where h is the height in meters of the wave above its starting position and t is the time in minutes after the machine is turned on. Find the first time the wave machine reaches its maximum height and state height of the wave at this time. *(6 marks)*

(8) (a) Show the equation $f(x) = x^3 - \dfrac{1}{x} - 6$ has a root α such that $1 < \alpha < 2$. *(2 marks)*

(b) Show that the equation $x^3 - \dfrac{1}{x} - 6 = 0$ can be written in the form $x = \left(6x + 1\right)^{\frac{1}{4}}$ *(2 marks)*

(c) Using the iterative formula $x_{n+1} = \left(6x_n + 1\right)^{\frac{1}{4}}$, $x_1 = 1$ to find the value of x_2, x_3 and x_4 giving your answers to 3 decimal places. *(3 marks)*

(9) The amount of a drug present in a patient's body after ingestion is modelled by the equation $M = 300e^{-kt}$ where M is the mass in mcg (micrograms) of the drug and t is the time in hours after ingestion. k is a positive constant.

(a) Write down the initial mass present in the patient's body at the time of ingestion. *(1 marks)*

(b) 5 hours after ingestion there is 200mcg of the drug present in the patient's body. Find the value of k giving your answer to 3 significant figures. *(4 marks)*

(c) Find the time taken for there to be less than 150mcg of the drug present in the patient's body giving your answer to the nearest minute. *(3 marks)*

(d) Sketch a graph showing the amount of the drug present in the patient's body for $0 \leq t \leq 6$. *(2 marks)*

End of Questions