

**www.m4ths.com – C2 –
Logarithms and Exponentials**

(1) Write the following in the form $\log_a b = c$:

- (a) $2^3 = 8$
- (b) $7^2 = 49$
- (c) $4^2 = 16$
- (d) $3^{-2} = \frac{1}{9}$
- (e) $6^0 = 1$
- (f) $10^{-3} = 0.001$
- (g) $p^q = r$

(2) Write the following in the form $a^b = c$:

- (a) $\log_5 25 = 2$
- (b) $\log_2 16 = 4$
- (c) $\log \frac{1}{10} = -1$
- (d) $\log_t s = r$

(3) Evaluate the following without a calculator:

- (a) $\log_2 32$
- (b) $\log_5 5$
- (c) $\log_8 1$
- (d) $\log 1000$
- (e) $\log_9 27$
- (f) $\log_4 \frac{1}{64}$
- (g) $\log_8 \frac{1}{16}$
- (h) $\left(\log_{27} \frac{1}{81}\right)^2$
- (i) $\log_{0.5} \sqrt{16}$

(4) Find the value of x in the following. Give your answers to 3 S.F where appropriate:

- (a) $\log_3 16 = x$
- (b) $\log_6 x = -2$
- (c) $\log_4 9 = x$
- (d) $\log_{\frac{1}{3}} x = -2$
- (e) $\log_{2.4} 13 = x$
- (f) $\log_4 x = -0.17$
- (g) $\log_x 16 = 2$

(5) Simplify the following:

- (a) $\log_p p^2$
- (b) $3\log_r r^5$
- (c) $(\log_2 8) \times \left(\log_p \frac{1}{p}\right)$

(6) Write the following in the form $a \log_b c$:

- (a) $\log_b c^4$
- (b) $3\log_b c^3$
- (c) $5\log_b \frac{1}{c}$
- (d) $0.25\log_b \sqrt{c}$

(7) Write the following in the form $\log_b c^a$:

- (a) $2\log_b c^5$
- (b) $4\log_b \sqrt[3]{c}$
- (c) $-2\log_b \frac{1}{c^3}$
- (d) $0.75\log_b c^{0.25}$

(8) Rewrite the following as single logarithms:

- (a) $\log 2 + \log 3$
- (b) $\log_b a + \log_b c^2$
- (c) $2\log_b p + \log_b 5c$
- (d) $\log 5 - \log 2$
- (e) $2\log 3 - 5\log 2$
- (f) $3\log_b p - 2\log_b r$
- (g) $\log 3 + \log \frac{1}{9}$
- (h) $2\log a + 5\log b - \log \sqrt{c}$
- (i) $0.5\log_8 x - \log_8 3y + \log_8 \sqrt{x}$

(9) Express the following in the form $\log a + \log b$:

- (a) $\log p^2 q$
- (b) $\log 2x^3$
- (c) $\log \frac{p}{r^4}$
- (d) $3\log p\sqrt{q}$
- (e) $-\log \frac{\sqrt[4]{p}}{q^{0.4}}$

(10) Express the following in the form $a \log x + b \log y$:

- (a) $\log y^2 x^3$
- (b) $\log \frac{\sqrt{x}}{y^5}$
- (c) $3\log \left(\frac{y^{\frac{1}{6}}}{\sqrt[3]{x}}\right)$

(11) Simplify the following:

- (a) $\log 8 + \log 12.5$
- (b) $\log_5 100 - \log_5 4$
- (c) $\log_6 2 + \log_6 108 + 2\log_6 6$
- (d) $2\log_6 2 + \log_6 9$
- (e) $\log_2 80 - \log_2 5 + 3\log_2 32$

(12) Given $\log_2 p = a$ and $\log_2 q = b$, simplify the following giving your answers in terms of a and b :

- (a) $\log_2 pq^2$
- (b) $\log_2 \frac{8q}{p}$
- (c) $0.5\log_2 \sqrt{32p^3q^4}$

(13) Solve the following giving your answers to 3 S.F:

- (a) $3^x = 14$
- (b) $5^{x-1} = 9.4$
- (c) $2 \times 6^{2x+3} = 3.4$
- (d) $2^{1-3x} + 3.1 = 9.7$

(14) Solve the following giving your answers to 3 S.F:

- (a) $3^{x-1} = 2^{x+2}$
- (b) $5^{2x-3} = 7^{x+1}$
- (c) $7 \times 5^{2x-3} = 7^{x+1}$
- (d) $10 \times 7^{x-3} = 9^{x+1}$

(15) Solve the following equations giving your answers to 3 S.F where appropriate:

- (a) $3^{2x} - 3^x - 2 = 0$
- (b) $2^{2x} = 7(2^x) - 12$
- (c) $6(4^{2x}) + 13(4^x) = 5$
- (d) $2^{2x+1} - 1 = 2^x$

(16) Solve the following equations giving your answers to 3 S.F where appropriate:

(a) $\log_2(x-4) = 3$

(b) $\log_3(2x-1) = \log_3(x+1) + 2$

(c) $\log_2(x) = 4 - \log_2(x+6)$

(d) $\log_4(x-1) = 1.7 - \log_4(x+2)$

(e) $2\log_5(x+1) = \log_5(x+2) + 1.9$

(f) $2\log_2(x-3) = \frac{3}{\log_2(x-3)}$

(17) Solve the following equation giving your answers to 3 S.F where appropriate:

$$\log_2(2x-1) = \log_4(x+3) + 0.5$$

(18) Solve the simultaneous equations:

$$\log_2\left(\frac{x}{y^2}\right) = -3$$

$$3\log_8(4x\sqrt{y}) = 4$$

(19) Sketch the following graphs stating the coordinates of any points of intersection with the coordinate axis and the equations of any asymptotes:

(a) $y = 2^x$

(b) $y = 5^x$

(c) $y = \left(\frac{1}{2}\right)^x$

(d) $y = 3^{x-1}$

(e) $y = 4^x + 2$

(f) $y = 1 - 2^x$

(20) Given that $\log_5 p = a$ and $\log_5 q = b$, find an expression in terms of a and b for:

$$2\log_5\left(\frac{p^3}{25\sqrt{q}}\right)$$

(21) Show there is only one real solution to the equation $\log_4(x+5) = 1.5 - \log_4(x-2)$ and find the solution to the equation.

(22) (a) Sketch the graphs of $y = 2^{x-1}$ and $y = 0.5^x$ on the same set of axis showing any points of intersection with the coordinate axis and state the equation of any asymptotes.

(b) Solve the equation $2^{x-1} = 0.5^x$

(c) State fully the two transformations that map the curve $y = 0.5^x$ onto the curve $y = 3 - 0.5^x$.