www.m4ths.com – C2 – Integration

(1) Evaluate the following integrals giving your answers to 3 significant figures where appropriate:

(a) $\int_1^4 (2x+1)\,dx$

(b) $\int_{-2}^0 (x^2+3)\,dx$

(c) $\int_2^4 \left(3-\frac{1}{x^2}\right)dx$

(d) $\int_1^8 \left(3x-\frac{1}{2x^2}-\sqrt{x}\right)dx$

(e) $\int_2^4 \left(x^{-2}-3x^{-4}\right)dx$

(f) $\int_3^8 \left(x^{\frac{1}{2}}-2x^{-\frac{3}{2}}\right)dx$

(g) $\int_{0.5}^{1.5} \left(\sqrt{x}-2\right)\left(\sqrt{x}+3\right)dx$

(2) Find the value of the following definite integrals giving your answers to 3 significant figures:

(a) $\int_2^4 \left(\frac{2x^3-3x}{\sqrt{x}}\right)dx$

(b) $\int_3^5 \left(\frac{3x^4-x+4}{x^{\frac{1}{3}}}\right)dx$

(3) Find the area enclosed by the x axis and the curve with equation $y = x(3-x)$ giving your answer as an exact fraction.

(4) Find the area enclosed by the by the curve with equation $y=\sqrt{x}$, the x axis and the lines $x=2$ and $x=4$ giving your answer to 3 significant figures.

(5) (a) Find the area enclosed by the x axis and the curve with equation $y = 2x - x^{1.5}$.

(b) Find the area trapped between the curve, the x axis and the lines $x=1$ and $x=4$.

(6) (a) Show that the equation $y = x^3 - x^2 - 6x$ can be written in the form $y = x(x+2)(x-3)$.

(b) Sketch the curve $y = x^3 - x^2 - 6x$ showing any points of intersection with the coordinate axis.

(c) Find the area trapped between the x axis and the curve with equation $y = x^3 - x^2 - 6x$.

(7) Find the area enclosed by the by the curve with equation $y=(x-1)(x-3)$, the x axis and the lines $x=0$ and $x=2$.

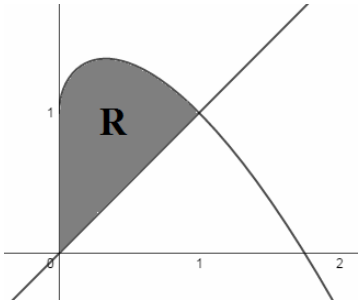
(8) (a) Sketch the curve $y = -x^2 + 7x - 6$ and the line $y = x+2$ on the same set of axis showing any points of intersection with the coordinate axis.

(b) Find the coordinates of the points where the line cuts the curve.

(c) Find the area trapped between the curve $y = -x^2 + 7x - 6$ and the line $y = x+2$ giving your answer as an exact fraction.

(9) Find the area trapped between the curve $y = x^2 + 2x + 5$ and the line $y = 5 - x$ giving your answer as an exact fraction.

(10) The diagram below shows a sketch of the line $y = x$ and the curve $y = -x^{1.5} + x^{0.5} + 1,\ x \geq 0$



(a) The coordinates of the point the where the line and the curve meet are $(1, p)$. Write down the value of p.

(b) The shaded region R is bounded by the line and the curve. Find the area of R.

(11) Given that $\int_1^k (3x-1)(x+1)\,dx = 9$, find the value of the constant k.

(12) The line $y=5$ meets the curve $y = -4x + x^2,\ x \leq 0$ at the point $P(a,b)$.

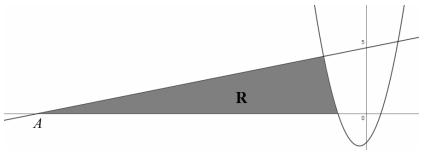
(a) Find the values of a and b.

(b) Find the area trapped between the curve, the line and the y axis to 2 decimal places.

(13) The normal to the curve $y=(x-1)(x+2)$ at the point $(-3,4)$ crosses the x axis at the point A shown in the diagram below.

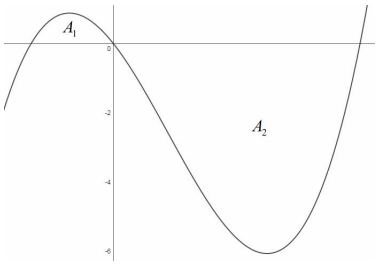
(a) Find the coordinates of A.

(b) Find the area R trapped between the normal, the curve and the x axis.



(14) Part of the curve $y = ax^3 + bx^2 + cx$ is shown below. The solutions to the equation $y=0$ are $x=-1$, $x=0$ and $x=3$.

(a) Find the values of a, b and c.



(b) The areas A_1 and A_2 are the two regions trapped between the curve and the x axis. Find the ratio $A_1 : A_2$