www.m4ths.com - C2 -

## Algebra and Functions

(1) Simplify the following algebraic fractions:
(a) $\frac{x^{2}+2 x}{x}$
(b) $\frac{2 x^{2}-3 x+5}{x}$
(c) $\frac{x^{2}-x-12}{x-4}$
(d) $\frac{2 x^{2}-5 x-12}{2 x+3}$
(e) $\frac{4 x^{2}-25}{2 x-5}$
(f) $\frac{a^{2}-b^{2}}{2(a+b)}$
(2) Explain whether or not you can use long division to simplify the following fractions:
(a) $\frac{x^{2}+2 x+4}{2 x^{2}+3 x+1}$
(b) $\frac{4 x^{3}-2 x^{2}+3}{x+1}$
(c) $\frac{3 x^{2}+x-4}{x^{3}+7 x+4}$
(3) Find the quotient when $x^{3}+2 x^{2}-4 x+1$ is divided by $x-1$.
(4) Find the quotient and remainder when
$x^{4}+3 x^{3}+x^{2}-2 x+1$ is divided by $x-2$.
(5) Find the quotient and remainder when
$2 x^{4}+3 x^{2}+x-3$ is divided by $x+3$.
(6) Simplify $\frac{4 x^{3}-7 x^{2}+2 x+1}{2 x-3}$
(7) Show that $(x+2)$ is a factor of $x^{3}-x^{2}+x+14$
(8) State which of the following are factors of
$2 x^{4}+3 x^{3}-24 x^{2}-13 x+12$ :
(i) $(x-3)$
(ii) $(x-1)$
(iii) $(2 x-1)$
(iv) $(x+4)$
(9) Explain why $(3 x-2)$ is not a factor of $x^{4}+5 x^{2}+2 x-1$.
(10) Given that $(x-2)$ is a factor of $2 x^{3}-x^{2}+2 p+3$ find the value of $p$.
(11) $\mathrm{f}(x)=x^{3}+p x^{2}+q x+6$

Given that $(x-3)$ and $(x+1)$ are factors of $\mathrm{f}(x)$, find the values of $p$ and $q$.
(12) $\mathrm{g}(x)=2 x^{3}-7 x^{2}-10 x+24$

Given that $(x-4)$ is a factor of $\mathrm{g}(x)$, fully factorise $\mathrm{g}(x)$.
(13) Solve the equation $x^{3}+x^{2}-17 x+15=0$.
(14) Find the remainder when $x^{3}+2 x^{2}-4 x+2$ is divided by $(x-1)$.
(15) When $4 x^{3}-p x^{2}+3$ is divided by $(x+1)$ the remainder is 4 . Find the value of $p$.
(16) $\mathrm{f}(x)=2 x^{3}+p x^{2}+x+q$ When $\mathrm{f}(x)$ is divided by $(x+3)$ the remainder is -12 . Given also $(x-1)$ is a factor of $\mathrm{f}(x)$ find the values of $p$ and $q$.
(17) Given when $4 x^{2}-a x+3$ is divided by $(x+1)$ the remainder is the same as when it's divided by $(x-2)$, find the value of the constant $a$.
(18) The graph below shows part of the curve
$y=2 x^{3}+x^{2}-13 x+6$. Given that $A=-3$, find the values of $B \& C$.

(19) $\mathrm{f}(x)=3 x^{3}+4 x^{2}+p x-2$
(a) Given $(x-1)$ is a factor
of $\mathrm{f}(x)$ show that $p=-5$.
(b) Find all of the solutions to the equation $\mathrm{f}(x)=0$.
(20) Given that
$\frac{x^{4}-x^{3}-19 x^{2}-11 x+30}{(x+2)}$
can be written in the form
$\left(A x^{3}+B x^{2}+C x+D\right)$
show that $A+B+C+D=0$.
(21) When $4 x^{3}+a x^{2}+b x-2$ is divided by $(1-2 x)$ the remainder is 6 .
(a) Find a linear relationship between $a$ and $b$.
(b) Given further that $\frac{a}{3}=b$, find the value of $(a b)^{0.5}$ in the form $k \sqrt{3}$ where $k$ is a constant to be found.
(22) Sketch the graph of $y=2 x^{3}-5 x^{2}-x-6$ showing any points of intersection with the coordinate axis.

