www.m4ths.com – C2 – Differentiation

(1) Find the values of x for which each function below is an increasing function:

(a) $f(x) = x^2 - 2x + 4$

(b) $f(x) = \frac{1}{3}x^3 - x^2 - 8x + 9$

(c) $f(x) = 2 - 4x + 6x^2$

(d) $f(x) = \left(x^2 - 8\right)\left(x + 8\right)$

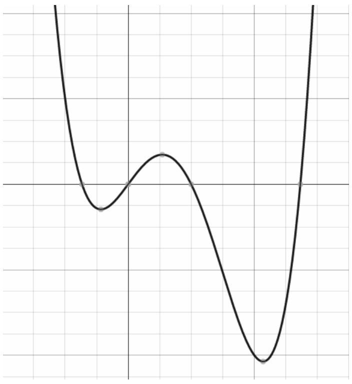
(e) $f(x) = 2x - 1$

(2) Find the values of x for which each function below is decreasing:

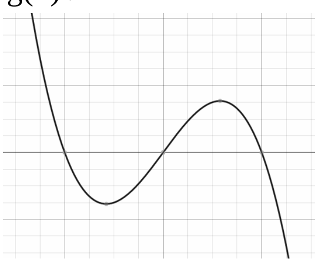
(a) $f(x) = 3x + x^{-1},\ x > 0$

(b) $f(x) = (3x^2 - 8)x$

(3) The graph below shows part of the curve $y = f(x)$. Mark the points $x = a$ and $x = b$, $0 < a < b$, such that $f(x)$ is a decreasing function for $a < x < b$.



(4) Below is part of the graph $y = g(x)$.



(a) Label the stationary points on the curve A and B, $A < B$.

(b) Determine the nature of the stationary points A and B.

(c) Explain why A and B are not points of inflexion.

(d) Draw a tangent to the curve at any point where the function is *(i)* increasing and *(ii)* decreasing.

(5) (a) Use differentiation to show that the coordinates of the stationary point on the curve $y = x^2 - 8x + 6$ are $(4, -10)$

(b) Show that this result is true by completing the square to find the stationary point.

(c) Determine the nature of the stationary point by sketching the graph of $y = x^2 - 8x + 6$.

(d) Show that your answer to part (c) is correct by evaluating the second derivative at $x = 4$.

(6) Find the coordinates of any stationary points for each of the following functions and determine their nature:

(a) $f(x) = x^2 + 4x$

(b) $f(x) = 3x - 0.5x^2$

(c) $f(x) = 2x^3 + 3x^2 - 72x$

(d) $f(x) = x^3 - 3x^2$

(e) $f(x) = 4 - 5x^3$

(f) $f(x) = 16x + x^{-2},\ x \neq 0$

(g) $f(x) = px^2 - qx,\ \ p < 0 < q$

(7) (a) Find the maximum value of the function

$f(x) = \left(1 - 2x\right)\left(3x^2 - 5\right),\ \ x \geq 0$

giving your answer to 3 significant figures.

(b) Prove that the answer found in part (a) is a maximum value by evaluating $f''(x)$.

(c) Is the function increasing or decreasing when $x = 3$?

(8) (a) Sketch the curve of $y = x^3 - 6x^2 + 5x + 12$ showing any points of intersection with the coordinate axes.

(b) Find the x coordinates of the stationary points giving your answers as exact fractions.

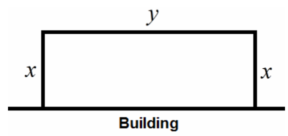
(c) Using the graph, determine the nature of the stationary points.

(d) On the same set of axes draw a different cubic equation that only has one stationary point. Write a possible equation for the curve.

(9) $g(x) = \frac{1}{x} + 16x,\ \ x \neq 0$

Show that the distance between the two stationary points on the curve $y = g(x)$ is $\frac{5\sqrt{41}}{2}$ units.

(10) James is making a pig pen. He has 240m of fencing and would like to make a rectangular shaped pen with the largest possible area. He needs to fence 3 sides of the pen as a wall provides the 4th side.
The longer side of the fence is y meters long and the two shorter sides of the fence are both x meters long as shown below.



(a) Find an expression for y in terms of x

(b) Show that the area of the pen can be written as $A = 240x - 2x^2$

(c) Hence find the maximum area for the pen.

(11) A rectangular sheet of metal measuring $4cm$ by $5cm$ has a square of side length xcm cut from each corner and is made into an open topped tank.

(a) Find the maximum volume of the tank.

(b) Prove that the answer found in part (a) gives a maximum value for the volume of the tank

(12) An open top cylinder with radius rcm and height hcm is being made out of metal must hold exactly $20\pi cm^3$ of liquid.

(a) Find the length of the radius that minimises the amount of metal required to make the cylinder.

(b) Hence find the minimum amount of metal used.

(13) Sally is standing on level ground and throws a small ball into the air. The flight path of the ball is modelled by the equation $h = 1 + 6t - 3t^2$, $t \geq 0$ where h is the height in meters above ground and t is the time in seconds after it's thrown.

(a) Write down the height the ball was released from.

(b) Find the time taken for the ball to hit the ground giving your answer to 3 significant figures.

(c) Find the time taken for the ball to reach its maximum height above the ground.

(d) Find the maximum height the ball reaches above the ground.

(e) Show that the value found in part (d) is a maximum by using differentiation.

(14) $\mathrm{t}(x) = px^4 + qx^3 - 3x^2 + 4$

Given that the curve $y = \mathrm{t}(x)$ has a minimum at the point with coordinates $\left(2, -\frac{4}{3}\right)$, find the value of the constants p and q.