

(1) Find the midpoint of the following points:

- (a) (3, 7) and (1, 3)  
(b) (-2, 3) and (4, -3)  
(c)  $\left(1, \frac{3}{4}\right)$  and  $\left(\frac{7}{4}, -2\right)$

(2) Find the length of line segment  $AB$  for the following:

- (a)  $A(1, 3)$  and  $B(2, 5)$   
(b)  $A(-3, 2)$  and  $B(2, -5)$   
(c)  $A(2, 0.5)$  and  $B(-1, 3)$

(d) Given that the line segment  $AB$  is a diameter of a circle, state the radius of each circle in questions a – c.

(3) The points (2, 1) and (-2, -1) are the end points of a diameter of a circle.

- (a) Find the centre of the circle  
(b) Find the length of the radius

(4) (a) The point  $A\left(1, \frac{5}{4}\right)$  is a point on the circle  $C$ . Given that the centre of the circle has coordinates (1.75, -1), find the coordinates of the point  $B$  such that  $AB$  is a diameter of the circle  $C$ .

(b) Find the length of the diameter  $AB$  to 3 significant figures.

(5) Write down the centre of each circle and the length of its radius:

- (a)  $x^2 + y^2 = 25$   
(b)  $(x-1)^2 + (y+2)^2 = 16$   
(c)  $(x+5)^2 + (y-0.5)^2 = 32$   
(d)  $(y+2)^2 + (x-14)^2 = 27$   
(e)  $(x-3)^2 + y^2 - 0.01 = 0$   
(f)  $(2x-3)^2 + (2y+5)^2 = 36$   
(g)  $(x-p)^2 + (y+q)^2 = r$   
(h)  $(px+4)^2 + (py+2)^2 = p^2$

(6) Find the centre of each circle and the length of its radius where possible:

- (a)  $x^2 + y^2 + 2x - 4y = 20$   
(b)  $x^2 + y^2 - 3x + y = 13.5$   
(c)  $x^2 - \sqrt{3}x + y^2 = 24.25$   
(d)  $x^2 + 2x = 4y + y^2$   
(e)  $x^2 + y^2 + px - 4py = 17p$   
(f)  $2x^2 + 2y^2 + 4x - 3y = 26$   
(g)  $y^2 + (x-3)^2 = 8y$   
(h)  $qx^2 - 6x - 2y + qy^2 = 0$   
(Can you spot the equation that isn't a circle?)

(7) The point  $A(2, 3)$  lies on the circle  $C$ . The centre of the circle has coordinates (8, 1). Find an equation for the circle.

(8) The points  $P(2, 3)$  and  $Q(6, 9)$  lie on the circle  $C$ .

Given  $PQ$  is a diameter of the circle, find an equation of the circle  $C$ .

(9) The circle with equation  $x^2 - 2x - 4y + y^2 = 48$  crosses the  $x$  axis at the points  $A$  and  $B$ .

- (a) Find the area of the triangle  $ABC$  where  $C$  is the centre of the circle.  
(b) The circle crosses the  $y$  axis at the points  $D$  and  $E$ . Find the length of the chord  $DE$  in the form  $p\sqrt{q}$  where  $q$  is a prime number.

(10) A circle has equation  $(x-4)^2 + (y+3)^2 = 20$ . State whether the following points are inside, on the circle or outside the circle:

- (a) (5, 1), (b) (0, 7), (c) (9, -2)

(11) A circle with centre (6, -1) passes through the point  $T(-3, 2)$ . Find an equation for the tangent to the circle at the point  $T$ .

(12) A tangent to the circle  $C$  at the point  $P$  passes through the point  $Q(10, -3)$ . Given that the equation of circle  $C$  is  $(x-2)^2 + (y+1)^2 = 16$ , find the length of the line  $PQ$ .

(13) A circle has equation  $(x-3)^2 + (y-5)^2 = 100$ .

(a) Show that the points  $P(11, -1)$  and  $Q(-3, -3)$  lie on the circle.

The line  $l$  is the perpendicular bisector of the chord  $PQ$ .

(b) Show that line  $l$  passes through the centre of the circle.

(14) The points  $A(-2, 12)$ ,  $B(-5, 11)$  and  $C(3, -3)$  lie on a circle. Find an equation of the circle.

(15) The points  $A(0, 4)$ ,  $B(-3, -5)$  and  $C(6, -8)$  lie on a circle.

- (a) Prove that  $AC$  is a diameter of the circle.  
(b) Find an equation of the circle.

(16) A circle has equation  $(x+4)^2 + (y-7)^2 = 90$ .

(a) Write down the coordinates of the centre and the length of the radius.

The points  $A(-13, 10)$  and  $B(-7, -2)$  lie on the circle.

(b) Find the area of the triangle  $ABC$  where  $C$  is the centre of the circle.

(17) A circle touches the  $y$  axis at the point (0, -8) and crosses the  $x$  axis at the points (-4, 0) and (-16, 0). Find an equation for the circle.

(18) A circle passes through the points  $A(6, 3)$  and  $B(-2, 11)$  and has centre  $C(-2, p)$ .

- (a) Find the value of  $p$
- (b) Find an equation of the tangent to the circle at the point  $(6, 3)$ .
- (c) The point  $D$  has coordinates  $(-10, 3)$ . Show that  $AD$  is a diameter of the circle.
- (d) Show that  $\angle ABD = 90^\circ$

(19) The line  $y = x + c$  is a tangent to a circle with the equation

$$(x-4)^2 + (y-1)^2 = 98.$$

- (a) Find the possible values of  $c$ .
- (b) Find the possible points where the tangent could touch the circle.

(20) The circle with equation

$$(x-6)^2 + (y+4)^2 = r^2$$
 does not cross either coordinate axis.

- (a) Find the set of value of  $r^2$  that satisfy this condition.
- (b) Given further that  $r^2 = 9$ , find the coordinates of the point  $P$  such that  $P$  is the furthest point on the circle from the  $x$  axis.
- (c) Write down the equation of the tangent to the circle at the point  $P$ .