www.m4ths.com – C2 – Binomial Expansion

(1) Use Pascal's Triangle to fully expand the following:

(a) $(x+2)^3$

(b) $(x-3)^4$

(c) $(2x+1)^3$

(d) $(a+b)^5$

(e) $(3-p)^5$

(f) $(1+x)(x+4)^3$

(2) (a) Find the expansion of $(3+x)^4$.

(b) Hence find the expansion of

(i) $(3-x)^4$

(ii) $(3+y^2)^4$

(3) Find the value of the following:

(a) ${}^3C_2$

(b) ${}^5C_3$

(c) ${}^4C_1$

(d) $\binom{7}{5}$

(4) Use the ${}^nC_r$ method for finding coefficients to find the first 4 terms in the expansion of the following:

(a) $(1+2x)^6$

(b) $(3-x)^{10}$

(c) $(2-3x)^8$

(d) $(a-2b)^{12}$

(e) $(2+x)(1-4x)^7$

(5) (a) Expand fully $(1+2x)^5$ in ascending powers of x.

(b) Hence write down the full expansion of $(1-2x)^5$.

(c) Simplify $(1+2x)^5+(1-2x)^5$

(6) (a) Find the term in x^5 in the expansion of $(5-x)^{12}$.

(b) Find the term in x^7 in the expansion of $\left(1+\frac{x}{3}\right)^9$.

(c) Find the term in x^{18} in the expansion of $(0.5+x^3)^{13}$.

(7) Find the term in x^3 in the expansion of $(3+2x)(1-x)^6$.

(8) Find the term independent of y in the expansion of:

(a) $\left(y+\frac{1}{y}\right)^6$

(b) $\left(2y-\frac{1}{y^2}\right)^{12}$

(9) The coefficient of the term in x^2 in the expansion of $(3+px)^5$ is 1080. Given that $p>0$, find the value of p.

(10) Given that the coefficient of the term in x in the expansion of $(2+ax)^4$ is 12, find the coefficient of the term in x^3.

(11) (a) Find the first four terms in the expansion of $(1+2x)^8$.

By using a suitable substitution for x and the answer found in part (a), approximate:

(b) $(1.01)^8$

(c) $(0.98)^8$

(Round each answer to 4 decimal places)

(d) Explain what would happen to the accuracy of your answer in parts (b) and (c) if you use (i) the first 3 terms and (ii) the first 7 terms instead of the first 4 terms found in part (a).

(12) Use the binomial expansion

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\ldots$$

to find the first 4 terms in the expansion of:

(a) $(1+2x)^9$

(b) $(1-0.5x)^7$

(c) $\left(1+\frac{x}{2}\right)^6$

(d) $(3-x)^{10}$

(e) $(1-5x)^{\frac{1}{2}}$

(13) Given that $n>0$ and the coefficient of the term in x^2 in the expansion of $(1+2x)^n$ is 40, find the value of n.

(14) Given that n and p are both positive integers and that $(1+px)^n = 1+12x+54x^2+\ldots$, find the coefficient of the term in x^3 in the expansion of $(1+px)^n$.

(15) (a) Expand $(1+y)^4$ in ascending powers of y.

(b) Using your answer to part (a) and a suitable substitution, find the value of $\left(1+\sqrt{2}\right)^4-\left(1-\sqrt{2}\right)^4$ in the form $p\sqrt{2}$.

(16) Find, in fully factored form, the first 3 terms in the expansion of $(2+x)(1+px)^n$ giving your answer in terms of n, p and x.

(17) (a) Find the first 4 terms in the expansion of $(1+3x)^6$.

(b) By using a suitable value of x and your answer to part (a) to find an approximate value of $(1.03)^6$ correct to 5 dp.

(c) Find the percentage error between the approximation found in part (b) and the actual value of $(1.03)^6$.