www.m4ths.com – C1 – Sequences and Series

(1) Find the first 4 terms for each of the sequences below:

(a) $a_{n+1} = 2a_n + 1,\ \ a_1 = 3$

(b) $a_{n+1} = (a_n)^2 - 2,\ \ a_1 = 1$

(c) $a_n = 3 - 2a_{n-1},\ \ a_2 = 2$

(d) $a_{n+2} = (a_{n+1})^2 - a_n,\ a_1 = 1$ and $a_2 = 3$.

(2) Write down a possible recurrence relationship for each of the following sequences:

(a) 4, 8, 12, 16....

(b) 3, 6, 12, 24...

(3) A sequence of terms is defined by the recurrence relationship:

$a_{n+1} = \frac{a_n}{2} - 2,\ a_1 = 4p$.

(a) Find an expression in terms of p for a_2, a_3 and a_4.

(b) Given $a_4 = 23$ find the value of p.

(4) A sequence of terms is defined by the recurrence relationship:

$a_{n+1} = 2a_n - 1,\ a_1 = p$.

(a) Find an expression in terms of p for a_2, a_3 and a_4.

(b) Given $\sum_{r=1}^{4} a_r = 49$, find the value of p.

(5) State whether the following sequences are arithmetic or not:

(a) 3, 5, 7, 9, 11

(b) 8, 5, 2, −1, −4, −6

(c) $0.25,\ -\frac{1}{2},\ -\frac{5}{4},\ -2,\ -\frac{11}{4}$

(6) Find the 12th term in each of the following sequences:

(a) 1, 4, 7, 10.....

(b) 6, 4, 2, 0....

(c) $0.25,\ -\frac{1}{2},\ -1.25,\ -2$

(7) Find the number of terms in the following sequences:

(a) 4, 8, 12, 16.....80, 84

(b) 3, 1, −1, −3.....−78, −81

(c) $\frac{1}{6},\ \frac{2}{3},\ \frac{7}{6},\ \frac{5}{3}.....\frac{67}{6},\ \frac{35}{3}$

(8) Find the first and 10th term of each sequence below, given:

(a) $2nd$ term $= 5$ & $5th$ term $= 2$.

(b) $6th$ term $= 17$ and $8th = 23$.

(9) Alfie is revising for his exams. He starts one evening by answering four questions from his book. He increases the number of questions that he answers each evening such that it follows an arithmetic sequence. On the 8th night he answers 39 questions. How many questions did he answer on the 11th night?

(10) The first 3 terms of an arithmetic sequence are $(x-1), (2x-4)$ and $(x+3)$. Which is the first term in the sequence that will exceed 98?

(11) Given p is an integer, find the value of p that makes the following sequence arithmetic:

$2p,\ 3p^2,\ 11p - 2$

(12) (a) Show that the sum of the first n terms of an arithmetic series with first term a and common difference d can be written as:

$S_n = \frac{n}{2}(2a + (n-1)d)$

(b) Show that the sum can also be written as:

$S_n = \frac{n}{2}(a + l)$

(13) Find the sum of the first 10 terms of the following arithmetic series:

(a) $4 + 7 + 10 + 13 + ...$

(b) $5 + 2.5 + 0 + (-2.5) + ...$

(c) $\frac{1}{6} + \frac{2}{3} + \frac{7}{6} + \frac{5}{3} + ...$

(d) Given the sum of the first 12 terms of the sequence $p + 2p + 3p + 4p + ..$ is 156, find the value of p.

(14) Find the number of terms in each arithmetic series below:

(a) $a = 3, d = 2$ and $S_n = 80$

(b) $a = -1, d = -4$ and $S_n = -66$

(15) Find the 5th term in the arithmetic series with first term a, given the sum of the first ten terms is −30 and the common difference is −2.

(16) An arithmetic series has first term $0.2x$ and a common difference of 6. Given $S_{12} = 42x$ find the value of x.

(17) Find the sum of the first 38 even numbers.

(18) Evaluate the following:

(a) $\sum_{n=1}^{8} 2n - 3$

(b) $\sum_{n=1}^{6} 1 - 3n$

(c) $\sum_{n=3}^{10} 5n + 1$

(19) Find the smallest value of m such that $\sum_{n=1}^{m} 3n - 1$ exceeds 155.

(20) Fred has a collection of 225 marbles. His son wants to start collecting marbles. Fred gives his son 1 marble on day 1, 3 the following day, 5 the day after such that the number of marbles he gives away follows an arithmetic sequence. Find how many days it will take until Fred can no longer give his son any marbles.