www.m4ths.com – C1 – Differentiation

(1) Find $\frac{dy}{dx}$ for the following:

(a) $y = 2x$

(b) $y = 3x^2$

(c) $y = 4$

(d) $y = 5x^2 - 3x + 1$

(2) Find $\frac{ds}{dt}$ for the following:

(a) $s = 4t^3$

(b) $s = 2t^5 - 3t + 1$

(c) $s = (t-1)(t+2)$

(3) Find $f'(x)$ for the following:

(a) $f(x) = 2x^{\frac{1}{2}}$

(b) $f(x) = 5x^{\frac{4}{3}} - x + 7$

(c) $f(x) = \frac{x^2 - 1}{x + 1}$

(4) Find $\frac{dy}{dx}$ for the following:

(a) $y = \frac{x^2 - 3x}{2x}$

(b) $y = x\sqrt{x} - \frac{3}{x}$

(c) $y = 3x^{0.5} - 3x^{-2} + \frac{4}{x^5}$

(d) $y = x^{\frac{1}{2}}\left(3 - 2x^2\right) + c$

(e) $y = \frac{x^{\frac{1}{2}} + 5x^{\frac{3}{4}}}{3x}$

(f) $y = \left(1 - \sqrt[3]{x}\right)\left(2x^2 - 3\right)$

(5) Find the gradient of the curve at the given point:

(a) $y = x^2 + x$ at $x = 3$

(b) $y = 2x^3 - 3x^2 + x$ at $x = 1$

(c) $y = x^5 - x^4 + 3$ at $x = -2$

(6) Find $g'(4)$ given $g(x) = 2x^{\frac{1}{2}}$

(7) Find the points on the curve $y = 2x^3 - 6x^2 + 3x$ where the gradient is = 21.

(8) Find the gradient of the tangent to the curve $s = 4t^3 - \frac{1}{3\sqrt{t}}$ at the point $\left(1, \frac{11}{3}\right)$

(9) Find the point where the tangent to the curve $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x + 3,\ x > 0$ is parallel to the x axis.

(10) Find the equation of the tangent to the curve $y = x^2 + 4x + 3$ at the point where $x = 1$ giving your answer in the form $y = mx + c$

(11) Find the equation of the tangent to the curve $y = \frac{2x - 4}{\sqrt{x}}$ at the point where $x = 4$ giving your answer in the form $ax + by + c = 0$.

(12) Find an equation of the normal to the curve $s = 4t^5 - 3t + 1$ at the point where $t = 1$.

(13) Find the equation of the normal to the curve $y = x^{\frac{1}{2}}\left(x^2 - 3\right)$ at the point where $x = 4$ giving your answer in the form $y = mx + c$

(14) Find $\frac{d^2y}{dx^2}$ for each of the following:

(a) $y = 3x^4 - 5x^2 + 1$

(b) $y = 3x^{\frac{3}{2}} - 4\sqrt{x}$

(c) $y = 5x\left(x^2 - 3\right)$

(d) $y = \frac{2x^3 - 3}{x^{0.5}}$

(15) Find $f''(2)$ for the each of the following:

(a) $f(x) = 2x^3 - 4x^2 + 3x$

(b) $f(x) = 4x^{\frac{3}{2}} - 5x$

(16) The normal to the curve $y = x^{\frac{1}{2}} - 3x^{\frac{1}{4}}$ at the point where $x = 1$ crosses the x at A and the y axis at B. Find the area of the triangle AOB where O is the origin.

(17) The tangent to the curve $y = 3x^2 + 4x + 1$ at the point where $x = -1$ intersects the line $y = 3x - 12$ at the point P. Find the coordinates of P.

(18) Find the coordinates of the point on the curve $y = x^3 + 2x^2 + x$ where $\frac{d^2y}{dx^2} = 16$.

(19) Show that the point $(5, 2)$ lies on the tangent to the curve $y = x\left(\sqrt{x} - 1\right)$ at the point where $x = 1$.

(20) Mike says that the 2nd derivative of the function $f(x) = x^2(2x^2 - 3x + 4)$ will produce a linear function.
(a) Explain why he is wrong.
(b) Find the value of the 2nd derivative at the point where $x = 0.5$.

(21) Find the value of $f'(\sqrt{3})$ given $f(x) = \frac{5x^6 + 2x^4}{x}$.

(22) The displacement of a particle (s) can be modelled by the equation $s = 4t^3 + 2t^2 + 3t - 1$ for $t \geq 0$.
(a) Find the displacement of the particle after 2 seconds.
(b) Find the velocity and the acceleration of the particle at $t = 1$.

(23) Show that if $f(x) = \frac{2}{3}x^3 + x^2 + 2x + 3$ $f'(x) = 0$ has no real solutions.