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Core Mathematics C1 and C2 Workbook and Exam Papers

Version 1.1 - August 2015

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This book is primarily designed to match the Edexcel spec but is equally valid for other boards.

Solutions to this workbook come in video form only and can be found at www.m4ths.com

This format was chosen to encourage independent study for my students.

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www.m4ths.com – C1 Indices

(1) Simplify the following:
(a) $m^5 \times m^2$
(b) $m^4 \times m^{-1}$
(c) $3m \times 2m^5$
(d) $5m^3 \times \frac{1}{2}m^5 \times 4m^{-2}$
(e) $m^3\left(1-m^4\right)$
(f) $m^5 \div m^3$
(g) $6m \div 3m^3$
(h) $\frac{8m^5}{4m^4}$
(i) $\frac{9m^5 \times 6m}{3m^{-2}}$
(j) $m^3 \times (-6m)^2$

(2) Simplify the following:
(a) $\left(2m^2\right)^3$
(b) $3\left(m^4n\right)^2$
(c) $\left(3m^{-1}n^2\right)^3$
(d) $\left(3m^2\right)\left(3m^4\right)^2$
(e) $\left(\frac{4m^3}{n^5}\right)^3$

(3) Simplify the following:
(a) 3^0
(b) a^0
(c) $3m^0$
(d) $(6m)^0$
(e) $\frac{m^5 \times 6m}{3m^6}$

(4) Evaluate the following:
(a) 2^{-1}
(b) 3^{-2}
(c) $\left(\frac{2}{5}\right)^{-3}$
(d) 4×2^{-4}
(e) $\frac{3}{4^{-2}}$
(f) $4^{-1} \times \frac{1}{3^2}$
(g) $5 \times \frac{1}{3^{-2}}$

(6) Write each of the following with a positive index:
(a) m^{-1}
(b) $3m^{-2}$
(c) $\frac{4}{m^{-3}}$
(d) $5m^{-4}$
(e) $\left(\frac{2m^3}{n^2}\right)^{-3}$

(7) Write each of the following values as a power of 10:
(a) 100
(b) $\frac{1}{10}$
(c) 0.01
(d) 0.002

(8) Evaluate the following:
(a) $25^{\frac{1}{2}}$
(b) $8^{\frac{1}{3}}$
(c) $81^{\frac{1}{4}}$
(d) $16^{0.25}$
(e) $0.01^{0.5}$
(f) $3 \times 125^{\frac{1}{3}}$
(g) $0.09^{0.5} \times \sqrt{10000}$

(9) Simplify the following:
(a) $\left(m^4\right)^{\frac{1}{2}}$
(b) $\left(8m^3\right)^{\frac{1}{3}}$
(c) $3\left(16m^{-1}\right)^{\frac{1}{2}}$
(d) $\left(625m^2n^3\right)^{\frac{1}{4}} \times (81mn)^{0.5}$

(10) Evaluate the following:
(a) $8^{\frac{2}{3}}$
(b) $16^{\frac{3}{2}}$
(c) $27^{\frac{4}{3}}$
(d) $81^{0.75}$
(e) $32^{0.6}$
(f) $\left(\frac{4}{81}\right)^{\frac{3}{2}}$
(g) $\left(3^{-2} \times 27^{\frac{1}{3}}\right)^2$
(h) $\left(1\frac{9}{16}\right)^{1.5}$

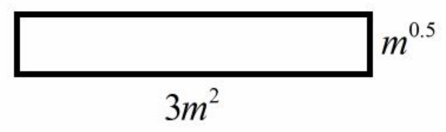
(11) Simplify the following:
(a) $\left(8m^3\right)^{\frac{4}{3}}$
(b) $9^{\frac{3}{2}} \times \left(m^4\right)^{-3}$

(12) Evaluate the following:
(a) $25^{-\frac{1}{2}}$
(b) $8^{-\frac{4}{3}}$
(c) $3 \times 16^{-\frac{3}{4}}$
(d) $\left(\frac{9}{25}\right)^{-\frac{3}{2}}$
(e) $(0.16)^{-\frac{1}{2}}$

(13) Simplify the following:
(a) $\left(4m^{-1}\right)^{-\frac{1}{2}} \times 5^2$
(b) $\left(\frac{9m^{0.5}n^2}{25}\right)^{-\frac{3}{2}}$
(c) $(2m)^{-3} \times \left(32m^2\right)^{2.5}$

(14) Find the values of a and n such that:
$\left(3m^3\right)^{-2} \times \left(9m^2\right)^{1.5} \equiv am^n$

(15) The area of the rectangle below is 96 square units. Find the perimeter of the rectangle.



(16) Given, when simplified, the expression
$(2x)^{-1} \times \frac{3}{\sqrt{x}} \times \left(6x^n\right)^2$ is independent of terms in x:
(a) Find the value of n.
(b) Write down the value of the expression.

www.m4ths.com – C1 – Surds

(1) Simplify the following:

(a) $\sqrt{36}$

(b) $\sqrt{8}$

(c) $\sqrt{12}$

(d) $\sqrt{27}$

(e) $\sqrt{50}$

(f) $\sqrt{1200}$

(g) $\sqrt{a^2}$

(h) $\left(\sqrt{3}\right)^3$

(i) $\left(36\sqrt{5^4}\right)^{\frac{1}{2}}$

(2) Simplify the following:

(a) $\sqrt{6}\times\sqrt{6}$

(b) $\sqrt{2}\times\sqrt{5}$

(c) $\sqrt{3}\times\sqrt{6}$

(d) $\sqrt{2}\times\sqrt{6}$

(e) $\sqrt{14}\times\sqrt{7}$

(f) $\sqrt{\frac{2}{3}}\times\sqrt{3}$

(g) $\sqrt{2}\times\sqrt{2}\times\sqrt{2}$

(h) $\sqrt{a}\times\sqrt{a}$

(i) $2\sqrt{5}\times3\sqrt{7}$

(j) $p\sqrt{q}\times3p^2\sqrt{q}$

(k) $\frac{\sqrt{12}}{\sqrt{3}}$

(3) Simplify the following:

(a) $\sqrt{3}+\sqrt{3}$

(b) $2\sqrt{5}+\sqrt{5}$

(c) $7\sqrt{3}-2\sqrt{3}$

(d) $\sqrt{2}+\sqrt{8}$

(e) $\sqrt{50}+3\sqrt{5}$

(f) $\sqrt{27}+2\sqrt{3}-\sqrt{12}$

(g) $\sqrt{a}+3\sqrt{4a^2}-5\sqrt{a\times a}$

(4) Simplify the following:

(a) $\sqrt{2}\left(3+\sqrt{2}\right)$

(b) $\sqrt{3}\left(4-\sqrt{12}\right)$

(c) $\sqrt{p}\left(4-3\sqrt{p}\right)$

(d) $2\sqrt{p}\left(1+3\sqrt{p^3}\right)$

(e) $\sqrt{3}\left(\sqrt{6}-\sqrt{27}\right)$

(5) Simplify the following:

(a) $\left(1+\sqrt{3}\right)\left(\sqrt{3}+4\right)$

(b) $\left(4+\sqrt{7}\right)\left(4-\sqrt{7}\right)$

(c) $\left(2-\sqrt{5}\right)\left(1-\sqrt{10}\right)$

(d) $\left(4-\sqrt{p}\right)\left(5-3\sqrt{p}\right)$

(e) $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)$

(f) $a\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$

(6) Rationalise the denominator of the following fractions:

(a) $\frac{1}{\sqrt{3}}$

(b) $\frac{2}{\sqrt{5}}$

(c) $\frac{3}{4\sqrt{7}}$

(d) $\frac{6}{5\sqrt{2}}$

(e) $\frac{3}{8\sqrt{t}}$

(f) $\frac{2\sqrt{27}}{5\sqrt{12}}$

(g) $\frac{\sqrt{3}}{\left(1-\sqrt{3}\right)\left(1+\sqrt{27}\right)}$

(7) Rationalise the denominator of the following fractions:

(a) $\frac{1}{1+\sqrt{5}}$

(b) $\frac{1}{1-\sqrt{3}}$

(c) $\frac{5}{2-\sqrt{7}}$

(d) $\frac{5+\sqrt{7}}{4-\sqrt{7}}$

(e) $\frac{2+\sqrt{12}}{5-\sqrt{3}}$

(f) $\frac{5+\sqrt{2}}{3-\sqrt{8}}$

(g) $\frac{\sqrt{2}}{1-\sqrt{32}}$

(8) Write $\frac{1}{\sqrt{27}}+\frac{2}{\sqrt{3}}$ as a single fraction in its lowest form.

(9) Given $\frac{a-\sqrt{b}}{1-\sqrt{8}}\equiv2+\sqrt{2}$ find the values of a and b .

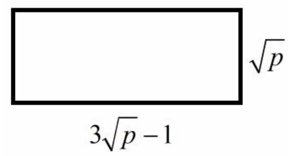
(10) Solve the equation $3x+4=\sqrt{2}x+6$ writing your answer as a rational fraction.

(11) Simplify $\left(4p\right)^{\frac{3}{2}}\times3\sqrt{p}$ leaving your answer in the form ap^n .

(12) Solve the equation

$$\sqrt{x}-\frac{6}{\sqrt{x}}=1$$

(13) (a) Find the perimeter of the rectangle below giving your answer in terms of p .

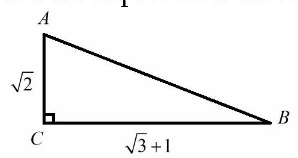


(b) Given the perimeter of the rectangle $38cm$ find the area of the rectangle.

(14) The triangle ABC is shown below.

$AC=\sqrt{2}$ & $BC=\sqrt{3}+1$.

(a) Find an expression for AB^2 .



(b) Show the area of the triangle can be written in the form $\frac{\sqrt{a}+\sqrt{b}}{a}$ stating the values of a and b .

(15) Solve the equation

$x^2-2\sqrt{3}x-7=0$

Give your answer in the form $x=\sqrt{a}\pm\sqrt{b}$.

www.m4ths.com – C1 – Coordinate Geometry 1

(1) Find the gradient of the line passing through each set of points given below:
(a) $(2,1)$ and $(6,9)$
(b) $(4,7)$ and $(2,5)$
(c) $(-3,2)$ and $(2,-1)$
(d) $\left(\frac{1}{2},3\right)$ and $\left(\frac{3}{2}.-\frac{3}{4}\right)$

(2) The gradient of the line passing through the points $(p,5)$ and $(1,-7)$ is 4. Find the value of p.

(3) State the gradient and y intercept of the following lines:
(a) $y=3x-1$
(b) $y=-2x+5$
(c) $y=\frac{1}{2}-x$
(d) $y=3(2x+1)$
(e) $y=-\frac{1}{3}x$

(4) The line $y=3x+c$ passes through the point $(1,5)$. Find the coordinates where the line crosses the x axis.

(5) State the gradient and y intercept of the following lines:
(a) $8x+4y-3=0$
(b) $3x-2y+5=0$
(c) $5x-6y=4$
(d) $ax+by+c=0$

(6) Find the equation of the straight line with the given gradient and point in the form $y=mx+c$:
(a) Gradient = 3, point $(2,1)$
(b) Gradient $=-1$, point $(3,-2)$
(c) Gradient $=\frac{1}{4}$, point $(-8,2)$
(d) Gradient $=0.2$, point $\left(5,\frac{1}{4}\right)$
(e) Gradient $=m$, point $(0,m)$

(7) Find the equation of the straight line passing through the given points in the form $y=mx+c$:
(a) $(2,1)$ and $(4,5)$
(b) $(-1,5)$ and $(2,-3)$
(c) $(5,-7)$ and the origin.
(d) $\left(2,\frac{1}{3}\right)$ and $\left(4,\frac{2}{3}\right)$

(8) Find the equation of the straight line passing through the given points in the form $ax+by+c=0$:
(a) $(6,10)$ and $(4,6)$
(b) $(3,0)$ and $(0.5,4)$
(c) $(-3,0)$ and $(0,-3)$
(d) $(-1,-3)$ and $\left(2,\frac{1}{4}\right)$

(9) The line l has gradient 4 and crosses the x axis at the point $(3,0)$. Find where it crosses the y axis.

(10) The line l with gradient 3 passing through the point $(2,4)$ intersects the line $2x-y=5$ at the point P. Find the coordinates of P.

(11) Find the distance between the two given points leaving your answer in exact form where appropriate:
(a) $(5,6)$ and $(1,3)$
(b) $(4,1)$ and $(10,9)$
(c) $(-1,-4)$ and the origin.
(d) $(-1,-1)$ and $(1,1)$
(e) $(5,3)$ and $(5,7)$

(12) Given the distance between the points $(p,3)$ and $(4,1)$ is $2\sqrt{5}$ find the possible values of p.

(13) The distance between the points $(10,q)$ and $(q,12)$ is 10. Find the possible values of q.

(14) Find the midpoint of the following pairs of coordinates:
(a) $(2,1)$ and $(6,9)$
(b) $(4,7)$ and $(2,5)$
(c) $(-1,5)$ and $(2,-3)$
(d) $(0.5,3)$ and $\left(\frac{1}{4},-\frac{1}{3}\right)$

(15) The midpoint of the points $(12,7)$ and $(p,3)$ is $(5,q)$. Find the values of p and q.

(16) Write down the gradient of a line *(i)* parallel to and *(ii)* perpendicular to the following lines:
(a) $y=3x-1$
(b) $y=4-2x$
(c) $x+y=0$
(d) $2x+3y=7$
(e) $px-qy-4=0$

(17) Find an equation of the line *(i)* parallel to and *(ii)* perpendicular to the line $y=5x+1$ that passes through the point $(2,4)$.

(18) The perpendicular bisector of the line segment AB crosses the x axis at the point P. Given the coordinates of A are $(2,1)$ and the coordinates of B are $(6,4)$ find the coordinates of the point P.

(19) The lines $x+3y-4=0$ and $y=mx+2$ are perpendicular. Find the value of m.

(20) Given the lines $px+y=0$ and $2y=3+5qx$ are parallel express p in terms of q.

(21) The line l passes through the point $(-1,5)$ and is perpendicular to the line $2x+4y+7=0$. Line l meets the line $y=3x+8$ at the point P. Find the coordinates of P.

www.m4ths.com – C1 – Quadratic Functions

(1) Factorise the following quadratic expressions:

(a) $x^2 - x - 12$

(b) $8 - 6x + x^2$

(c) $x^2 + 3x$

(2) Solve the following quadratic equations:

(a) $(x-2)(x+1) = 0$

(b) $(2x+3)(x+4) = 0$

(c) $x^2 - 2x - 8 = 0$

(d) $x(x-1) = 6$

(3) Factorise the following quadratic expressions:

(a) $2x^2 + x - 1$

(b) $3x^2 - 5x - 2$

(c) $12x^2 + 16x - 3$

(4) Factorise and solve the following quadratic equations:

(a) $2x^2 - 5x - 3 = 0$

(b) $5x^2 + 4x - 1 = 0$

(c) $6x^2 + 7x = 3$

(d) $x(2x-1) = 15$

(e) $0.4x^2 + x = 0.6$

(5) (a) Given that the quadratic equation $\mathrm{f}(x) = (2x-3)(3x-5)$ can be written in the form $\mathrm{f}(x) = ax^2 + bx + c$, find the values of a, b and c.

(b) Write down the solutions to the equation $\mathrm{f}(x) = 0$.

(c) Find the solutions to the equation $\mathrm{f}(x) = 15$.

(6) Write the following quadratic expressions in the form $(x+a)^2 + b$

(a) $x^2 - 4x - 3$

(b) $2 - 6x + x^2$

(c) $x^2 + 5x + 2$

(d) $x^2 + 3x$

(7) Solve the following quadratic equations by completing the square leaving your answers in exact form where appropriate:

(a) $x^2 - 2x - 8 = 0$

(b) $x^2 + 3x + 1 = 0$

(c) $x^2 + 8x = 12$

(d) $2x^2 + 7x - 1 = 0$

(8) Write the following quadratic expressions in the form $a(x+b)^2 + c$:

(a) $2x^2 + 4x + 7$

(b) $-x^2 + 5x - 2$

(c) $7x^2 + 3x + 1$

(d) $8x + 5x^2$

(9) Solve the following quadratic equations by completing the square leaving your answers in exact form where appropriate:

(a) $3x^2 + 6x - 1 = 0$

(b) $7x^2 + 5x - 2 = 0$

(c) $4x(x-6) = 7$

(10) (a) Sketch the graph of $y = x^2 + 4x + 1$ showing any points of intersection with the coordinate axes and the coordinates of the minimum point.

(b) Sketch the graph of $y = 2x^2 + 5x - 4$ showing any points of intersection with the coordinate axes and the coordinates of the minimum point.

(c) Sketch the graph of $y = 3 - 5x - x^2$ showing any points of intersection with the coordinate axes and the coordinates of the maximum point.

(11) (a) Given that the quadratic expression $2(x+0.75)^2 - 1$ can be written in the form $ax^2 + bx + c$. Find the values of a, b and c.

(b) Solve the equation $2(x+0.75)^2 - 1 = 0$ giving your answers in exact form.

(12) Use the quadratic formula to find the solutions to the following equations. Give your answers in exact form where possible:

(a) $x^2 - 3x - 8 = 0$

(b) $0 = 2 - 10x + x^2$

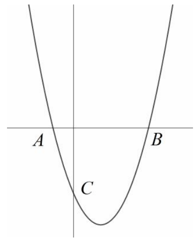
(c) $3x^2 - 2x - 4 = 0$

(d) $-x^2 + 7x - 1 = 0$

(e) $7x^2 = 1 + 5x$

(f) $0.3x + 1.2x^2 - 2.5 = 0$

(13) Part of the graph of $y = 4x^2 - 12x - 19$ is shown below. The curve crosses the x axis at the points A and B and the y axis at the point C.



(a) Write down the coordinates of the point C.

(b) Find the length of the line segment AB giving your answer in exact form.

(14) In completed square form the equation $y = x^2 + px + q$ can be written as $y = (x-2)^2 - 5$

(a) Find the values of p and q.

(b) Sketch the graph of $y = (x-2)^2 - 5$ showing any point of intersection with the coordinate axes.

(c) Label the minimum point M on the graph and write down its coordinates.

(d) The graph crosses the x axis at the points A and B. Find the area of the triangle AMB giving your answer in exact form.

(15) (a) Find the solutions to the equation $px^2 + qx + r = 0$ in terms of p, q and r.

(b) Given that $p < 0 < r < q$ draw a rough sketch of the curve $y = px^2 + qx + r$

www.m4ths.com – C1 – Simultaneous Equations

(1) Solve the following linear simultaneous equations for x and y:

(a) $2x+y=4$
$x-y=-1$

(b) $5x-3y=3$
$2x+2y=14$

(c) $x-\frac{1}{2}y=0$
$\frac{1}{3}x+2y=\frac{-13}{4}$

(2) Solve the following simultaneous equations:

(a) $y=x+1$
$y=x^2+x$

(b) $y=1-x$
$y=x^2-11$

(c) $y=3x^2+9x-1$
$y=x^2+4x+2$

(3) Sketch the graphs of $y=4-x$ and $y=x^2+5x$ on the same set of axes showing any points of intersection with the coordinate axes.
(b) Find the coordinates of the points where the graphs meet.

(4) Solve the following simultaneous equations:

(a) $x+y=3$
$x^2-y^2=21$

(b) $x+y=3$
$x^2+y^2=5$

(c) $x^2+2y^2=6$
$y-3x=5$

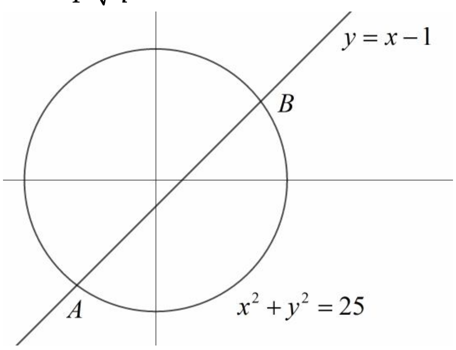
(5) Solve the following simultaneous equations:

(a) $x-2y=-8$
$xy+y^2=3$

(b) $xy+x^2=\frac{5}{18}$
$x+y-\frac{5}{6}=0$

(c) $x=\frac{12}{y}$
$2x-y-5=0$

(6) The diagram below shows the line with equation $y=x-1$ and the circle with equation $x^2+y^2=25$. The line intersects the circle at the points A and B. Find the length of the chord AB in the form $p\sqrt{q}$.



(7) (a) Sketch the graphs of $3x=4y$ and $y=\frac{48}{x}$ on the same set of axes showing any points of intersection with the coordinate axes.
(b) Find the coordinates of the points where the graphs meet.
(c) Write down the x coordinate that lies on the line $3x=4y$ but not on the curve $y=\frac{48}{x}$.

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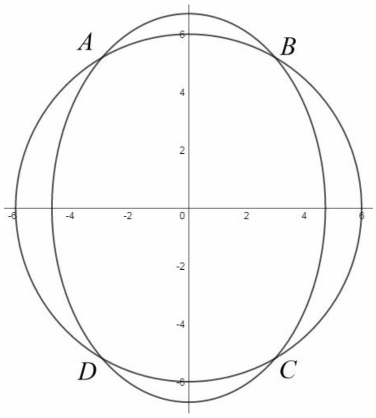
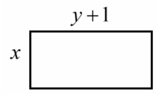
(8) The graphs of $x^2+y^2=36$ and $2x^2+y^2=45$ are shown below. The graphs intersect at the points A, B, C and D. Find the area of the rectangle $ABCD$ giving your answer in exact form.

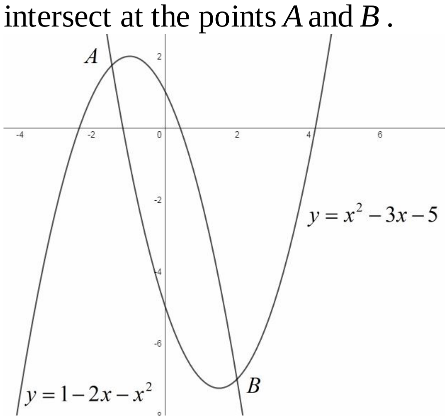


(9) The dimension of rectangular room are $y+1$ and x, $y>x$ as shown in the diagram below.



Given that the area of the room is $28m^2$ and the perimeter of the room is $22m$, find the length of the diagonals of the room giving your answer in exact form.

(10) The diagram below shows the graphs of $y=x^2-3x-5$ and $y=1-2x-x^2$. The graphs intersect at the points A and B.



Show that the straight line passing through the points A and B has gradient $-\frac{35}{14}$.

www.m4ths.com – C1 – Inequalities

(1) Find the set of values of x which satisfy the following:

(a) $2x > 1$

(b) $3x - 1 \leq 6$

(c) $2 - 5x < -3$

(d) $4 - 0.2x \geq 3.6$

(e) $2x - 1 < 3x - 5$

(f) $3(x-1) \geq 5(2-x)$

(g) $2(3 - px) \leq 4px - q$

(2) Solve the following inequalities:

(a) $(x-1)(x+2) < 0$

(b) $(2x-1)(x-3) \leq 0$

(c) $(1+4x)(2-3x) > 0$

(d) $(0.3x-1)(3x+5) \geq 0$

(3) Find the set of values of x which satisfy each inequality:

(a) $x^2 - x - 6 < 0$

(b) $x^2 + 2x - 8 > 0$

(c) $10x^2 + 20x - 80 > 0$

(d) $x^2 + 3x \leq 10$

(e) $2 - x - x^2 \geq 0$

(f) $x^2 > x$

(g) $2x^2 \leq 4x + 96$

(4) Factorise and solve the following inequalities:

(a) $2x^2 - 5x - 3 > 0$

(b) $10x^2 + 3x - 1 \leq 0$

(c) $4 - 7x - 2x^2 > 0$

(d) $6x^2 \leq 4 - 5x$

(5) Find the set of values of x which satisfy each inequality:

(a) $x^2 - 4x + 1 < 0$

(b) $2x^2 + 8x - 3 \geq 0$

(c) $3x - x^2 \geq 1$

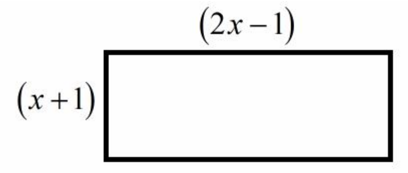
(6) (a) Find the set of values of x that satisfy $x + 1 < 0$.

(b) Find the set of values of x that satisfy $(x-1)(x+2) < 0$.

(c) Find the set of values of x that satisfy both $x + 1 < 0$ and $(x-1)(x+2) < 0$.

(7) Find the set of values of x that satisfy both $7x \geq 4 - 2x^2$ and $2x - 3 < 0$.

(8) The sketch below shows a plan of a living room. The length of the room is $(2x-1)$ and the width of the room is $(x+1)$ where x is measured in meters.



Given that the area of the room must be at least 135 square meters and the total length of the walls cannot exceed 54 meters

(a) Find the set of values of x that satisfy both constraints.

(b) Hence find the maximum and minimum values of the area and perimeter of the room.

www.m4ths.com – C1

Curve Sketching (1)

(1) Sketch the following curves showing any points where the curve crosses the coordinate axis:

(a) $y=(x+2)(x-1)(x-3)$

(b) $y=(x+3)(x+2)(2x-1)$

(c) $y=x(x+1)(x-2)$

(d) $y=-(x+3)(x+1)(x-1)$

(e) $y=(x+3)(x^2-4)$

(f) $y=x(4x^2-9)$

(g) $y=(x+3)(x+2)(1-x)$

(h) $y=(3-x)(x+2)(2-x)$

(i) $y=(x^2-2x-5)(2+x)$

(j) $y=x^3-x^2-6x$

(k) $y=-x^3-2x^2+8x$

(l) $y=-6x^3+x^2+x$

(m) $y=x^3-4x$

(2) Sketch the following curves showing the points where the curve touches or crosses the coordinate axis:

(a) $y=(x+1)(x-2)^2$

(b) $y=(x+1)^2(x-2)$

(c) $y=x(x-3)^2$

(d) $y=x^2(x-2)$

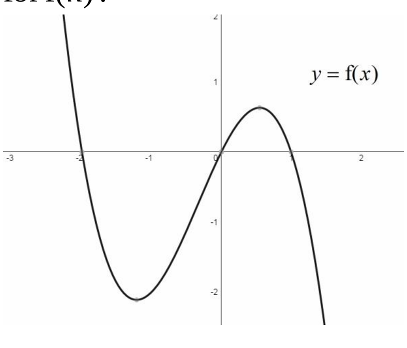
(e) $y=(1-x)x^2$

(f) $y=x^3-6x^2$

(g) $y=-x^3+2x^2$

(h) $y=-x(3-2x)^2$

(3) Part of the curve $y=\mathrm{f}(x)$ is shown below. Given $\mathrm{f}(x)$ is cubic, write down an equation for $\mathrm{f}(x)$.



(4) Sketch the following curves stating the equations of any asymptotes:

(a) $y=\dfrac{1}{x},\ \ x\neq 0$

(b) $y=\dfrac{3}{x},\ \ x<0$

(c) $y=-\dfrac{2}{x},\ \ x\neq 0$

(d) $y=-\dfrac{2}{3x},\ \ x>0$

(e) $y=\dfrac{1}{x^2},\ \ x\neq 0$

(5) (a) Sketch the graphs of $y=x^2$ and $y=2x$ on the same set of axis.

(b) State the number of solutions to the equation $x^2=2x$

(6) (a) Sketch the graphs of $y=x^3$ and $y=\dfrac{4}{x}$ on the same set of axis.

(b) State the number of solutions to the equation $x^3=\dfrac{4}{x}$

(7) (a) Sketch the graphs of $y=2x^3$ and $y=x^2$ on the same set of axis.

(b)State the number of solutions to the equation $x^2=2x^3$

(8) Explain how the graph of $y=\mathrm{f}(x)$ will be transformed when each of the following transformations are applied.

(a) $y=\mathrm{f}(x-1)$

(b) $y=\mathrm{f}(x+2)-3$

(c) $y=-\mathrm{f}(x)$

(d) $y=\mathrm{f}(-x)$

(e) $y=-\mathrm{f}(x-3)$

(f) $y=4-\mathrm{f}(x)$

(g) $y=\mathrm{f}(-x)-2$

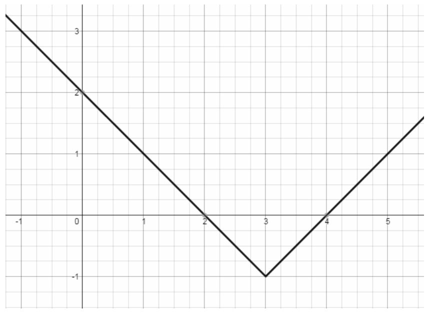
(h) $y=2\,\mathrm{f}(x)$

(i) $y=-\mathrm{f}(3x)$

(j) $y=2\,\mathrm{f}(4x)$

(k) $y=2+\mathrm{f}(0.5x)$

(9) Part of the graph $y=\mathrm{f}(x)$ is shown below.



(a) Write down the coordinates of the minimum point of $\mathrm{f}(x)$.

(b) Draw the graphs of the following stating the coordinates of the minimum point:

(i) $y=\mathrm{f}(x-1)$

(ii) $y=2\,\mathrm{f}(x)$

(iii) $y=1+\mathrm{f}(0.5x)$

(iv) $y=3\mathrm{f}(x+2)$

(v) $y=1-\mathrm{f}(2x)$

(10) $\mathrm{f}(x)=x(x+4)(x-2)$

(a) Draw the graph of $y=\mathrm{f}(x)$ showing any points of intersection with the coordinate axis.

(b) Sketch the curve for the following equations showing any points of intersection with the coordinate axis:

(i) $y=\mathrm{f}(x+2)$

(ii) $y=2\,\mathrm{f}(x-1)$

(iii) $y=-\mathrm{f}(-x)$

(c) $\mathrm{g}(x)=(x-3)(x+1)(x-5)$.

State the single transformation that maps $y=\mathrm{f}(x)$ to $y=\mathrm{g}(x)$.

(11) $\mathrm{f}(x)=-x^2-4x+14$

(a) Express in the form $\mathrm{f}(x)$ in the form $a(x+b)^2+c$.

(b) Sketch the graph of $y=\mathrm{f}(x)$ stating the coordinates of the maximum point.

(c) Sketch the graphs of the following equations stating the coordinates of the maximum point:

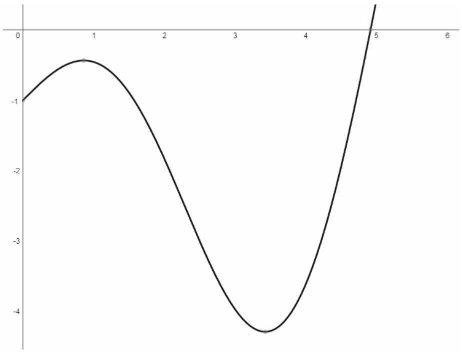
(a) $y=2\,\mathrm{f}(x-1)$

(b) $y=\mathrm{f}(-x)+1$

(c) $y=2+\mathrm{f}(0.25x)$

www.m4ths.com – C1
Curve Sketching (2)

(1) The graph of $y = \mathrm{f}(x)$, $0 \le x \le 6$ is shown below. The graph touches the y axis at the point $(0,-1)$ and crosses the x axis at the point $(5,0)$.



(a) State the coordinates of the point where the curve $y = -5\mathrm{f}(x)$ touches the y axis.

(b)The curve $y = \mathrm{f}(ax)$ crosses the x axis at the point $(12.5, 0)$. Find the value of a

(c) Write down the number of solutions to the equation $\mathrm{f}(x) + 3 = 0$.

(d) Write down the number of solutions to the equation $2\mathrm{f}(x) = -6$.

(2) $\mathrm{f}(x) = (2x-1)^2 - 4$.

(a) Sketch the graph of $y = \mathrm{f}(x)$ stating the coordinates of the points where the curve crosses the coordinate axis and the coordinates of the minimum point.

(b) Sketch the graphs of the following curves stating the coordinates of the maximum or minimum point:

(i) $y = 1 - \mathrm{f}(x)$

(ii) $y = \mathrm{f}(x-2) + 1$

(iii) $y = 3\mathrm{f}(-x)$

(c) The equation $\mathrm{f}(x) + k = 0$ has equal roots. State the value of k.

(3) Sketch the graph of $y = (2px-1)(2-x)(x+3)$ given $1 < p < 2$. Label clearly any points of intersection with the coordinate axis.

(4) (a) Sketch the graph of $y = \dfrac{2}{x} - 1$, $x \ne 0$ showing any points of intersection with the coordinate axis and stating the equations of any asymptotes.

(b) The curve $y = \dfrac{2}{x} - 1$ meets the straight line $y = x$ at the points A and B. Find the coordinate of A and B.

(c) State the single transformation that maps the graph of $y = \dfrac{2}{x} - 1$ onto the graph of $y = \dfrac{2}{x+3} - 1$.

(d) Write down the coordinates of the point where the graph of $y = \dfrac{2}{x+3} - 1$ crosses the y axis.

(5) (a) Sketch the graphs of $y = x\left(x^2 - 1\right)$ and $y = \dfrac{2}{x}$, $x \ne 0$ on the same set of axis.

(b) State the number of solutions to the equation $x\left(x^2-1\right) = \dfrac{2}{x}$

(c) Show the equation $x\left(x^2-1\right) = \dfrac{2}{x}$ can be written as $x^4 - x^2 - 2 = 0$.

(d) Using the substitution $p = x^2$, or otherwise, solve the equation $x^4 - x^2 - 2 = 0$ giving your answers in exact form.

www.m4ths.com – C1 – Sequences and Series

(1) Find the first 4 terms for each of the sequences below:

(a) $a_{n+1} = 2a_n + 1$, $a_1 = 3$

(b) $a_{n+1} = (a_n)^2 - 2$, $a_1 = 1$

(c) $a_n = 3 - 2a_{n-1}$, $a_2 = 2$

(d) $a_{n+2} = (a_{n+1})^2 - a_n$, $a_1 = 1$ and $a_2 = 3$.

(2) Write down a possible recurrence relationship for each of the following sequences:

(a) $4,\ 8,\ 12,\ 16....$

(b) $3,\ 6,\ 12,\ 24...$

(3) A sequence of terms is defined by the recurrence relationship:

$a_{n+1} = \frac{a_n}{2} - 2$, $a_1 = 4p$.

(a) Find an expression in terms of p for a_2, a_3 and a_4.

(b) Given $a_4 = 23$ find the value of p.

(4) A sequence of terms is defined by the recurrence relationship:

$a_{n+1} = 2a_n - 1$, $a_1 = p$.

(a) Find an expression in terms of p for a_2, a_3 and a_4.

(b) Given $\sum_{r=1}^{4} a_r = 49$, find the value of p.

(5) State whether the following sequences are arithmetic or not:

(a) $3,\ 5,\ 7,\ 9,\ 11$

(b) $8,\ 5,\ 2,\ -1,\ -4,\ -6$

(c) $0.25,\ -\frac{1}{2},\ -\frac{5}{4},\ -2,\ -\frac{11}{4}$

(6) Find the 12th term in each of the following sequences:

(a) $1,\ 4,\ 7,\ 10.....$

(b) $6,\ 4,\ 2,\ 0....$

(c) $0.25,\ -\frac{1}{2},\ -1.25,\ -2$

(7) Find the number of terms in the following sequences:

(a) $4,\ 8,\ 12,\ 16.....80,\ 84$

(b) $3,\ 1,\ -1,\ -3.....-78,\ -81$

(c) $\frac{1}{6},\ \frac{2}{3},\ \frac{7}{6},\ \frac{5}{3}.......\frac{67}{6},\ \frac{35}{3}$

(8) Find the first and 10th term of each sequence below, given:

(a) $2nd$ term $=5$ & $5th$ term $=2$.

(b) $6th$ term $=17$ and $8th = 23$.

(9) Alfie is revising for his exams. He starts one evening by answering four questions from his book. He increases the number of questions that he answers each evening such that it follows an arithmetic sequence. On the 8th night he answers 39 questions. How many questions did he answer on the 11th night?

(10) The first 3 terms of an arithmetic sequence are $(x-1), (2x-4)$ and $(x+3)$. Which is the first term in the sequence that will exceed 98?

(11) Given p is an integer, find the value of p that makes the following sequence arithmetic:

$2p,\ 3p^2,\ 11p-2$

(12) (a) Show that the sum of the first n terms of an arithmetic series with first term a and common difference d can be written as:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

(b) Show that the sum can also be written as:

$$S_n = \frac{n}{2}(a + l)$$

(13) Find the sum of the first 10 terms of the following arithmetic series:

(a) $4 + 7 + 10 + 13 + ...$

(b) $5 + 2.5 + 0 + (-2.5) + ...$

(c) $\frac{1}{6} + \frac{2}{3} + \frac{7}{6} + \frac{5}{3} + ...$

(d) Given the sum of the first 12 terms of the sequence $p + 2p + 3p + 4p + ..$ is 156, find the value of p.

(14) Find the number of terms in each arithmetic series below:

(a) $a = 3, d = 2$ and $S_n = 80$

(b) $a = -1, d = -4$ and $S_n = -66$

(15) Find the 5th term in the arithmetic series with first term a, given the sum of the first ten terms is -30 and the common difference is -2.

(16) An arithmetic series has first term $0.2x$ and a common difference of 6. Given $S_{12} = 42x$ find the value of x.

(17) Find the sum of the first 38 even numbers.

(18) Evaluate the following:

(a) $\sum_{n=1}^{8} 2n - 3$

(b) $\sum_{n=1}^{6} 1 - 3n$

(c) $\sum_{n=3}^{10} 5n + 1$

(19) Find the smallest value of m such that $\sum_{n=1}^{m} 3n - 1$ exceeds 155.

(20) Fred has a collection of 225 marbles. His son wants to start collecting marbles. Fred gives his son 1 marble on day 1, 3 the following day, 5 the day after such that the number of marbles he gives away follows an arithmetic sequence. Find how many days it will take until Fred can no longer give his son any marbles.

www.m4ths.com – C1 – Differentiation

(1) Find $\frac{dy}{dx}$ for the following:

(a) $y = 2x$

(b) $y = 3x^2$

(c) $y = 4$

(d) $y = 5x^2 - 3x + 1$

(2) Find $\frac{ds}{dt}$ for the following:

(a) $s = 4t^3$

(b) $s = 2t^5 - 3t + 1$

(c) $s = (t-1)(t+2)$

(3) Find $f'(x)$ for the following:

(a) $f(x) = 2x^{\frac{1}{2}}$

(b) $f(x) = 5x^{\frac{4}{3}} - x + 7$

(c) $f(x) = \frac{x^2 - 1}{x + 1}$

(4) Find $\frac{dy}{dx}$ for the following:

(a) $y = \frac{x^2 - 3x}{2x}$

(b) $y = x\sqrt{x} - \frac{3}{x}$

(c) $y = 3x^{0.5} - 3x^{-2} + \frac{4}{x^5}$

(d) $y = x^{\frac{1}{2}}\left(3 - 2x^2\right) + c$

(e) $y = \frac{x^{\frac{1}{2}} + 5x^{\frac{3}{4}}}{3x}$

(f) $y = \left(1 - \sqrt[3]{x}\right)\left(2x^2 - 3\right)$

(5) Find the gradient of the curve at the given point:

(a) $y = x^2 + x$ at $x = 3$

(b) $y = 2x^3 - 3x^2 + x$ at $x = 1$

(c) $y = x^5 - x^4 + 3$ at $x = -2$

(6) Find $g'(4)$ given $g(x) = 2x^{\frac{1}{2}}$

(7) Find the points on the curve $y = 2x^3 - 6x^2 + 3x$ where the gradient is = 21.

(8) Find the gradient of the tangent to the curve $s = 4t^3 - \frac{1}{3\sqrt{t}}$ at the point $\left(1, \frac{11}{3}\right)$

(9) Find the point where the tangent to the curve $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x + 3$, $x > 0$ is parallel to the x axis.

(10) Find the equation of the tangent to the curve $y = x^2 + 4x + 3$ at the point where $x = 1$ giving your answer in the form $y = mx + c$

(11) Find the equation of the tangent to the curve $y = \frac{2x - 4}{\sqrt{x}}$ at the point where $x = 4$ giving your answer in the form $ax + by + c = 0$.

(12) Find an equation of the normal to the curve $s = 4t^5 - 3t + 1$ at the point where $t = 1$.

(13) Find the equation of the normal to the curve $y = x^{\frac{1}{2}}\left(x^2 - 3\right)$ at the point where $x = 4$ giving your answer in the form $y = mx + c$

(14) Find $\frac{d^2y}{dx^2}$ for each of the following:

(a) $y = 3x^4 - 5x^2 + 1$

(b) $y = 3x^{\frac{3}{2}} - 4\sqrt{x}$

(c) $y = 5x\left(x^2 - 3\right)$

(d) $y = \frac{2x^3 - 3}{x^{0.5}}$

(15) Find $f''(2)$ for the each of the following:

(a) $f(x) = 2x^3 - 4x^2 + 3x$

(b) $f(x) = 4x^{\frac{3}{2}} - 5x$

(16) The normal to the curve $y = x^{\frac{1}{2}} - 3x^{\frac{1}{4}}$ at the point where $x = 1$ crosses the x at A and the y axis at B. Find the area of the triangle AOB where O is the origin.

(17) The tangent to the curve $y = 3x^2 + 4x + 1$ at the point where $x = -1$ intersects the line $y = 3x - 12$ at the point P. Find the coordinates of P.

(18) Find the coordinates of the point on the curve $y = x^3 + 2x^2 + x$ where $\frac{d^2y}{dx^2} = 16$.

(19) Show that the point $(5, 2)$ lies on the tangent to the curve $y = x\left(\sqrt{x} - 1\right)$ at the point where $x = 1$.

(20) Mike says that the 2nd derivative of the function $f(x) = x^2(2x^2 - 3x + 4)$ will produce a linear function.

(a) Explain why he is wrong.

(b) Find the value of the 2nd derivative at the point where $x = 0.5$.

(21) Find the value of $f'(\sqrt{3})$ given $f(x) = \frac{5x^6 + 2x^4}{x}$.

(22) The displacement of a particle (s) can be modelled by the equation $s = 4t^3 + 2t^2 + 3t - 1$ for $t \geq 0$.

(a) Find the displacement of the particle after 2 seconds.

(b) Find the velocity and the acceleration of the particle at $t = 1$.

(23) Show that if $f(x) = \frac{2}{3}x^3 + x^2 + 2x + 3$

$f'(x) = 0$ has no real solutions.

www.m4ths.com – C1 – Integration

(1) Integrate the following expressions with respect to t:

(a) $3t$

(b) $4t^3$

(c) t^{-2}

(d) $\frac{1}{2}t^3$

(e) 1

(2) Find the following indefinite integrals:

(a) $\int (2x+1)\ dx$

(b) $\int (3x^3-2x)\ dx$

(c) $\int (4x^{-3}-2x^{-5})\ dx$

(d) $\int (-x^{-5}+\sqrt{x})\ dx$

(e) $\int (7-x-x^{-5})\ dx$

(f) $\int \left(4x-\frac{c}{2}\right) dx$

(c is a constant)

(g) $\int (ax^3+bx^2)\ dx$

(a and b are constants)

(3) Find an expression for $f(x)$ for each of the following:

(a) $f'(x)=3x-4$

(b) $f'(x)=x^3-4x^{-5}$

(c) $f'(x)=x^{\frac{1}{2}}+\sqrt{2}x^{-2}$

(d) $f''(x)=\frac{1}{2}x^{\frac{1}{3}}+8x^{-5}$

(4) Find an expression for y given the following:

(a) $\frac{dy}{dx}=4x-x^2$

(b) $\frac{dy}{dx}=3+4x-2x^3$

(c) $\frac{dy}{dx}=9x+c$

(c is a constant)

(d) $\frac{d^2y}{dx^2}=9-x$

(e) $\frac{dy}{dx}=(ax+b)^2$

(a and b are constants)

(5) Find the following indefinite integrals in the form $y=f(x)$:

(a) $\int (2+3x)(1-x)\ dx$

(b) $\int x\left(\sqrt{x}-3\right) dx$

(c) $\int \left(\frac{3x-1}{x^{0.5}}\right) dx$

(d) $\int \left(\frac{2\sqrt[3]{x}-3}{x^{0.25}}\right) dx$

(e) $\int \left(1-4\sqrt{x^5}\right) dx$

(6) Given $y=x\sqrt{x}-3,\ x>0$ find $\int y\ dx$.

(7) Find an equation for each of curves with the derivates given below:

(a) $\frac{dy}{dx}=2x+3$
Point $(3,20)$

(b) $\frac{dy}{dx}=6x^2+10x-3$
Point $(1,11)$

(c) $\frac{dy}{dx}=(1-x)(2+x)$
Point $\left(2,-\frac{8}{3}\right)$

(d) $\frac{dy}{dx}=\frac{3x^2-1}{\sqrt{x}}$
Point $(1,3)$

(e) $\frac{dy}{dx}=\frac{1}{\sqrt{x}}+0.25x^{-0.75}$
Point $(16,11)$

(f) $\frac{dm}{dt}=t(1-t)^2$
Point $(0,1)$

(g) $\frac{dp}{dt}=\sqrt[4]{t}\left(\frac{t-1}{2}\right)$
Point $(1,1)$

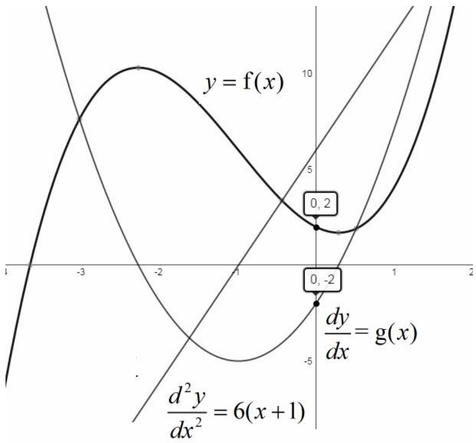
(8) The curve $y=f(x)$ passes through the point $(2,6)$.
Given $f'(x)=\frac{1}{2}x^3-2x+3$ find an equation for y in terms of x.

(9) The curve C passes through the point $(4,3)$. Find an equation for y in terms of x given $\frac{dy}{dx}=3x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}+2x+1$

(10) The curve $y=f(x)$ passes through the origin.
Given $f'(x)=3x^2-2x-6$ find the solutions to the equation $f(x)=0$.

(11) The gradient of a particular curve is given by $\frac{dy}{dx}=2x+c$ where c is a constant.
Given the curve passes through the points $(1,2)$ and $(3,14)$ find an equation for y in terms of x.

(12) The diagram below shows the line with equation
$\frac{d^2y}{dx^2}=6(x+1)$
and the curves of $f(x)$ and $g(x)$.
Find an equation for $y=f(x)$ given $g(0)=-2$ and $f(0)=2$.



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Core Mathematics C1

Advanced Subsidiary

Practice Paper 1

Time: 1 hour 30 minutes

(1) (a) Simplify $\left(1+2\sqrt{a}\right)\left(2-\sqrt{a}\right)$ giving your answer in terms of a. *(3 marks)*

(b) Solve the equation $\left(1+2\sqrt{a}\right)\left(2-\sqrt{a}\right)=0$ *(4 marks)*

(2) Simplify fully $\left(25p^3q^{\frac{1}{2}}\right)^{-0.5}$ *(3 marks)*

(3) The line l passes through the points $A(-1,-1)$ and $B(5, p)$. The gradient of line l is -1.

(a) Find the value of p *(2 marks)*

(b) Line l crosses the x axis at the point C and the y axis at the point D. Find the area of the triangle DOC where O is the origin. *(6 marks)*

(4) (a) The sum of the first 10 terms of an arithmetic sequence is -95. Given the common difference of the sequence is -3 find the first term of the sequence. *(4 marks)*

(b) Find the 8th term of the sequence. *(2 marks)*

(5) The curve $y = \mathrm{f}(x)$ passes through the point $(2, 20)$ and it's gradient function $\frac{dy}{dx} = 3x^2 + x + 3$.

(a) Find $\mathrm{f}(x)$ *(6 marks)*

(b) Show that the gradient of $\mathrm{f}(x)$ is never negative. *(4 marks)*

(6) A sequence is defined by

$$a_{n+1} = 2a_n - 1,\ a \geq 1$$
$$a_1 = 3p$$

(a) Find an expression for a_2 and a_3 in terms of p. *(2 marks)*

(b) Given $\sum_{i=1}^{4} a_i = 79$ find the value of p. *(4 marks)*

(7) $\mathrm{f}(x) = \frac{1}{3}x^3 - x^2 - 8x + 5$

(a) Draw the graph of $y = \mathrm{f}\,'(x)$ showing any points of intersection with the coordinate axis. *(4 marks)*

(b) Draw the graph of $y = 2\mathrm{f}\,'(x)$ showing any points of intersection with the coordinate axis. *(2 marks)*

(c) Draw the graph of $y = -\mathrm{f}\,'(x)$ showing any points of intersection with the coordinate axis. *(2 marks)*

(8) Find the equation of the normal to the curve $y = \frac{2x+\sqrt{x}}{x^{0.5}}$ at the point where $x = 4$ giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. *(6 marks)*

(9) (a) Solve the simultaneous equations

$$xy = 6$$
$$x^2 + y^2 = 13.$$

(8 marks)

(b) Draw the graph of $y = \frac{1}{x}$ stating the equations of any asymptotes. *(3 marks)*

(c) Hence draw the graph of $xy = 6$ stating the equations of any asymptotes. *(2 marks)*

(10) (a) Given the line $y = 2x - p$ is a tangent to the curve $y = x^2 + 4x - 11$ find the value of p. *(5 marks)*

(b) Find the point where the line $y = 2x - p$ meets the curve $y = x^2 + 4x - 11$ *(3 marks)*

End of Questions

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Core Mathematics C1

Advanced Subsidiary

Practice Paper 2

Time: 1 hour 30 minutes

(1) Find (a) $\int\left(2x^{\frac{3}{2}}-x+5x^{-2}\right)dx$ *(3 marks)*

(b) $\frac{d}{dx}\left(4x^3-3\sqrt{x}+\frac{5}{x^4}\right)$ *(3 marks)*

(2) (a) Express $-x^2+4x-7$ in the form $a(x+b)^2+c$ *(3 marks)*

(b) Hence sketch the graph $y=-x^2+4x-7$ stating the coordinates of the minimum point. *(3 marks)*

(c) Show the equation $0=-x^2+4x-7$ has no real roots. *(2 marks)*

(3) Find the values of a,b and c such that $\frac{1+\sqrt{12}}{1+\sqrt{3}}\equiv\frac{a+b\sqrt{3}}{c}$ *(4 marks)*

(4) Solve the equation $x^{0.5}-\frac{6}{x^{0.5}}=1$ *(5 marks)*

(5) (a) Find the equation of the line l which is perpendicular to the line $3x+4y-7=0$ and passes through the point $A(1,2)$ in the form $y=mx+c$ *(4 marks)*

(b) The line l crosses the x axis at C and the y axis at D. Find the length of CD giving your answer as a simplified fraction. *(5 marks)*

(6) Find the set of values of k for which the equation $2x^2+kx+(k-3)=-1$ has two distinct real roots. *(5 marks)*

(7) John starts running each day. He runs 2 miles on the first day and doubles the distance each day after.

(a) Explain why John's mileage doesn't form an arithmetic sequence. *(1 marks)*

Fred also starts running on the same day as John. Fred runs 1 mile on the first day and then increases the number of miles he runs each day by m miles such that his mileage forms and arithmetic sequence.

(b) Given John and Fred run the same distance on the 4th day find the value of m. *(4 marks)*

(c) Find the total number of miles Fred had run after the 6th day. *(3 marks)*

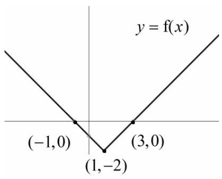
(8) The diagram below shows part of the graph $y=\mathrm{f}(x)$. On separate diagrams draw the graphs of:

(a) $y=2\,\mathrm{f}(x)$ *(3 marks)*

(b) $y=\mathrm{f}(-x)$ *(3 marks)*

(c) $y=\mathrm{f}(x+1)+2$ *(3 marks)*

State the coordinates on each graph.



(9) (a) Solve the simultaneous equations

$$y+9=6x$$
$$x^2-y=0.$$
(7 marks)

(b) Sketch the graphs $y+9=6x$ and $x^2-y=0$ on the same set of axis. Using your answer from part (a) label any points where the 2 graphs meet. *(4 marks)*

(10) (a) Find an equation of the tangent to the curve $y=x^2-2\sqrt{x}$ at the point where $x=4$ *(6 marks)*

(b) By drawing two separate graphs on the same set of axis state the number of solutions to the equation $0=x^2-2\sqrt{x}$ *(4 marks)*

End of Questions

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Core Mathematics C1

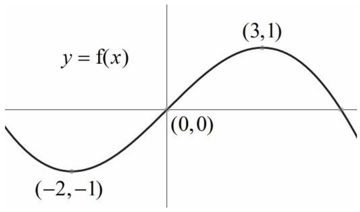
Advanced Subsidiary

Practice Paper 3

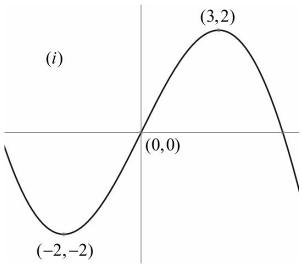
Time: 1 hour 30 minutes

(1) (a) Part of the curve $y = \mathrm{f}(x)$ is shown in *Figure 1* below. The curve has a minimum point at $(-2,-1)$ a maximum point at $(3,1)$ and passes through the origin. State fully the **single** transformation that maps $\mathrm{f}(x)$ onto the graphs of (a) (i) and (b) (ii) shown below. *(4 marks)*

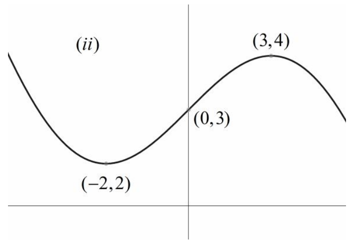
Figure 1



(i)



(ii)



(c) Sketch the graph of $y = \mathrm{f}(-x)$ stating the coordinates of the maximum and minimum points. *(3 marks)*

(2) (a) Find the values of a and b such that $x^4 - 16 \equiv (x^2+4)(x+a)(x+b)$ *(3 marks)*

(b) Hence of otherwise write down the 2 real solutions to the equation $x^4 - 16 = 0$ *(2 marks)*

(3) Simplify $2^{-1} pq^{0.5} \times 2pq^{\frac{3}{2}}$ *(3 marks)*

(4) The line l_1 is parallel to the line $3x - 2y = 4$ and passes through the origin O.

(a) Find an equation for the line l_1. *(3 marks)*

(b) Show that the point $A(8,12)$ lies on the line l_1. *(1 mark)*

(c) Find an equation for the line perpendicular to l_1 that passes through the midpoint of OA. *(4 marks)*

(5) (a) Show that $\dfrac{2}{3-\sqrt{8}}$ can be written in the form $6 + a\sqrt{2}$ where a is an integer to be found. *(3 marks)*

(b) A rectangle has an area of 2 and side lengths $3-\sqrt{8}$ and l. Hence or otherwise write down the length of l giving your answer as a simplified surd. *(2 marks)*

(6) An arithmetic sequence has first term a and common difference d. Given the 5th term of the sequence is 16 and the sum of the first ten terms is 175, find the 12th term. *(6 marks)*

(7) The point $(4,11)$ lies on the curve $y = \mathrm{f}(x)$. Given $\dfrac{dy}{dx} = \dfrac{2x^{\frac{3}{2}} - 3}{\sqrt{x}}$ find an expression for $\mathrm{f}(x)$. *(7 marks)*

(8) The circle $(x-3)^2 + (y-2)^2 = 5$ and the line $y = x+2$ intersect at the points A and B. Find the length AB giving your answer in the form $\sqrt{k}$ where k is an integer to be found. *(7 marks)*

(9) (a) Find an equation of the normal to the curve $y = x\left(3 - x^{\frac{3}{2}}\right)$ at the point where $x = 1$. *(5 marks)*

(b) The normal to the curve meets the line $y = 2x$ at the point A. Find the coordinates of A. *(3 marks)*

(c) Show the line $y = 2x$ pass through the origin O. *(1 marks)*

(d) The normal to the curve crosses the y axis at the point B. Find the area of the triangle OBA. *(4 marks)*

(10) Show that the function $\mathrm{f}(x) = x^2 - 2x + 3$ is positive for all values of x. *(5 marks)*

(11) (a) $\mathrm{f}(x) = 2x^2 - kx + (k-1)$. Given the discriminant of $\mathrm{f}(x) = -8$, find the value of k. *(4 marks)*

(b) Sketch the graph of $y = \mathrm{f}(x)$ showing the coordinates of the minimum point. *(5 marks)*

End of Questions

Core 1 Formulae

Mensuration

Volume of a Sphere $= 4\pi r^3$

Area of curved surface area of a cone $= \pi r \times slant\ height$

Arithmetic Series

$u_n = a + (n-1)d$

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\left[2a + (n-1)d\right]$

www.m4ths.com – C2 – Algebra and Functions

(1) Simplify the following algebraic fractions:

(a) $\frac{x^2+2x}{x}$

(b) $\frac{2x^2-3x+5}{x}$

(c) $\frac{x^2-x-12}{x-4}$

(d) $\frac{2x^2-5x-12}{2x+3}$

(e) $\frac{4x^2-25}{2x-5}$

(f) $\frac{a^2-b^2}{2(a+b)}$

(2) Explain whether or not you can use long division to simplify the following fractions:

(a) $\frac{x^2+2x+4}{2x^2+3x+1}$

(b) $\frac{4x^3-2x^2+3}{x+1}$

(c) $\frac{3x^2+x-4}{x^3+7x+4}$

(3) Find the quotient when x^3+2x^2-4x+1 is divided by $x-1$.

(4) Find the quotient **and** remainder when $x^4+3x^3+x^2-2x+1$ is divided by $x-2$.

(5) Find the quotient **and** remainder when $2x^4+3x^2+x-3$ is divided by $x+3$.

(6) Simplify $\frac{4x^3-7x^2+2x+1}{2x-3}$

(7) Show that $(x+2)$ is a factor of x^3-x^2+x+14

(8) State which of the following are factors of $2x^4+3x^3-24x^2-13x+12$:

(i) $(x-3)$

(ii) $(x-1)$

(iii) $(2x-1)$

(iv) $(x+4)$

(9) Explain why $(3x-2)$ is not a factor of x^4+5x^2+2x-1.

(10) Given that $(x-2)$ is a factor of $2x^3-x^2+2p+3$ find the value of p.

(11) $f(x)=x^3+px^2+qx+6$ Given that $(x-3)$ and $(x+1)$ are factors of $f(x)$, find the values of p and q.

(12) $g(x)=2x^3-7x^2-10x+24$ Given that $(x-4)$ is a factor of $g(x)$, fully factorise $g(x)$.

(13) Solve the equation $x^3+x^2-17x+15=0$.

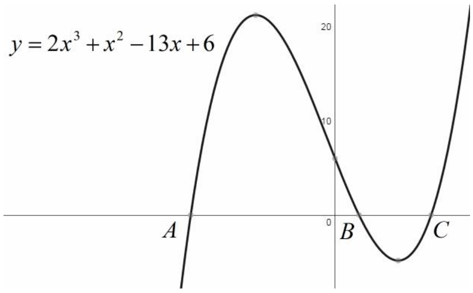
(14) Find the remainder when x^3+2x^2-4x+2 is divided by $(x-1)$.

(15) When $4x^3-px^2+3$ is divided by $(x+1)$ the remainder is 4. Find the value of p.

(16) $f(x)=2x^3+px^2+x+q$ When $f(x)$ is divided by $(x+3)$ the remainder is -12. Given also $(x-1)$ is a factor of $f(x)$ find the values of p and q.

(17) Given when $4x^2-ax+3$ is divided by $(x+1)$ the remainder is the same as when it's divided by $(x-2)$, find the value of the constant a.

(18) The graph below shows part of the curve $y=2x^3+x^2-13x+6$. Given that $A=-3$, find the values of B & C.



(19) $f(x)=3x^3+4x^2+px-2$

(a) Given $(x-1)$ is a factor of $f(x)$ show that $p=-5$.

(b) Find all of the solutions to the equation $f(x)=0$.

(20) Given that $\frac{x^4-x^3-19x^2-11x+30}{(x+2)}$ can be written in the form (Ax^3+Bx^2+Cx+D) show that $A+B+C+D=0$.

(21) When $4x^3+ax^2+bx-2$ is divided by $(1-2x)$ the remainder is 6.

(a) Find a linear relationship between a and b.

(b) Given further that $\frac{a}{3}=b$, find the value of $(ab)^{0.5}$ in the form $k\sqrt{3}$ where k is a constant to be found.

(22) Sketch the graph of $y=2x^3-5x^2-x-6$ showing any points of intersection with the coordinate axis.

www.m4ths.com – C2 – Coordinate Geometry

(1) Find the midpoint of the following points:
(a) $(3,7)$ and $(1,3)$
(b) $(-2,3)$ and $(4,-3)$
(c) $\left(1,\frac{3}{4}\right)$ and $\left(\frac{7}{4},-2\right)$

(2) Find the length of line segment AB for the following:
(a) $A\ (1,3)$ and $B\ (2,5)$
(b) $A\ (-3,2)$ and $B\ (2,-5)$
(c) $A\ (2,0.5)$ and $B\ (-1,3)$
(d) Given that the line segment AB is a diameter of a circle, state the radius of each circle in questions a – c.

(3) The points $(2,1)$ and $(-2,-1)$ are the end points of a diameter of a circle.
(a) Find the centre of the circle
(b) Find the length of the radius

(4) (a) The point $A\left(1,\frac{5}{4}\right)$ is a point on the circle C. Given that the centre of the circle has coordinates $(1.75,-1)$, find the coordinates of the point B such that AB is a diameter of the circle C.
(b) Find the length of the diameter AB to 3 significant figures.

(5) Write down the centre of each circle and the length of its radius:
(a) $x^2+y^2=25$
(b) $(x-1)^2+(y+2)^2=16$
(c) $(x+5)^2+(y-0.5)^2=32$
(d) $(y+2)^2+(x-14)^2=27$
(e) $(x-3)^2+y^2-0.01=0$
(f) $(2x-3)^2+(2y+5)^2=36$
(g) $(x-p)^2+(y+q)^2=r$
(h) $(px+4)^2+(py+2)^2=p^2$

(6) Find the centre of each circle and the length of its radius where possible:
(a) $x^2+y^2+2x-4y=20$
(b) $x^2+y^2-3x+y=13.5$
(c) $x^2-\sqrt{3}x+y^2=24.25$
(d) $x^2+2x=4y+y^2$
(e) $x^2+y^2+px-4py=17p$
(f) $2x^2+2y^2+4x-3y=26$
(g) $y^2+(x-3)^2=8y$
(h) $qx^2-6x-2y+qy^2=0$
(Can you spot the equation that isn't a circle?)

(7) The point $A\ (2,3)$ lies on the circle C. The centre of the circle has coordinates $(8,1)$. Find an equation for the circle.

(8) The points $P\ (2,3)$ and $Q\ (6,9)$ lie on the circle C. Given PQ is a diameter of the circle, find an equation of the circle C.

(9) The circle with equation $x^2-2x-4y+y^2=48$ crosses the x axis at the points A and B.
(a) Find the area of the triangle ABC where C is the centre of the circle.
(b) The circle crosses the y axis at the points D and E. Find the length of the chord DE in the form $p\sqrt{q}$ where q is a prime number.

(10) A circle has equation $(x-4)^2+(y+3)^2=20$. State whether the following points are inside, on the circle or outside the circle:
(a) $(5,1)$, (b) $(0,7)$, (c) $(9,-2)$

(11) A circle with centre $(6,-1)$ passes through the point $T(-3,2)$. Find an equation for the tangent to the circle at the point T.

(12) A tangent to the circle C at the point P passes through the point $Q(10,-3)$. Given that the equation of circle C is $(x-2)^2+(y+1)^2=16$, find the length of the line PQ.

(13) A circle has equation $(x-3)^2+(y-5)^2=100$.
(a) Show that the points $P(11,-1)$ and $Q(-3,-3)$ lie on the circle.
The line l is the perpendicular bisector of the chord PQ.
(b) Show that line l passes through the centre of the circle.

(14) The points $A\ (-2,12)$, $B\ (-5,11)$ and $C\ (3,-3)$ lie on a circle. Find an equation of the circle.

(15) The points $A\ (0,4)$, $B\ (-3,-5)$ and $C\ (6,-8)$ lie on a circle.
(a) Prove that AC is a diameter of the circle.
(b) Find an equation of the circle.

(16) A circle has equation $(x+4)^2+(y-7)^2=90$.
(a) Write down the coordinates of the centre and the length of the radius.
The points $A\ (-13,10)$ and $B\ (-7,-2)$ lie on the circle.
(b) Find the area of the triangle ABC where C is the centre of the circle.

(17) A circle touches the y axis at the point $(0,-8)$ and crosses the x axis at the points $(-4,0)$ and $(-16,0)$. Find an equation for the circle.

(18) A circle passes through the points $A\ (6,3)$ and $B\ (-2,11)$ and has centre $C\ (-2,p)$.

(a) Find the value of p

(b) Find an equation of the tangent to the circle at the point $(6,3)$.

(c) The point D has coordinates $(-10,3)$. Show that AD is a diameter of the circle.

(d) Show that $\angle ABD = 90^0$

(19) The line $y = x + c$ is a tangent to a circle with the equation $\left(x-4\right)^2 + \left(y-1\right)^2 = 98$.

(a) Find the possible values of c.

(b) Find the possible points where the tangent could touch the circle.

(20) The circle with equation $\left(x-6\right)^2 + \left(y+4\right)^2 = r^2$ does not cross either coordinate axis.

(a) Find the set of value of r^2 that satisfy this condition.

(b) Given further that $r^2 = 9$, find the coordinates of the point P such that P is the furthest point on the circle from the x axis.

(c) Write down the equation of the tangent to the circle at the point P.

www.m4ths.com – C2 – Logarithms and Exponentials

(1) Write the following in the form $\log_a b = c$:

(a) $2^3 = 8$

(b) $7^2 = 49$

(c) $4^2 = 16$

(d) $3^{-2} = \frac{1}{9}$

(e) $6^0 = 1$

(f) $10^{-3} = 0.001$

(g) $p^q = r$

(2) Write the following in the form $a^b = c$:

(a) $\log_5 25 = 2$

(b) $\log_2 16 = 4$

(c) $\log \frac{1}{10} = -1$

(d) $\log_t s = r$

(3) Evaluate the following without a calculator:

(a) $\log_2 32$

(b) $\log_5 5$

(c) $\log_8 1$

(d) $\log 1000$

(e) $\log_9 27$

(f) $\log_4 \frac{1}{64}$

(g) $\log_8 \frac{1}{16}$

(h) $\left(\log_{27} \frac{1}{81}\right)^2$

(i) $\log_{0.5} \sqrt{16}$

(4) Find the value of x in the following. Give your answers to 3 S.F where appropriate:

(a) $\log_3 16 = x$

(b) $\log_6 x = -2$

(c) $\log_4 9 = x$

(d) $\log_{\frac{1}{3}} x = -2$

(e) $\log_{2.4} 13 = x$

(f) $\log_4 x = -0.17$

(g) $\log_x 16 = 2$

(5) Simplify the following:

(a) $\log_p p^2$

(b) $3\log_r r^5$

(c) $(\log_2 8) \times \left(\log_p \frac{1}{p}\right)$

(6) Write the following in the form $a \log_b c$:

(a) $\log_b c^4$

(b) $3\log_b c^3$

(c) $5\log_b \frac{1}{c}$

(d) $0.25\log_b \sqrt{c}$

(7) Write the following in the form $\log_b c^a$:

(a) $2\log_b c^5$

(b) $4\log_b \sqrt[3]{c}$

(c) $-2\log_b \frac{1}{c^3}$

(d) $0.75\log_b c^{0.25}$

(8) Rewrite the following as single logarithms:

(a) $\log 2 + \log 3$

(b) $\log_b a + \log_b c^2$

(c) $2\log_b p + \log_b 5c$

(d) $\log 5 - \log 2$

(e) $2\log 3 - 5\log 2$

(f) $3\log_b p - 2\log_b r$

(g) $\log 3 + \log \frac{1}{9}$

(h) $2\log a + 5\log b - \log \sqrt{c}$

(i) $0.5\log_8 x - \log_8 3y + \log_8 \sqrt{x}$

(9) Express the following in the form $\log a + \log b$:

(a) $\log p^2 q$

(b) $\log 2x^3$

(c) $\log \frac{p}{r^4}$

(d) $3\log p\sqrt{q}$

(e) $-\log \frac{\sqrt[4]{p}}{q^{0.4}}$

(10) Express the following in the form $a\log x + b\log y$:

(a) $\log y^2 x^3$

(b) $\log \frac{\sqrt{x}}{y^5}$

(c) $3\log\left(\frac{y^{\frac{1}{6}}}{\sqrt[3]{x}}\right)$

(11) Simplify the following:

(a) $\log 8 + \log 12.5$

(b) $\log_5 100 - \log_5 4$

(c) $\log_6 2 + \log_6 108 + 2\log_6 6$

(d) $2\log_6 2 + \log_6 9$

(e) $\log_2 80 - \log_2 5 + 3\log_2 32$

(12) Given $\log_2 p = a$ and $\log_2 q = b$, simplify the following giving your answers in terms of a and b:

(a) $\log_2 pq^2$

(b) $\log_2 \frac{8q}{p}$

(c) $0.5\log_2 \sqrt{32p^3q^4}$

(13) Solve the following giving your answers to 3 S.F:

(a) $3^x = 14$

(b) $5^{x-1} = 9.4$

(c) $2 \times 6^{2x+3} = 3.4$

(d) $2^{1-3x} + 3.1 = 9.7$

(14) Solve the following giving your answers to 3 S.F:

(a) $3^{x-1} = 2^{x+2}$

(b) $5^{2x-3} = 7^{x+1}$

(c) $7 \times 5^{2x-3} = 7^{x+1}$

(d) $10 \times 7^{x-3} = 9^{x+1}$

(15) Solve the following equations giving your answers to 3 S.F where appropriate:

(a) $3^{2x} - 3^x - 2 = 0$

(b) $2^{2x} = 7(2^x) - 12$

(c) $6(4^{2x}) + 13(4^x) = 5$

(d) $2^{2x+1} - 1 = 2^x$

(16) Solve the following equations giving your answers to 3 S.F where appropriate:

(a) $\log_2(x-4)=3$

(b) $\log_3(2x-1)=\log_3(x+1)+2$

(c) $\log_2(x)=4-\log_2(x+6)$

(d)
$\log_4(x-1)=1.7-\log_4(x+2)$

(e)
$2\log_5(x+1)=\log_5(x+2)+1.9$

(f) $2\log_2(x-3)=\dfrac{3}{\log_2(x-3)}$

(17) Solve the following equation giving your answers to 3 S.F where appropriate:
$\log_2(2x-1)=\log_4(x+3)+0.5$

(18) Solve the simultaneous equations:

$$\log_2\left(\frac{x}{y^2}\right)=-3$$

$$3\log_8\left(4x\sqrt{y}\right)=4$$

(19) Sketch the following graphs stating the coordinates of any points of intersection with the coordinate axis and the equations of any asymptotes:

(a) $y=2^x$

(b) $y=5^x$

(c) $y=\left(\frac{1}{2}\right)^x$

(d) $y=3^{x-1}$

(e) $y=4^x+2$

(f) $y=1-2^x$

(20) Given that $\log_5 p=a$ and $\log_5 q=b$, find an expression in terms of a and b for:

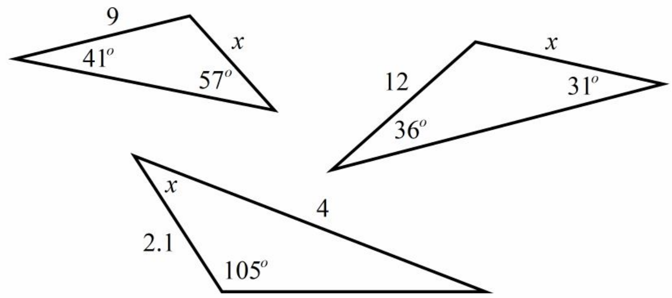
$$2\log_5\left(\frac{p^3}{25\sqrt{q}}\right)$$

(21) Show there is only one real solution to the equation
$\log_4(x+5)=1.5-\log_4(x-2)$
and find the solution to the equation.

(22) (a) Sketch the graphs of $y=2^{x-1}$ and $y=0.5^x$ on the same set of axis showing any points of intersection with the coordinate axis and state the equation of any asymptotes.
(b) Solve the equation $2^{x-1}=0.5^x$
(c) State fully the two transformations that map the curve $y=0.5^x$ onto the curve $y=3-0.5^x$.

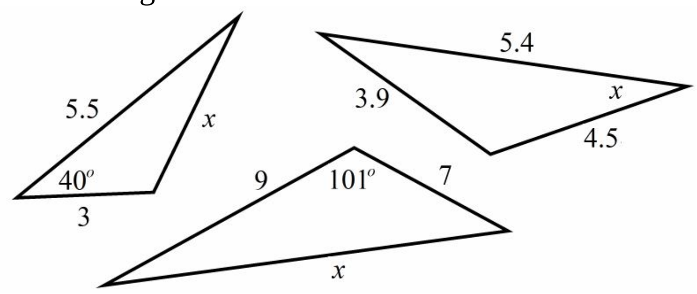
www.m4ths.com – C2 – Sine and Cosine Rule

(1) Use the sine rule to find the value of x in each of the triangles below:

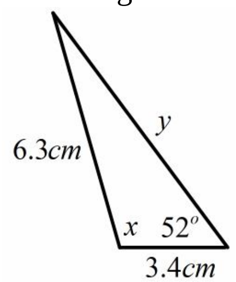


(2) In triangle ABC, $AB = 10cm$, $BC = 6cm$ and $\angle BAC = 35^o$.
Find the two possible sizes of $\angle ACB$ giving you answer to 3 significant figures.

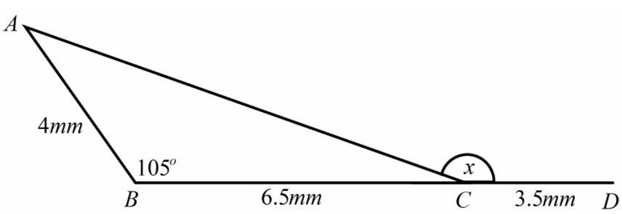
(3) Use the cosine rule to find the value of x in each of the triangles below:



(4) Find the value of x and y in the triangle below giving each answer to 3 significant figures.

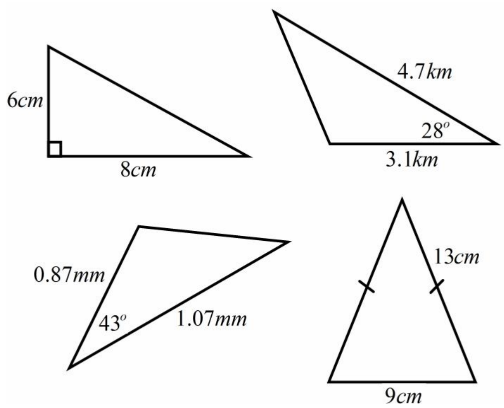


(5) In the diagram below BCD is a straight line.
(a) Find the size of the obtuse angle x.
(b) Find the length AD giving your answers to 3 significant figures.



(c) A line from point A is drawn such that it's perpendicular to the line BCD. Find the shortest distance from point B to the line.

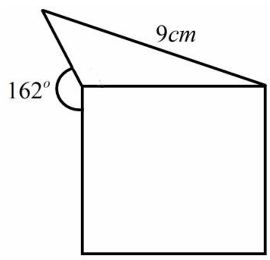
(6) Find the area of each triangle giving your answers to 1 decimal place where appropriate.



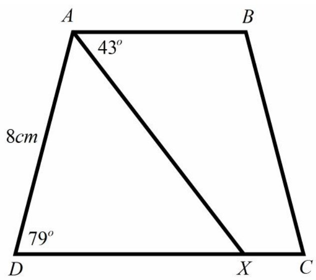
(7) Fred is standing at a point looking north. He walks on a bearing of 056^o for $9.8km$ before stopping. He then walks an additional $3.5km$ on a bearing of 112^o before stopping again. Find out how far he is away from his starting point.

(8) Sue walks around the perimeter of a triangular field. She walks west from one corner of the field for $480m$ before stopping at the second corner. She then walks an additional $312m$ on a bearing of 072^o to complete the second side of the field.
(a) How long is the third side of the field?
(b) Find the total area of the enclosed field.

(9) The diagram below shows a square with a triangle attached to one side. The triangle and the square share one side length. Given that the area of the square is $49cm^2$, find the area of the triangle as a percentage of the area of the square. Round your answer to the nearest one percent.



(10) Below is a picture of the isosceles trapezium $ABCD$. The line BX is perpendicular to the line DC, $\angle BAX = 43^o$ and $\angle ADX = 79^o$.



(a) Find the length of the line AX.
(b) Find the area of ΔADX
(c) Find the area of the quadrilateral

www.m4ths.com – C2 – Binomial Expansion

(1) Use Pascal's Triangle to fully expand the following:

(a) $(x+2)^3$

(b) $(x-3)^4$

(c) $(2x+1)^3$

(d) $(a+b)^5$

(e) $(3-p)^5$

(f) $(1+x)(x+4)^3$

(2) (a) Find the expansion of $(3+x)^4$.

(b) Hence find the expansion of

(i) $(3-x)^4$

(ii) $(3+y^2)^4$

(3) Find the value of the following:

(a) ${}^3C_2$

(b) ${}^5C_3$

(c) ${}^4C_1$

(d) $\binom{7}{5}$

(4) Use the ${}^nC_r$ method for finding coefficients to find the first 4 terms in the expansion of the following:

(a) $(1+2x)^6$

(b) $(3-x)^{10}$

(c) $(2-3x)^8$

(d) $(a-2b)^{12}$

(e) $(2+x)(1-4x)^7$

(5) (a) Expand fully $(1+2x)^5$ in ascending powers of x.

(b) Hence write down the full expansion of $(1-2x)^5$.

(c) Simplify $(1+2x)^5+(1-2x)^5$

(6) (a) Find the term in x^5 in the expansion of $(5-x)^{12}$.

(b) Find the term in x^7 in the expansion of $\left(1+\frac{x}{3}\right)^9$.

(c) Find the term in x^{18} in the expansion of $\left(0.5+x^3\right)^{13}$.

(7) Find the term in x^3 in the expansion of $(3+2x)(1-x)^6$.

(8) Find the term independent of y in the expansion of:

(a) $\left(y+\frac{1}{y}\right)^6$

(b) $\left(2y-\frac{1}{y^2}\right)^{12}$

(9) The coefficient of the term in x^2 in the expansion of $(3+px)^5$ is 1080. Given that $p>0$, find the value of p.

(10) Given that the coefficient of the term in x in the expansion of $(2+ax)^4$ is 12, find the coefficient of the term in x^3.

(11) (a) Find the first four terms in the expansion of $(1+2x)^8$.

By using a suitable substitution for x and the answer found in part (a), approximate:

(b) $(1.01)^8$

(c) $(0.98)^8$

(Round each answer to 4 decimal places)

(d) Explain what would happen to the accuracy of your answer in parts (b) and (c) if you use (i) the first 3 terms and (ii) the first 7 terms instead of the first 4 terms found in part (a).

(12) Use the binomial expansion

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\ldots$$

to find the first 4 terms in the expansion of:

(a) $(1+2x)^9$

(b) $(1-0.5x)^7$

(c) $\left(1+\frac{x}{2}\right)^6$

(d) $(3-x)^{10}$

(e) $(1-5x)^{\frac{1}{2}}$

(13) Given that $n>0$ and the coefficient of the term in x^2 in the expansion of $(1+2x)^n$ is 40, find the value of n.

(14) Given that n and p are both positive integers and that $(1+px)^n=1+12x+54x^2+\ldots$, find the coefficient of the term in x^3 in the expansion of $(1+px)^n$.

(15) (a) Expand $(1+y)^4$ in ascending powers of y.

(b) Using your answer to part (a) and a suitable substitution, find the value of $\left(1+\sqrt{2}\right)^4-\left(1-\sqrt{2}\right)^4$ in the form $p\sqrt{2}$.

(16) Find, in fully factored form, the first 3 terms in the expansion of $(2+x)(1+px)^n$ giving your answer in terms of n, p and x.

(17) (a) Find the first 4 terms in the expansion of $(1+3x)^6$.

(b) By using a suitable value of x and your answer to part (a) to find an approximate value of $(1.03)^6$ correct to 5 dp.

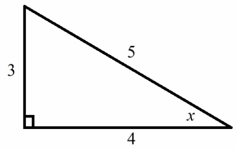
(c) Find the percentage error between the approximation found in part (b) and the actual value of $(1.03)^6$.

www.m4ths.com – C2 – Trigonometry (1)

(1) Using the triangle below, show that:

(a) $\dfrac{\sin x}{\cos x} = \tan x$

(b) $\sin^2 x + \cos^2 x = 1$



(2) Simplify the following expressions:

(a) $\dfrac{\sin 3\theta}{\cos 3\theta}$

(b) $4\sin^2 2x + 4\cos^2 2x$

(c) $3 - 3\cos^2 5x$

(d) $\dfrac{3\sin^2 4p}{\sin 4p\sqrt{1-\sin^2 4p}}$

(3) Show the expression $(\sin x + \cos x)^2 - (\sin x - \cos x)^2$ can be written as $k\sin x\cos x$ stating the value of k.

(4) Prove the following identities:

(a) $\dfrac{\sin^2 x - \cos^2 x}{\sin x + \cos x} \equiv \sin x - \cos x$

(b) $\dfrac{\sqrt{1-\cos^2 3x}}{\sqrt{1-\sin^2 3x}} \equiv \tan 3x$

(c) $\sin^4 x - \cos^4 x \equiv 1 - 2\cos^2 x$

(d) $\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x} \equiv \dfrac{1}{\sin x\cos x}$

(5) Given $\sin A = \dfrac{3}{5}$ and that A is obtuse, find the values of:

(a) $\cos A$

(b) $\tan A$

(6) Given $x = 3\cos A$ and $y = 2\sin A$, write an equation connecting y and x.

Give all answers to 1 decimal place where appropriate.

(7) Solve the following equations in the interval $0 \le x \le 360^o$:

(a) $\sin x = 0.5$

(b) $\cos x = \dfrac{1}{\sqrt{2}}$

(c) $\tan x = 1$

(d) $\cos x = \dfrac{\sqrt{3}}{2}$

(e) $\sin x = -\dfrac{\sqrt{3}}{2}$

(8) Solve the following equations in the interval $0 \le x \le 2\pi$ giving your answers as multiples of π:

(a) $\sin x = \dfrac{\sqrt{3}}{2}$

(b) $\tan x = \dfrac{1}{\sqrt{3}}$

(c) $\cos x = -\dfrac{1}{2}$

(d) $\tan x = -1$

(9) Solve the following equations for $0 \le x \le 360^o$:

(a) $\sin x = 0.24$

(b) $\cos x = 0.83$

(c) $3\tan x - 1 = 2.12$

(d) $4\sin x = -1.08$

(10) Solve the following equations for $0 \le x \le 2\pi$:

(a) $\cos x = -0.54$

(b) $\tan x = 3.7$

(c) $1 - \sin x = 0.43$

(d) $2\cos x = \sin x$

(11) Solve the following equations in the interval $-180^o \le x \le 180^o$:

(a) $\sin\left(x - 30^o\right) = \dfrac{\sqrt{3}}{2}$

(b) $\cos\left(x + 45^o\right) = \dfrac{1}{2}$

(c) $3\tan\left(x - 15^o\right) = \sqrt{3}$

(d) $2\sin(x + 60^o) = \sqrt{2}$

(12) Solve the following equations in the interval $0 \le x \le 2\pi$:

(a) $2\cos\left(x + \dfrac{\pi}{3}\right) = \sqrt{3}$

(b) $\tan\left(x - \dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{3}}$

(13) Solve the following equations for $0 \le x \le 180^o$.

(a) $\sin(3x) = \dfrac{\sqrt{3}}{2}$

(b) $\cos(2x) = 0.45$

(c) $\sin\left(3x - 20^o\right) = 0.3$

(d) $\tan\left(2x + 12^o\right) = 1.3$

(14) Solve the following equations the interval $0 \le x \le \pi$

(a) $\tan\left(3x - 1.2^c\right) = 0.4$

(b) $\sin\left(2x - 0.2^c\right) = -0.12$

(c) $2\cos\left(3x + 0.65^c\right) = 1.87$

(15) Solve the following equations in the interval $0 \le x \le 360^o$:

(a) $2\sin x = \sin x\cos x$

(b) $\sin\left(2x - 10^o\right) = \sin\left(50^o\right)$

(c) $\tan\left(3\theta - 20^o\right) = \tan\left(30^o\right)$

(16) Solve the following equations for $0 \le x \le 360^o$:

(a) $\sin^2 x = \dfrac{1}{2}$

(b) $\tan^2 2x = 1$

(c) $2\sin^2 x - \sin x = 1$

(d) $\left(\cos x - 1\right)\left(2\sin x - 1\right) = 0$

(e) $4\sin^2 x - 4\cos x - 1 = 0$

(f) $2\cos\dfrac{1}{2}x = \tan\dfrac{1}{2}x$

(17) Solve the following equations for $0 \le x \le 2\pi$:

(a) $\cos^2 x - \sin\left(\dfrac{\pi}{2} - x\right) = 2$

(b) $3\tan^2 x + 5\tan x - 2 = 0$

(c) $\sin x\left(2\sin x + 1\right) = 0$

www.m4ths.com – C2 – Trigonometry (2)

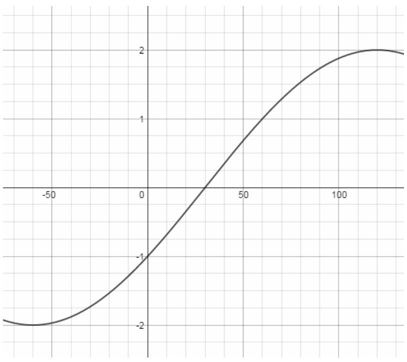
(1) Solve the equation
$2\cos^2 x - 9\sin x + 3 = 0$
in the interval $0 < x \le 2\pi$ giving your answers as multiples of π.

(2) (a) Show the equation $2\cos x + 3\sin x = 0$ can be written as $\tan x = -\frac{2}{3}$.
(b) Hence or otherwise solve the equation
$2\cos\frac{\theta}{2} + 3\sin\frac{\theta}{2} = 0$
in the interval $0 \le \theta \le 360^o$ giving your answer to 3 significant figures.

(3) (a) If $x = 3\cos\theta - 1$ and $y = 3\sin\theta + 2$ show that
$(x+1)^2 + (y-2)^2 = r^2$
stating the value of r.
(b) Sketch the graph of $(x+1)^2 + (y-2)^2 = r^2$ showing any points of intersection with the coordinate axis in exact from.

(4) Show that the equation $2\sin x = 3\tan x$ has 2 solutions in the interval $0 < x \le 2\pi$ giving the solutions as multiples of π.

(5) The graph below shows part of the curve $y = p\sin(x - q^o)$.
(a) Write down the values of p and q.
(b) Solve the equation
$p\sin(x - q^o) = \sqrt{3}$
in the interval $0 \le x \le 360^o$.



(6) (a) Show the expression $\sin^4 x - \cos^4 x$ can be written in the form $a\sin^2 x - 1$ stating the value of a.
(b) Hence or otherwise solve the equation
$\sin^4 2x - \cos^4 2x = -\frac{1}{2}$
in the interval $0 \le x \le 180^o$

(7) (a) Show the equation $1 + \tan x = 2\left(\frac{\cos x}{\sin x}\right)$ can be written as $\tan^2 x + \tan x - 2 = 0$.
(b) Solve the equation
$\tan^2 x + \tan x - 2 = 0$
for $-180^o < x \le 180^o$ giving your answers to 3 significant figures where appropriate.

(8) Given $\sin\alpha = 0.8$ and $90^o < \alpha < 180^o$ find the value of:
(a) $\cos\alpha$
(b) $\tan^3\alpha$
(c) $\sin\alpha\cos^2\alpha$

(9) (a) Sketch the graphs of $y = \sin 2x$ and $y = \cos 2x$ for $0 \le x \le 2\pi$ on the same set of axis.
(b) Using your graph show there are 4 solutions to the equation $\sin 2x = \cos 2x$ in the interval $0 \le x \le 2\pi$.
(c) Solve the equation $\sin 2x = \cos 2x$ for $0 \le x \le 2\pi$ giving your answers as multiples of π.

www.m4ths.com – C2 – Radian Measures

(1) Convert the following exact values into degrees:

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{4}$

(c) $\frac{3\pi}{5}$

(d) $\frac{7\pi}{12}$

(2) Convert the following into degrees giving your answers to 3 significant figures:

(a) 2.13^c

(b) 4.65^c

(c) 5.1^c

(3) Convert the following values into radians giving your answers in exact form:

(a) 180^o

(b) 135^o

(c) 270^o

(d) 60^o

(4) Convert the following into radians giving your answers to 3 significant figures:

(a) 134^o

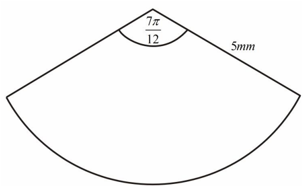
(b) 97^o

(c) 12^o

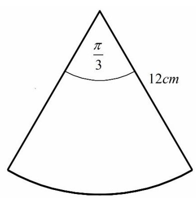
(For Q5 onwards all diagrams of sectors show the centre of the sector and 2 radii.)

(5) Find the arc length for each sector below giving your answer to 3 significant figures:

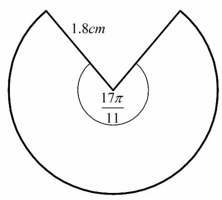
(a)



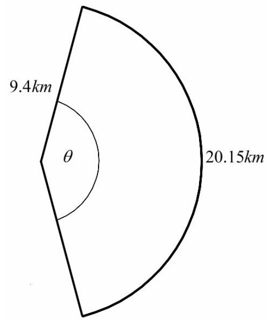
(b)



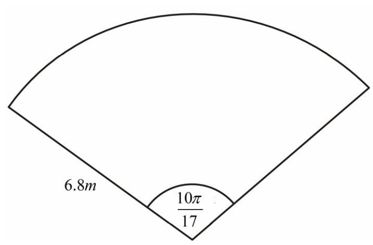
(c)



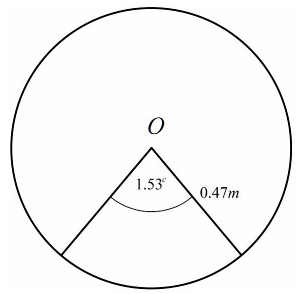
(6) Find the value of θ in the diagram below, given that the radius of the sector is $9.4km$ and the arc length is $20.15km$



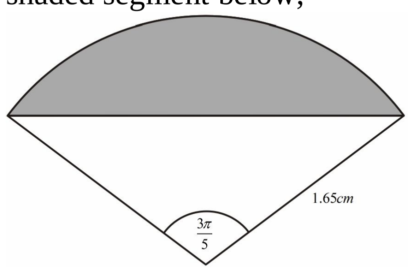
(7) (a) Find the area of the sector below:



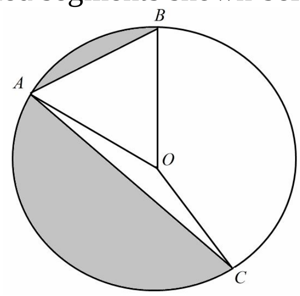
(b) Find the area of the major sector below:



(8) (a) Find the area of the shaded segment below;



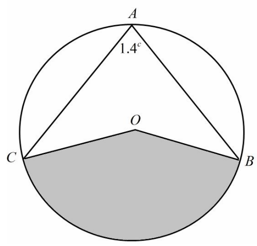
(b) A circle with centre O and radius $8m$ has the points A, B & C on its circumference. Given that $\angle AOB = 1.1^c$ and $\angle BOC = 2.05^c$, find the combined area of the two shaded segments shown below.



(9) A circle has centre O and radius $7.2cm$. A, B and C lie on the circumference of the circle. Given that $\angle CAB = 1.4^c$, find:

(a) The length of the minor arc BC.

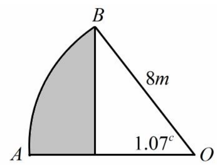
(b) The area of shaded sector.



(10) The diagram below shows a sector with centre O and radius $8m$. A line is drawn vertically from B to the line AO such that the two lines are perpendicular. Find:

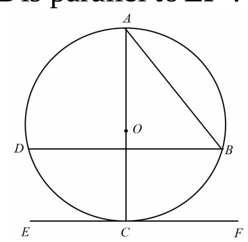
(a) The area of the shaded region.

(b) |The perimeter of the shaded region.



(11) Below is a picture of a circle with centre O and diameter $AC = 12cm$.

The line EF is a tangent and the line DB is parallel to EF.



Given that $\angle AOB = 2.01^c$ find:

(a) The arc length BC.

(b) The area of the ΔODB.

www.m4ths.com – C2 – Geometric Sequences/Series

(1) State which of the following are geometric sequences giving a reason for your answer.
(a) $2, 5, 8, 11...$
(b) $1, 3, 9, 27...$
(c) $0.5, 0.25, 0.125, 0.625...$
(d) $-2, 4, -8, 16...$
(e) $25ab, 5a^2b^2, a^3b^3, 0.2a^4b^4...$

(2) Find the common ratio for each of the following geometric sequences and write down the next two terms.
(a) $2, 6, 18, 54...$
(b) $80, 40, 20, 10...$
(c) $-3, 12, -48, 192...$
(d) $\frac{1}{5}, \frac{4}{15}, \frac{16}{45}, \frac{64}{135}$
(e) $t, 2t^3, 4t^5, 8t^7...$

(3) Find the 7th and 12th terms in each of the sequences below:
(a) First term: $a = 4$
Ratio: $r = 2$
(b) First term: $a = 0.5$
Ratio: $r = -3$

(4) Find the 9th and 14th terms in each of the sequences below:
(a) $5, 15, 45, 135...$
(b) $8, -4, 2, -1...$
(c) $35, 7, 1.4, 0.28$

(5) Find the 1st term of the geometric sequence with 2nd term 9 and 5th term $\frac{243}{8}$.

(6) A geometric sequence with a positive ratio has 3rd term 18 & 7th term 1458. Find the value of the 10th term.

(7) A geometric sequence has the first 3 terms $2, 2k, 9k+5...$, Given that $k > 0$, find:
(a) The value of k.
(b) The 7th term of the sequence

(8) A geometric sequence has the first 3 terms $2p, \frac{1}{2}, p^{-4}...$
(a) Find the value of p
(b) Write down the *nth* term for the sequence.
(c) Find the value of $a_8 - a_6$.

(9) A ball is dropped from a height of 5m above the floor. After bouncing once it reaches a height of 4m above the floor. The height reached by the ball after each subsequent bounce forms a geometric sequence.
(a) Find maximum the height above the floor the ball reaches after the 3rd bounce?
(b) Find the minimum number of times the ball will bounce before the maximum height reached above the floor is less than 1.18m.

(10) Find the sum of the first 8 terms for each geometric series
(a) 1st term $a = 4$ ratio $r = 0.1$
(b) 1st term $a = 0.4$ ratio $r = -3$
(c) 1st term $a = -5$ ratio $r = -0.3$

(11) Find the sum of the first 10 terms for each geometric series:
(a) $2 + 6 + 18, +54 + ...$
(b) $5 + 10 + 20 + 40 + ...$
(c) $8 - 2 + 0.5 - 0.125 + ...$

(12) Show that the sum of the first n terms of a geometric series with first term a and common ratio r is:
$$S_n = \frac{a(1-r^n)}{1-r}$$

(13) Evaluate the following:
(a) $\sum_{r=1}^{6} 3^r$
(b) $\sum_{r=1}^{8} 2 \times 0.5^r$
(c) $\sum_{r=1}^{9} 2^{r-1}$
(d) $\sum_{r=0}^{11} (2^r + 1)$

(14) Find the least value of n such that the sum of the first n terms of the geometric series $2 + \frac{5}{2} + \frac{25}{8} + \frac{125}{32} + ...$ exceeds 65.

(15) Fred starts a new job. He is paid £32000 in his first year and each year he works for the company he is paid 9% more than the previous year.
(a) Find how much Fred is paid in the 5th year.
(b) Find how much Fred earns in total by the end of the 12th year working for the company.

(16) Find the sum to infinity of the following geometric series:
(a) $4 + 2 + 1 + 0.5 + ...$
(b) $-10 + 2 - 0.4 + 0.08 + ...$
(c) $2p + \frac{1}{2} + p^{-4}...$

(17) Evaluate $\sum_{r=1}^{\infty} 3 \times (0.5)^r$

(18) A geometric series has first term 3.15 and the sum to infinity is 14.2. Find the ratio of the series as an exact fraction.

(19) Peter is doing his Maths homework. It takes him 4 minutes to do the 1st question and each subsequent question takes him 8% less time than the question before.
(a) Find out how long it takes him to complete the 12th question.
(b) Find out how long it takes him to complete the first 20 questions.
Give your answers to the nearest second.

(20) Sue pays £250 into a savings account each year that pays a fixed rate of 3.7% interest.
Find the total amount in the account, to the nearest penny, at the end of the 14th year.

www.m4ths.com – C2 – Differentiation

(1) Find the values of x for which each function below is an increasing function:

(a) $f(x) = x^2 - 2x + 4$

(b) $f(x) = \frac{1}{3}x^3 - x^2 - 8x + 9$

(c) $f(x) = 2 - 4x + 6x^2$

(d) $f(x) = \left(x^2 - 8\right)\left(x + 8\right)$

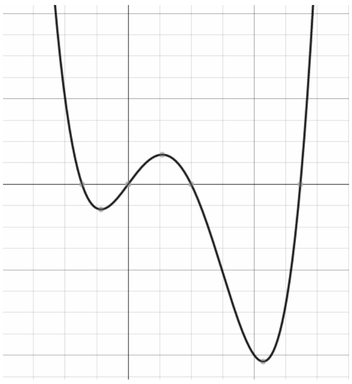
(e) $f(x) = 2x - 1$

(2) Find the values of x for which each function below is decreasing:

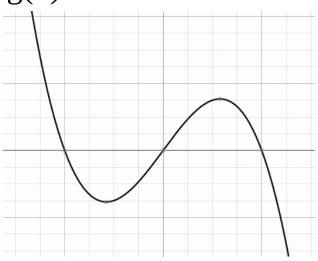
(a) $f(x) = 3x + x^{-1},\ x > 0$

(b) $f(x) = (3x^2 - 8)x$

(3) The graph below shows part of the curve $y = f(x)$. Mark the points $x = a$ and $x = b$, $0 < a < b$, such that $f(x)$ is a decreasing function for $a < x < b$.



(4) Below is part of the graph $y = g(x)$.



(a) Label the stationary points on the curve A and B, $A < B$.
(b) Determine the nature of the stationary points A and B.
(c) Explain why A and B are not points of inflexion.
(d) Draw a tangent to the curve at any point where the function is *(i)* increasing and *(ii)* decreasing.

(5) (a) Use differentiation to show that the coordinates of the stationary point on the curve $y = x^2 - 8x + 6$ are $(4, -10)$
(b) Show that this result is true by completing the square to find the stationary point.
(c) Determine the nature of the stationary point by sketching the graph of $y = x^2 - 8x + 6$.
(d) Show that your answer to part (c) is correct by evaluating the second derivative at $x = 4$.

(6) Find the coordinates of any stationary points for each of the following functions and determine their nature:

(a) $f(x) = x^2 + 4x$

(b) $f(x) = 3x - 0.5x^2$

(c) $f(x) = 2x^3 + 3x^2 - 72x$

(d) $f(x) = x^3 - 3x^2$

(e) $f(x) = 4 - 5x^3$

(f) $f(x) = 16x + x^{-2},\ x \neq 0$

(g) $f(x) = px^2 - qx,\ \ p < 0 < q$

(7) (a) Find the maximum value of the function

$f(x) = \left(1 - 2x\right)\left(3x^2 - 5\right),\ \ x \geq 0$

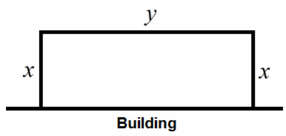
giving your answer to 3 significant figures.
(b) Prove that the answer found in part (a) is a maximum value by evaluating $f''(x)$.
(c) Is the function increasing or decreasing when $x = 3$?

(8) (a) Sketch the curve of $y = x^3 - 6x^2 + 5x + 12$ showing any points of intersection with the coordinate axes.
(b) Find the x coordinates of the stationary points giving your answers as exact fractions.
(c) Using the graph, determine the nature of the stationary points.
(d) On the same set of axes draw a different cubic equation that only has one stationary point. Write a possible equation for the curve.

(9) $g(x) = \frac{1}{x} + 16x,\ x \neq 0$

Show that the distance between the two stationary points on the curve $y = g(x)$ is $\frac{5\sqrt{41}}{2}$ units.

(10) James is making a pig pen. He has 240m of fencing and would like to make a rectangular shaped pen with the largest possible area. He needs to fence 3 sides of the pen as a wall provides the 4th side. The longer side of the fence is y meters long and the two shorter sides of the fence are both x meters long as shown below.



(a) Find an expression for y in terms of x
(b) Show that the area of the pen can be written as $A = 240x - 2x^2$
(c) Hence find the maximum area for the pen.

(11) A rectangular sheet of metal measuring $4cm$ by $5cm$ has a square of side length xcm cut from each corner and is made into an open topped tank.
(a) Find the maximum volume of the tank.
(b) Prove that the answer found in part (a) gives a maximum value for the volume of the tank

(12) An open top cylinder with radius rcm and height hcm is being made out of metal must hold exactly $20\pi cm^3$ of liquid.
(a) Find the length of the radius that minimises the amount of metal required to make the cylinder.
(b) Hence find the minimum amount of metal used.

(13) Sally is standing on level ground and throws a small ball into the air. The flight path of the ball is modelled by the equation $h = 1 + 6t - 3t^2$, $t \geq 0$ where h is the height in meters above ground and t is the time in seconds after it's thrown.

(a) Write down the height the ball was released from.

(b) Find the time taken for the ball to hit the ground giving your answer to 3 significant figures.

(c) Find the time taken for the ball to reach its maximum height above the ground.

(d) Find the maximum height the ball reaches above the ground.

(e) Show that the value found in part (d) is a maximum by using differentiation.

(14) $\mathrm{t}(x) = px^4 + qx^3 - 3x^2 + 4$

Given that the curve $y = \mathrm{t}(x)$ has a minimum at the point with coordinates $\left(2, -\frac{4}{3}\right)$, find the value of the constants p and q.

www.m4ths.com – C2 – Integration

(1) Evaluate the following integrals giving your answers to 3 significant figures where appropriate:

(a) $\int_{1}^{4}(2x+1)\,dx$

(b) $\int_{-2}^{0}\left(x^2+3\right)dx$

(c) $\int_{2}^{4}\left(3-\frac{1}{x^2}\right)dx$

(d) $\int_{1}^{8}\left(3x-\frac{1}{2x^2}-\sqrt{x}\right)dx$

(e) $\int_{2}^{4}\left(x^{-2}-3x^{-4}\right)dx$

(f) $\int_{3}^{8}\left(x^{\frac{1}{2}}-2x^{-\frac{3}{2}}\right)dx$

(g) $\int_{0.5}^{1.5}\left(\sqrt{x}-2\right)\left(\sqrt{x}+3\right)dx$

(2) Find the value of the following definite integrals giving your answers to 3 significant figures:

(a) $\int_{2}^{4}\left(\frac{2x^3-3x}{\sqrt{x}}\right)dx$

(b) $\int_{3}^{5}\left(\frac{3x^4-x+4}{x^{\frac{1}{3}}}\right)dx$

(3) Find the area enclosed by the x axis and the curve with equation $y=x(3-x)$ giving your answer as an exact fraction.

(4) Find the area enclosed by the by the curve with equation $y=\sqrt{x}$, the x axis and the lines $x=2$ and $x=4$ giving your answer to 3 significant figures.

(5) (a) Find the area enclosed by the x axis and the curve with equation $y=2x-x^{1.5}$.

(b) Find the area trapped between the curve, the x axis and the lines $x=1$ and $x=4$.

(6) (a) Show that the equation $y=x^3-x^2-6x$ can be written in the form $y=x(x+2)(x-3)$.

(b) Sketch the curve $y=x^3-x^2-6x$ showing any points of intersection with the coordinate axis.

(c) Find the area trapped between the x axis and the curve with equation $y=x^3-x^2-6x$.

(7) Find the area enclosed by the by the curve with equation $y=(x-1)(x-3)$, the x axis and the lines $x=0$ and $x=2$.

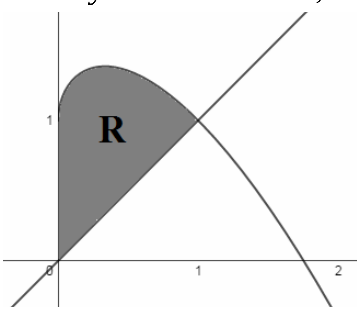
(8) (a) Sketch the curve $y=-x^2+7x-6$ and the line $y=x+2$ on the same set of axis showing any points of intersection with the coordinate axis.

(b) Find the coordinates of the points where the line cuts the curve.

(c) Find the area trapped between the curve $y=-x^2+7x-6$ and the line $y=x+2$ giving your answer as an exact fraction.

(9) Find the area trapped between the curve $y=x^2+2x+5$ and the line $y=5-x$ giving your answer as an exact fraction.

(10) The diagram below shows a sketch of the line $y=x$ and the curve $y=-x^{1.5}+x^{0.5}+1,\ x\geq 0$



(a) The coordinates of the point the where the line and the curve meet are $(1,p)$. Write down the value of p.

(b) The shaded region R is bounded by the line and the curve. Find the area of R.

(11) Given that $\int_{1}^{k}(3x-1)(x+1)\,dx=9$, find the value of the constant k.

(12) The line $y=5$ meets the curve $y=-4x+x^2,\ x\leq 0$ at the point $P(a,b)$.

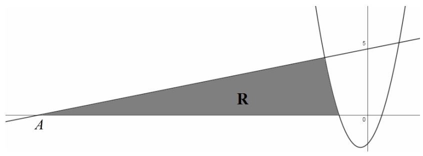
(a) Find the values of a and b.

(b) Find the area trapped between the curve, the line and the y axis to 2 decimal places.

(13) The normal to the curve $y=(x-1)(x+2)$ at the point $(-3,4)$ crosses the x axis at the point A shown in the diagram below.

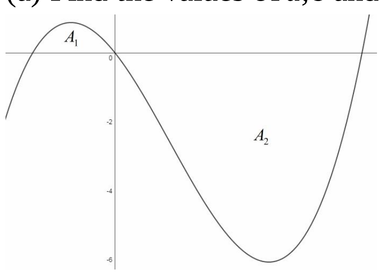
(a) Find the coordinates of A.

(b) Find the area R trapped between the normal, the curve and the x axis.



(14) Part of the curve $y=ax^3+bx^2+cx$ is shown below. The solutions to the equation $y=0$ are $x=-1,\ x=0$ and $x=3$.

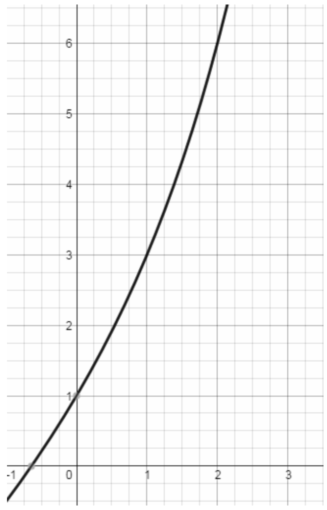
(a) Find the values of a,b and c.



(b) The areas A_1 and A_2 are the two regions trapped between the curve and the x axis. Find the ratio $A_1:A_2$

www.m4ths.com – C2 – Trapezium Rule

(1) The diagram below shows part of the curve with equation $y = x + 2^x$



(a) Fill in the missing values in the table below for $y = x + 2^x$

x	0	1	2
y			

(b) Use the trapezium rule with 2 strips to find an estimate for the area trapped under the curve $y = x + 2^x$ between the lines $x = 0$ and $x = 2$ using the values found in part (a).

(c) Fill in the missing values in the table below for $y = x + 2^x$

x	0	0.5	1	1.5	2
y					

(d) Use the trapezium rule with 4 strips to find an estimate for the area trapped under the curve $y = x + 2^x$ between the lines $x = 0$ and $x = 2$ using the values found in part (c).

(e) Explain which of the two estimations found in part (b) and (d) is closest to the actual area trapped under the curve $y = x + 2^x$ between the lines $x = 0$ and $x = 2$ giving a reason for your answer.

(f) How could you make the estimation ever more accurate?

(g) State whether the answers found in part (b) and (d) are overestimates or underestimates for the area under the curve giving a reason for your answer.

(2) (a) Sketch the curve $y = x^2 - 4x + 2$ showing any points of intersection with the coordinate axes.

(b) Using the trapezium rule with 4 strips, find an estimate for the area trapped under the curve $y = x^2 - 4x + 2$ between the lines $x = 5$ and $x = 13$.

(c) State with the aid of a sketch whether the estimate found in part (b) is an overestimate or an underestimate.

(d) Write down an equation for a quadratic function that would give an underestimate when using the trapezium rule.

(e) Use integration to find the exact area trapped under the curve $y = x^2 - 4x + 2$ between the lines $x = 5$ and $x = 13$.

(f) Find the percentage error between the answers found in part (b) and part (e).

(g) Explain what would happen to the percentage error in part (f) if 12 strips had been used in part (b).

(3) The trapezium rule is used with 2 strips to estimate the area trapped under the curve with equation $y = \frac{x-1}{x}$, $x \geq 1$, between the lines $x = k$ and $x = 3k$. Given that the estimate obtained using the trapezium rule is $\frac{17}{6}$ find the value of the constant k.

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Core Mathematics C2

Advanced Subsidiary

Practice Paper 1

Time: 1 hour 30 minutes

(1) $g(x) = px^3 - 2qx^2 + 4$

(a) When $g(x)$ is divided by $(x+1)$ the remainder is 2. Write down a linear relation between p and q. *(3 marks)*
(b) Given $g(2) = -28$ find the values of p and q. *(4 marks)*

(2) (a) State the two transformations that map the graph of $y = \cos(x)$ *to* $y = \cos(x-30)+1$ *(2 marks)*
(b) Solve the equation $2 = \cos(x-30)+1$ for $-360 < x < 180$ *(4 marks)*

(3) (a) Sketch the graph of $y = x^3 - x^2 - 6x$ showing any points of intersection with coordinate axis. *(3 marks)*
(b) Show the area trapped between the curve and the x axis to the left of the axis y is $\frac{16}{3}$. *(5 marks)*

(4) Fred is playing a computer game. The strength he has in each round is such that in round 1 he has a strength of 3 units, in round 2 he has a strength of 6 units, in round 3 he has a strength of 12 units and so on such that his strength in each level forms a geometric series.

(a) Find his strength in the 8th round. *(2 marks)*
(b) Find the number of rounds completed before his strength exceeds 8000 units. *(4 marks)*

(5) A circle has equation $x^2 + y^2 - 8x - 6y - 25 = 0$

(a) Find the centre of the circle. *(3 marks)*
(b) Find the length of the radius giving your answer in the form $p\sqrt{q}$ *(3 marks)*
(c) Show that the point $R(10,7)$ lies outside the circle. *(3 marks)*
(d)Find the equation of the tangent to the circle at the point $S(11,4)$ *(5 marks)*

(6) Company X is designing a mini rollercoaster. The path of the roller coaster is modelled by the equation $h = 18t - 2t^2$ where h is the height above ground level and t the time in seconds after the ride has started. The model is valid for $0 \le t \le 12$

(a) Find the time taken for the rollercoaster to return to ground level once in the ride has started. *(2 marks)*
(b) Find the maximum height of the rollercoaster above the ground and justify it's a maximum. *(5 marks)*

(7) (a) Show that the equation $\tan(x) = 2\sin(x)$ can be written as $(1-2\cos x)\sin(x) = 0$ *(3 marks)*
(b) Hence solve the equation $\tan(x) = 2\sin(x)$ for $0 \le x \le 2\pi$ giving your answers in terms of π *(4 marks)*

(8) $f'(x) = 3x^2 - \frac{2}{\sqrt{x}} + 4$ Given the point $(4,0)$ lies on $f(x)$ find an equation for $f(x)$ *(7 marks)*

(9) (a) Solve the equation $2^{1-3x} = 17$ giving your answer to 3 significant figures. *(3 marks)*
(b) Given $\log_3 p = a$ and $\log_3 q = b$ simplify $\log_3 27p^2q^3$ giving your answer in terms of a and b. *(5 marks)*

(10) Find the x coordinate of the stationary point of the curve with equation $m = x\left(\sqrt{x} - 12\right)$ giving your answer in the form 2^n where n is an integer to be found and determine the nature of the stationary point. *(5 marks)*

End of Questions

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Core Mathematics C2

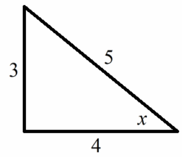
Advanced Subsidiary

Practice Paper 2

Time: 1 hour 30 minutes

(1) (a) Find the remainder when $f(x) = 2x^3 - 7x^2 + 2x + 3$ is divided by $(x-1)$ *(2 marks)*
(b) Hence or otherwise solve the equation $f(x) = 0$ *(4 marks)*
(c) State the maximum number of stationary points of graph $y = f(x)$ *(1 mark)*

(2) (a) Using the triangle below show that $\cos^2(x) + \sin^2(x) \equiv 1$ *(3 marks)*



(b) Given $0 < x < 90^\circ$ write down the value of $\tan(x)$ *(1 mark)*
(c) Solve the equation $2\sin^2(x) - 5\cos(x) + 1 = 0$ for $0 < x < 360^\circ$ *(6 marks)*

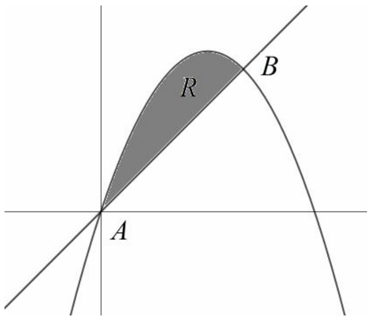
(3) Fred invests £2000 in a bank account that pays 4% compound interest at the end of each year.
(a) Show that at the end of the 3rd year his investment will be worth less than £2500. *(2 marks)*
(b) Find the number of years it will take for Fred's investment to be worth more than £6200. *(5 marks)*

(4) A circle has equation $(x-3)^2 + (y-2)^2 = 20$. The circle has centre C and crosses the x axis at the points A and B.
(a) Write down the coordinates of C. *(1 mark)*
(b) Find the coordinates of A and B. *(4 marks)*
(c) Find the size of angle ACB giving your answer in radians to 3 significant figures. *(4 marks)*

(5) (a) Solve the equation $\log_6(x+3) = 1 - \log_6(x-2)$ *(5 marks)*
(b) Using the substitution $p = 2^x$ or otherwise solve the equation $2^x - 2 = \dfrac{8}{2^x}$ *(5 marks)*

(6) Find the stationary points on the curve $y = 2x^3 + 7x^2 - 12x + 3$ and determine their nature. *(7 marks)*

(7) The diagram below shows part of the curve $y = 3x - x^2$ and the line $x = y$. The curve and the line intersect at the points A and B. The shaded region R is the area bound by the line $x = y$ and the curve $y = 3x - x^2$.



(a) Find the coordinates of A and B. *(4 marks)*
(b) Show the area of the shaded region R is $\dfrac{4}{3}$. *(5 marks)*

(8) Find the coefficient of term in x^3 in the expansion of $(1+x)^2(2-x)^5$ *(6 marks)*

(9) (a) Sketch the graphs of $y = \sin(x)$ and $y = \cos(x)$ for $0 \le x \le 2\pi$ on the same set of axis showing any points of intersection with the coordinate axis. *(4 marks)*
(b) State with a reason the number of solutions to the equation $\sin(x) = \cos(x)$ for $0 \le x \le 2\pi$ *(2 marks)*
(c) Solve the equation $\sin(x) = \cos(x)$ for $0 \le x \le 2\pi$ giving your answers in terms of π. *(4 marks)*

End of Questions

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Core Mathematics C2

Advanced Subsidiary

Practice Paper 3

Time: 1 hour 30 minutes

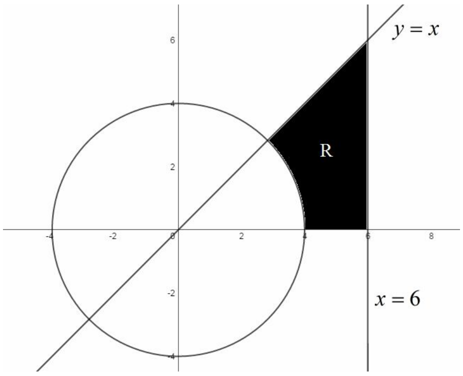
(1) (a) Solve the equation $2\cos(x)\sin(x) = \sin(x)$ for $0^\circ \le x \le 360^\circ$ *(5 marks)*

(b) Hence or otherwise solve the equation $2\cos(2\theta)\sin(2\theta) = \sin(2\theta)$ for $0^\circ \le \theta \le 180^\circ$ *(3 marks)*

(2) $f(x) = px^3 + 2qx^2 - 3x - 5$. Given $(x-1)$ is a factor of $f(x)$ and the remainder is 9 when $f(x)$ is divided by $(x+2)$ find the values of p and q. *(6 marks)*

(3) Show that $\int_1^4 \sqrt{x}(5-2x)dx = \frac{-22}{15}$. *(7 marks)*

(4) In the diagram below the circle has centre $(0,0)$ and passes through the points $(4,0), (0,4), (-4,0)$ and $(0,-4)$. Also pictured in the diagram are the lines $y = x$ and $x = 6$.



(a) Find the equation of the circle. *(3 marks)*

(b) Find the area of the shaded region R in the form $a + b\pi$. *(7 marks)*

(5) (a) Sketch the graph of $y = 2^x$ showing any points of intersection with the coordinate axis. Write down the equation of the asymptote of the curve. *(3 marks)*

(b) Solve the equation $2^x = 3$ giving your answer to 3 significant figures. *(3 marks)*

(c) Using a sketch show there are no real solutions to the equation $2^x = -1$ *(2 marks)*

(6) (a) Find the terms up to and including the term in x^3 of the binomial expansion $(1-2x)^7$. *(4 marks)*

(b) Using your answer to part (a) find an approximation for 0.98^7 correct to 4 decimal places. *(3 marks)*

(c) Explain how you could increase the accuracy of your approximation. *(1 mark)*

(7) (a) Given $y = \frac{1}{4}x^4 - x^2 - 3x + 5$ find $\frac{dy}{dx}$. (3 marks)

(b) Given $x^3 - 2x - 3 \equiv (x-1)(x+1)(x+3)$ find the stationary points of the curve $y = \frac{1}{4}x^4 - x^2 - 3x + 5$ and determine their nature. *(6 marks)*

(8) (a) Fred starts from home (H) and walks on a bearing of 050° for 4km to the point (A). He then walks on a bearing of 120° for 6km to reach the point (B). Find the distance HB to 3 significant figures. *(5 marks)*

(b) Fred is marking out a field HAB. Find the area of the field to 3 significant figures. *(3 marks)*

(9) (a) Show that $\frac{\log_2 32 + \log_8 16}{\log_4 8} = \frac{38}{9}$. *You must show each step of your workings.* *(4 marks)*

(b) Hence or otherwise solve the equation $\frac{\log_2 32 + \log_8 16}{\log_4 8} = \log_6 x$ correct to 3 significant figures *(3 marks)*

(c) Given $\log_x y = p$ and $\log_x z = q$ fully simplify $\log_x \frac{x^3y^4}{z}$. *(4 marks)*

End of Questions

Core 2 Formulae

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^{n}_{r}C = \dfrac{n!}{r!(n-r)!}$

$$(1+x^n) = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \quad (|x|<1, n \in \mathbb{R})$$

Logarithms and Exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric Series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{a}{1-r} \quad \text{for } |r|<1$$

Numerical Integration

$$\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}, \quad \text{where} \quad h = \frac{b-a}{n}$$