www.m4ths.com – C1 – Integration

(1) Integrate the following expressions with respect to t:

(a) $3t$

(b) $4t^3$

(c) t^{-2}

(d) $\frac{1}{2}t^3$

(e) 1

(2) Find the following indefinite integrals:

(a) $\int (2x+1)\ \mathrm{d}x$

(b) $\int (3x^3-2x)\ \mathrm{d}x$

(c) $\int (4x^{-3}-2x^{-5})\ \mathrm{d}x$

(d) $\int (-x^{-5}+\sqrt{x})\ \mathrm{d}x$

(e) $\int (7-x-x^{-5})\ \mathrm{d}x$

(f) $\int \left(4x-\frac{c}{2}\right)\ \mathrm{d}x$

(c is a constant)

(g) $\int (ax^3+bx^2)\ \mathrm{d}x$

(a and b are constants)

(3) Find an expression for $\mathrm{f}(x)$ for each of the following:

(a) $\mathrm{f}'(x)=3x-4$

(b) $\mathrm{f}'(x)=x^3-4x^{-5}$

(c) $\mathrm{f}'(x)=x^{\frac{1}{2}}+\sqrt{2}x^{-2}$

(d) $\mathrm{f}''(x)=\frac{1}{2}x^{\frac{1}{3}}+8x^{-5}$

(4) Find an expression for y given the following:

(a) $\frac{dy}{dx}=4x-x^2$

(b) $\frac{dy}{dx}=3+4x-2x^3$

(c) $\frac{dy}{dx}=9x+c$

(c is a constant)

(d) $\frac{d^2y}{dx^2}=9-x$

(e) $\frac{dy}{dx}=(ax+b)^2$

(a and b are constants)

(5) Find the following indefinite integrals in the form $y=\mathrm{f}(x)$:

(a) $\int (2+3x)(1-x)\ \mathrm{d}x$

(b) $\int x\left(\sqrt{x}-3\right)\ \mathrm{d}x$

(c) $\int \left(\frac{3x-1}{x^{0.5}}\right)\ \mathrm{d}x$

(d) $\int \left(\frac{2\sqrt[3]{x}-3}{x^{0.25}}\right)\ \mathrm{d}x$

(e) $\int \left(1-4\sqrt{x^5}\right)\ \mathrm{d}x$

(6) Given $y=x\sqrt{x}-3,\ x>0$ find $\int y\ \mathrm{d}x$.

(7) Find an equation for each of curves with the derivates given below:

(a) $\frac{dy}{dx}=2x+3$

Point $(3,20)$

(b) $\frac{dy}{dx}=6x^2+10x-3$

Point $(1,11)$

(c) $\frac{dy}{dx}=(1-x)(2+x)$

Point $\left(2,-\frac{8}{3}\right)$

(d) $\frac{dy}{dx}=\frac{3x^2-1}{\sqrt{x}}$

Point $(1,3)$

(e) $\frac{dy}{dx}=\frac{1}{\sqrt{x}}+0.25x^{-0.75}$

Point $(16,11)$

(f) $\frac{dm}{dt}=t(1-t)^2$

Point $(0,1)$

(g) $\frac{dp}{dt}=\sqrt[4]{t}\left(\frac{t-1}{2}\right)$

Point $(1,1)$

(8) The curve $y=\mathrm{f}(x)$ passes through the point $(2,6)$.

Given $\mathrm{f}'(x)=\frac{1}{2}x^3-2x+3$ find an equation for y in terms of x.

(9) The curve C passes through the point $(4,3)$. Find an equation for y in terms of x given $\frac{dy}{dx}=3x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}+2x+1$

(10) The curve $y=\mathrm{f}(x)$ passes through the origin.

Given $\mathrm{f}'(x)=3x^2-2x-6$ find the solutions to the equation $\mathrm{f}(x)=0$.

(11) The gradient of a particular curve is given by $\frac{dy}{dx}=2x+c$ where c is a constant.

Given the curve passes through the points $(1,2)$ and $(3,14)$ find an equation for y in terms of x.

(12) The diagram below shows the line with equation $\frac{d^2y}{dx^2}=6(x+1)$ and the curves of $\mathrm{f}(x)$ and $\mathrm{g}(x)$.

Find an equation for $y=\mathrm{f}(x)$ given $\mathrm{g}(0)=-2$ and $\mathrm{f}(0)=2$.