

GCSE Maths Foundation help sheet V1.5 – Circle the ☺ & date it if you are 100% happy with the topic.

Number

Topic/Skill	Tips/Facts	Example
Integers ☺	An integer is a whole number . It can be positive or negative.	Integers: 2, 5, 100, 6345.... Non Integers: ¼, 12.3, 0.76
Square Number ☺	When you × a number by itself you get a square number. This number has to be an integer. Squaring a number is NOT the same as multiplying a number by 2.	The first 6 square numbers are: 1, 4, 9, 16, 25, 36... (a) $3^2 = 3 \times 3 = 9$ (NOT 6) (b) $5^2 = 5 \times 5 = 25$
Square Roots ☺	This is the inverse (reverse process) of squaring a number. $\sqrt{\quad}$ is used.	(a) $6^2 = 36$ so $\sqrt{36} = 6$ (b) $9^2 = 81$ so $\sqrt{81} = 9$
Cube Number ☺	A number multiplied by itself three times . (The cube root $\sqrt[3]{\quad}$ is the inverse).	(a) $4^3 = 4 \times 4 \times 4 = 64$ (NOT 12) (b) $2^3 = 8$ (NOT 6)
A Prime Number ☺	A number that has only 2 factors, 1 and itself . 2 is the only even prime.	2, 3, 5, 7, 11, 13, 17, 19... (1 is not a prime number!)
Reciprocal ☺	The reciprocal is 1 divided by the number. Often it's easier to think about turning the fraction upside down (inverting the fraction).	The reciprocal of 5 is $\frac{1}{5}$ The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$
Factors ☺	The smaller whole numbers that go into a larger number with no remainder	The factors of 8 are 1, 2, 4 and 8.
Multiples ☺	Think Times Tables. Just write out the times tables for that number.	The first 6 multiples of 4 are 4, 8, 12, 16, 20 and 24
HCF ☺ (Highest Common Factor)	The largest number that goes into 2 or more different numbers. Just list the factors of each! Don't confuse this with LCM	"Find the HCF of 8 and 28" Factors of 8: 1, 2, 4 & 8 Factors of 28: 1, 2, 4, 7, 14 & 28 The HCF of 8 and 28 is 4
LCM ☺ (Lowest Common Multiple)	The lowest number that 2 or more different numbers go in to. Just list out the times tables of each number and see which is the lowest number that appears in both lists. Note: The LCM is never 1 . Do not confuse this with HCF	"Find the LCM of 4 and 6" Multiples of 4: 4, 8, 12 , 16, .. Multiples of 6: 6, 12 , 18.. The LCM of 4 and 6 is 12
Product of Prime Factors ☺	Numbers can be made up by multiplying prime numbers . To find the Product of Primes start with a factor tree . (Shown to the right) Product means multiply so don't forget to put the × sign in between the numbers you found in your factor tree. If you are struggling with the factor tree just keep trying to divide by the prime number in order. Does it divide by 2? If so pick 2. If it doesn't divide by 2 does it divide by 3? By 5? By 7? By 11? etc	
Rounding to 1 DP ☺ (Decimal Place)	You are rounding the number to the nearest 10th . If the number after the first decimal value is 5 or more, round up. If it's 4 or less keep the value the same.	(a) 2.43 = 2.4 (3 is less than 5) (b) 5.67 = 5.7 (c) 1.09 = 1.1 (9 is more than 5) (d) 2.98 = 3.0
Rounding to 2 DP ☺	Same as before but now it's the 100^{ths} that you are rounding to.	(a) 3.562 = 3.56 (b) 0.785 = 0.79 (c) 1.499 = 1.50
Rounding to 1 SF ☺ (Significant Figure)	When reading a number from left to right the 1st value that is not 0 is the 1st significant figure. Round using the same techniques as used with decimals.	(a) 243 to 1 SF = 200 (b) 5.6 to 1 SF = 6 (c) 47 to 1 SF = 50 (d) 0.48 to 1 SF = 0.5
Rounding to 2 SF ☺	Same as before but now it's the second significant figure. 0's now count!	(a) 243 = 240 (b) 40.8 = 41 (c) 0.546 = 0.55
Estimations & Approximations ☺	Round to one significant figure and estimate. You must show workings! Estimating doesn't require the exact value. It's non calculator!	98 × 51.2 becomes 100 × 50 which = 5000 4.6 + 104.7 becomes 5 + 100 which = 105

Fractions to Decimals ☺	Some are obvious such as $\frac{3}{4}$ is 0.75. For those that are not simply divide the numerator by the denominator using short division OR SD on your Casio. Common error! $\frac{1}{3}$ is not 0.3. £1 shared between 3 people is not 30p each.	(a) $\frac{1}{8} = 0.125$ (c) $\frac{7}{100} = 0.07$	(b) $\frac{3}{10} = 0.3$ (d) $\frac{43}{100} = 0.43$
Decimals to Fractions ☺	Some are obvious $0.1 = \frac{1}{10}$, $0.75 = \frac{3}{4}$, $0.5 = \frac{1}{2}$ etc If it's not obvious write it as a fraction over 10, 100 or 1000 and simplify.	(a) $0.7 = \frac{7}{10}$ (c) $0.46 = \frac{46}{100}$ or $\frac{23}{50}$	(b) $0.23 = \frac{23}{100}$
% to Decimals ☺	Simply divide by 100 and \times by 100 when converting decimals to percentages.	(a) $0.23 \times 100 = 23\%$	(b) $47\% \div 100 = 0.47$
Fractions to Percentages ☺	Percentage is just a fraction out of 100. Without a calculator just 'scale' the denominator up to 100 with equivalent fraction. On a calculator just \times by 100	(a) $\frac{3}{25} = \frac{12}{100} = 12\%$	(b) $\frac{9}{17} \times 100 = 52.9\%$
Simplifying Fractions ☺	If they are not obvious like $\frac{5}{10} = \frac{1}{2}$ look for common factors to divide by.	(a) $\frac{6}{8} = \frac{3}{4}$ (divide by 2)	(b) $\frac{20}{35} = \frac{4}{7}$ (divide by 5)
Mixed Numbers ☺	See how many times the denominator goes into the numerator. This gives you the integer part and then just write the remainder over the original denominator.	(a) $\frac{9}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = 2\frac{1}{4}$	(b) $\frac{17}{5} = 3\frac{2}{5}$ (c) $\frac{5}{3} = 1\frac{2}{3}$
Ordering fractions ☺	Get a common denominator and find equivalent fractions. You must use the original fractions in your answer. Ascending means smallest to largest.	Order: $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}, \frac{1}{2}$ equiv $\frac{9}{12}, \frac{8}{12}, \frac{10}{12}, \frac{6}{12}$ so $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$	
Finding a Fraction of a Quantity ☺	'Divide by the bottom, times by the top'. If you need $\frac{3}{8}$ of a number, divide the number in the question by 8 then multiply the answer by 3.	"Find $\frac{2}{5}$ of £60" Start with $\text{£}60 \div 5 = \text{£}12$. Now simply multiply by two. $2 \times 12 = \text{£}24$	
Adding Fractions ☺	You must have a common denominator to add fractions. When you do, simply add the numerators. Use equivalent fractions to find common denominators. If you have forgotten! Numerator = top, Denominator = bottom.	(a) $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$ (c) $\frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15}$	(b) $\frac{2}{5} + \frac{3}{10} = \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$ (d) $\frac{3}{2} + \frac{2}{7} = \frac{21}{14} + \frac{4}{14} = 1\frac{11}{14}$
Subtracting Fractions ☺	You must have a common denominator to subtract fractions. When you do, simply subtract the numerators. Use equivalent fractions to find common denominators. Please note: You can cross multiply when adding and subtracting fractions although it's a long way round for some examples.	(a) $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$ (c) $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$	(b) $\frac{4}{5} - \frac{1}{10} = \frac{8}{10} - \frac{1}{10} = \frac{7}{10}$ (d) $\frac{2}{7} - \frac{5}{6} = \frac{12}{42} - \frac{35}{42} = -\frac{23}{42}$
Multiplying Fractions ☺	Multiply the numerators and multiply the denominators then simplify. You do not need a common denominator unlike adding or subtracting.	(a) $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$	(b) $\frac{1}{8} \times \frac{4}{9} = \frac{4}{72} = \frac{1}{18}$ (simplified)
Dividing Fractions ☺	Turn the second fraction upside down (invert) and multiply as above. "Dividing by a fraction is the same as multiplying its reciprocal!!" You do not need a common denominator unlike adding or subtracting. How many halves of pizza can you cut from a whole pizza? $1 \div \frac{1}{2} = 2$ of course!	(a) $\frac{1}{8} \div \frac{4}{9}$ is the same as $\frac{1}{8} \times \frac{9}{4} = \frac{9}{32}$	(b) $\frac{3}{4} \div \frac{5}{6}$ is the same as $\frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$ (simplified)
Finding 10%, 5%, 1% of a quantity ☺	To find 10% without a calculator just divide the original number by 10, to find 1% divide it by 10 again. 5% is half of 10%, 2.5% is half of that!	£36: 10% = £3.60 5% = £1.80 and 1% = £0.36 or 36p From this we could get 11% = £3.96, 12%, 16% etc etc	
Increase or Decrease by a % ☺	Find the % required (see above) and add it on (increase) or take it off (decrease) If it's a calculator question just multiply the quantity by the %	Q: Increase £30 by 10% A: 10% = £3 so 30+3 = £33 Q: Decrease 20 by 40% A: 10% = 2, 40% = 8, 20-8=12	

Percentage Change ☺	You are looking at the increase or decrease as a % of the original value. 'Difference divided by the original and multiplied by 100.'	Bought £200/Sold £250. $\frac{50}{200} = 25\%$ increase in value								
Reverse Percentage ☺	Use your multipliers (some shown below), set up an equation and solve. <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>1% increase 1.01</td> <td>1% decrease 0.99</td> </tr> <tr> <td>5% increase 1.05</td> <td>10% decrease 0.90</td> </tr> <tr> <td>10% increase 1.10</td> <td>25% off 0.75</td> </tr> <tr> <td>25% increase 1.25</td> <td>40% of original value 0.40</td> </tr> </table> <p>You are not finding the % of the amount given in the question! Be careful!</p>	1% increase 1.01	1% decrease 0.99	5% increase 1.05	10% decrease 0.90	10% increase 1.10	25% off 0.75	25% increase 1.25	40% of original value 0.40	Q: A jumper was priced at £48.60 after a 10% reduction. Find its original price. $J \times 0.9 = 48.60$ A: $J = 48.60 \div 0.9$ (I have used 0.9 you can use $\frac{90}{100}$) $J = 54$ The jumper was originally £54. Check working back!
1% increase 1.01	1% decrease 0.99									
5% increase 1.05	10% decrease 0.90									
10% increase 1.10	25% off 0.75									
25% increase 1.25	40% of original value 0.40									
Growth and Decay ☺	Find the starting quantity, \times this by the multiplier to increase or decrease the quantity and raise that to the required power. See worked example! The multiplier for growth will be greater than 1, for decay less than 1.	Q: A bank pays 5% compound interest a year. Bob invests £3000. How much will he have after 7 years? A: $3000 \times 1.05^7 = 4221.3012\dots$ (about £4221.30)								
Negative Numbers ☺ (\times and \div)	If you are either multiplying or dividing with negative numbers and the signs are the same the answer is positive , if they are different the answer is negative .	(a) $-2 \times 4 = -8$ (b) $-3 \times -5 = 15$ (c) $3 \div -3 = -1$ (d) $-16 \div -4 = 4$								
Negative Numbers ☺ (+ and -)	Adding a negative decreases the value. Subtracting a negative increases the value. Start on a number line and either move up or down from your start point.	(a) $2 - 4 = -2$ (b) $3 - -5 = 8$ (c) $-2 + -5 = -7$ (d) $-4 - -5 = 1$								
BODMAS/BIDMAS (Order of Operations) ☺	Brackets then Powers (BO/BI) comes first. Multiplication then Division (DM) comes next. Addition then Subtraction (AS) comes last.	(a) $3 + 4 \times 2 = 11$ (do the multiplication first) (b) $3 + (4 + 1)^2$ Brackets: $(5)^2 = 25$ and then add $3 = 28$ (c) $12 \div 0.5 - 3 = 21$ (division first!)								
Multiplying Decimals ☺	Method 1: Count the total digits after the decimals. The number you start with is the number you finish with. You may have to add 0's. Method 2: Consider place value. Tenths \times Tenths = Hundredths.	0.4×0.2 (2 digits after the decimals in total) $4 \times 2 = 8$ so my answer is 0.08 as I need to finish with 2 digits after the decimals. $0.3 \times 0.15 = 0.045$								
Standard Form ☺ (Scientific Notation)	The number must be between 1 and 9.9 and multiplied by a power of ten. + powers of 10 for 'large numbers' and - powers of 10 for 'small numbers'	(a) $8400 = 8.4 \times 10^3$ (b) $671000 = 6.71 \times 10^5$ (c) $0.00036 = 3.6 \times 10^{-4}$ (d) $0.097 = 9.7 \times 10^{-2}$								
Calculating With Standard Form	When multiplying , multiply the numbers and add the powers. When dividing , divide the numbers and subtract the powers. Make sure your answer is in standard form. You may need to adjust at the end as shown in example (c) to the right!	(a) $(1.2 \times 10^3) \times (4 \times 10^6) = 8.8 \times 10^9$ (b) $(4.5 \times 10^5) \div (3 \times 10^2) = 1.5 \times 10^3$ (c) $(4.1 \times 10^6) \times (3 \times 10^9) = 12.3 \times 10^{15} = 1.23 \times 10^{16}$								

Ratio, Proportion and Rates of Change


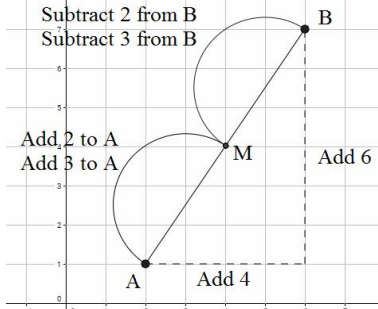
Simplifying Ratio ☺	Simplify them like fractions by dividing by common factors.	(a) 5:10 is 1:2 in its simplest form (b) 14:21 is 2:3
Ratios in the form 1:n and n:1 ☺	Divide both parts of the ratio by one of numbers to leave one as 1.	5:7 would be $1:\frac{7}{5}$ as $1:n$ or $\frac{5}{7}:1$ as $n:1$
Ratio Sharing ☺	Add the total parts. A ratio of 4:2:1 has 7 parts (not 3 parts as $4 + 2 + 1 = 7$) Divide the amount to be shared to find the value of one part . Simply multiply this value by the each part of the ratio. Be careful for already shared amounts!	"Share £60 in a 3:2:1 ratio" 6 total parts. £60 divided by 6 = £10. Each part is worth £10 $3 \times £10 = \mathbf{£30}$ $2 \times £10 = \mathbf{£20}$ $1 \times £10 = \mathbf{£10}$

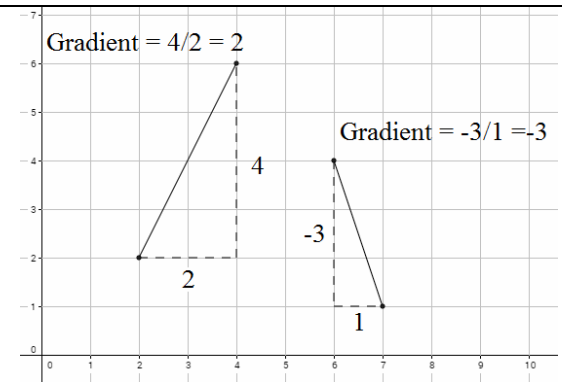
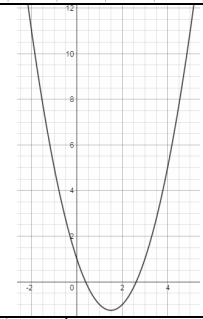
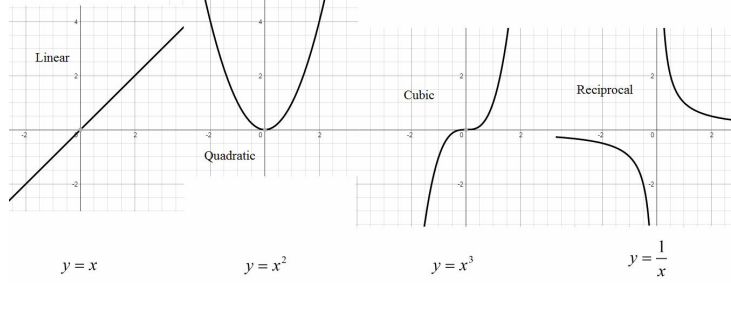
Ratios Already Shared ☺	Sometimes a ratio is already shared and you will need to work backwards. Simply find what one part is worth and then answers the questions given.	Q: Money was shared in the ratio 3:2:5 between Ann, Bob and Cat. Given that Bob had £16, find out how much was shared. A: Bob has 2 parts. This means £16 = 2 parts. One part will be worth £8. There are 10 parts in total so $10 \times 8 = £80$. A total of £80 was shared.
Ratios to Fractions ☺	Add the total parts. This is the denominator of the fraction. Simply write each part over that denominator. You should now be able to convert to decimals too!	$2:3$ has 5 parts so this would be $\frac{2}{5}$ and $\frac{3}{5}$.
Best Buys ☺	Find the unit cost by dividing the price by the quantity . The lowest number is the item that is the best buy. Be careful and don't round too early!	8 cakes for £1.28 = 16p each (this is the unit cost) 13 cakes for £2.05 = 15.8p each (so pack of 13 is better)
Basic Proportion ☺	Find out the value of one item and then multiply it by the number you need. Some of these are recipe type questions and others are just shopping type scenarios. Just find the cost, weight or size of 1 and then multiply up.	3 cakes require 450g of sugar to make. Find how much sugar 5 cakes require. $450 \div 3 = 150\text{g}$ per cake. Now multiply this by 5 to give 750g required for 5 cakes.
Direct and Inverse Proportion ☺	Direct: $y = kx$ or $y \propto x$ (This just reads y is directly proportional to x) Inverse $y = \frac{k}{x}$ or $y \propto \frac{1}{x}$ (This just reads y is inversely proportional to x) Solve for k using values given at the start of the question. Rewrite the equation with the correct value of k you have just found. Substitute the second given value in for x or y to find missing value.	Q: “ p is directly proportional to q . When $p = 12, q = 4$. Find p when $q = 20$ ” $p = kq$ $\therefore p = 3q$ A: Step 1, solve for k : $12 = k(4)$ Step 2: $p = 3(20)$ $k = 3$ $p = 60$

Algebra

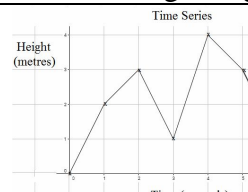
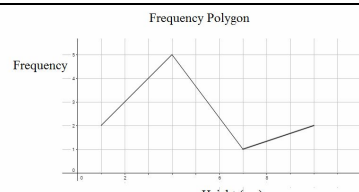
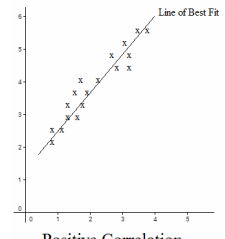
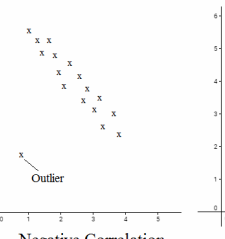
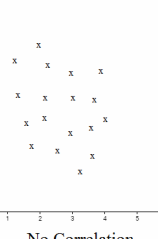
Simplifying Expressions ☺	Just collect the ‘like terms’ Be careful with negatives! x^2 and x are not like terms. $4x+1$ is not $5x$ as 1 is not an x . It's a known as a constant.	(a) $2x+3y+4x-5y+3$ becomes $6x-2y+3$ (b) $3x+4-x^2+2x-1$ becomes $5x-x^2+3$
x times x ☺	The answer is x^2 not $2x$ Just use numbers to check! This is a common error!	Squaring is multiplying by itself and not by 2. (Ahhh!)
$p \times p \times p$ ☺	This p^3 not $3p$ Just use numbers to check!	$3 \times 3 \times 3 = 27$ and not 9 $5 \times 5 \times 5 = 125$ and not 15
$p + p + p$ ☺	This $3p$ not p^3 Just use numbers to check!	$2 + 2 + 2 = 6$ and not 8 $4 + 4 + 4 = 12$ and not 64
Powers ☺ (\times and \div)	When multiplying with the same letter or number (base) just add the powers. When dividing just subtract the powers. Remember $p = p^1$ and $p^0 = 1$.	(a) $p^5 \times p^3 = p^8$ (b) $p^7 \div p^4 = p^3$ (c) $p^{12} \times p = p^{13}$
Powers (Negative Powers) ☺	$a^{-m} = \frac{1}{a^m}$. Don't panic! If you have a negative power just write the value as “1 over that value to the positive power” Easy in practice as shown to the right!	(a) $3^{-2} = \frac{1}{3^2}$ which is $\frac{1}{9}$ (b) $5^{-3} = \frac{1}{5^3}$ which is $\frac{1}{125}$
Expanding Single Brackets ☺	Multiply the value on the outside by each term inside the brackets. Be careful with negatives! The question may ask you to ‘multiply out’	(a) $5(3x+2) \equiv 15x+10$ A common error is $15x+5$. (b) $2x(3x-4) \equiv 6x^2-8x$

Expanding Double Brackets ☺	Multiply each term by one another. You can use F.O.I.L and then simplify. First, Outer, Inner, Last. Remember to simplify! $+4x - x$ is $+3x$. Don't forget x times x is x^2 not $2x$. Also remember $5x \times 3x = 15x^2$ not $8x$ or $8x^2$	$(x+2)(x-3)$ $(2x-1)(3x+2)$ (a) $x^2 - 3x + 2x - 6$ (b) $6x^2 + 4x - 3x - 2$ $x^2 - x - 6$ $6x^2 + x - 2$
Factoring Single Brackets ☺	HCF of 'letters' and 'numbers' outside, the rest inside. Expand to check if its right when you finish. You are looking for multiples! Remember $p^2 = p \times p$	(a) $6x - 3 \equiv 3(2x - 1)$ (b) $15x + 10 \equiv 5(3x + 2)$ (c) $6x^2 + 8x \equiv 2x(3x + 4)$
Factoring Quadratics ☺	When a quadratic expression is in the form $ax^2 + bx + c$ find the two numbers that ADD to give b and MULTIPLY to give c . Be careful with negatives.	(a) $x^2 + 7x + 10 \equiv (x + 5)(x + 2)$ (b) $x^2 + 2x - 8 \equiv (x + 4)(x - 2)$
Solving Linear Equations ☺	Get the x 's (unknowns or letters) on one side and the numbers on the other. Use either the balance method. Simply do the opposite operation to what the equation gives until you have only x 's on one side and only numbers on other. If you have a $+$, subtract this value from both sides. If you have a $-$ then add it to both sides, a \times then divide both sides by this quantity and a \div then multiply both sides by this quantity. What you do to one side, you just do to the other!	$2x - 3 = 7$ $5p + 6 = 2p + 18$ $\frac{y}{2} - 1 = 5$ (a) $2x = 10$ (b) $3p + 6 = 18$ (c) $\frac{y}{2} = 6$ $x = 5$ $3p = 12$ $y = 12$ $p = 4$
Solving Linear Simultaneous Equations (Algebraically) ☺	If you have 2 unknowns (x and y for example) you need two equations to find the value of both x and y . To do this we set up simultaneous equations. Either make the value in front of x 's the same or the y 's the same. Once they are the same (eg both 5) if the signs in front are the same subtract if they are different add . You will have now eliminated one unknown. (x or y) Solve the equation you have for either x or y . (This will be a simple equation) Finally substitute that value back in to any of the other equations to solve for the other unknown. Check your answers work for both!	(a) $2x + y = 7$ $3x - y = 3$ <div style="border: 1px solid black; padding: 5px; width: fit-content;">Add them to get $5x = 10$ & $x = 2$. Substitute in: $2(2) + y = 7$ so $y = 3$</div> (b) $5x + 2y = 9$ $10x + 3y = 16$ <div style="border: 1px solid black; padding: 5px; width: fit-content;">Multiply 1st equation by 2. $10x + 4y = 18$. Subtract to eliminate x's to give $y = 2$. Substitute in: $5x + 2(2) = 9$ so $x = 1$</div>
Solving Linear Simultaneous Equations (Graphically) ☺	These equations are solved by drawing the graphs (straight lines) of the two equations given. The solutions (answer to the question) will be where the lines meet. The graph to the right shows the solutions of the simultaneous equations $y = 5 - x$ and $y = 2x - 1$. They intersect (meet) at the point with coordinates $(2, 3)$. This means the solutions will be $x = 2$ and $y = 3$.	
Rearranging Equations and Formulae ☺	Changing the subject of an equation is like solving one without a pretty answer at the end. Instead of your answer being a number, it's usually an expression containing other letters and possibly numbers. Don't panic; just apply the same rules as for solving. If you have a $+$, subtract this value from both sides. If you have a $-$ then add it to both sides, a \times then divide both sides by this quantity and a \div then multiply both sides by this quantity. What you do to one side, you just do to the other! If there are no $+$ or $-$ subtract signs then it will be \times or \div . Remember $ut = u \times t$ and not $u + t$. Brackets means multiply too!	Q: Make x the subject of the equation $y = \frac{2x-1}{z}$: A: $y = \frac{2x-1}{z}$. Start by multiplying both sides by z to give $yz = 2x - 1$. Now add 1 to both sides so $yz + 1 = 2x$ and finally divide both sides by 2 to give $\frac{yz+1}{2} = x$. We now have x as the subject.

Formulae ☺	Simply substitute numbers in to formula as shown below. You may be asked to write and use a formula given a scenario, An example is shown to the right. You will generally have a constant such as +10 or -4.	“Bob charges £3 per window and a £5 call out charge” $C = 3N + 5$ with N being the number of windows cleaned and C the cost.												
Substituting into a Formulae ☺	Just follow the rules and substitute the numbers in. Be careful on the order if $x = 3$ and you need $2x^2$ square 3 first and then multiply by 2. There is a difference between $2x^2$ and $(2x)^2$. Careful with negatives. Squaring makes it positive! Also, subtract a - and the value increases	$a = 3, b = 2$ and $c = 5$ Find: (i) $2a$ which is just $2(3) = 6$ (ii) $3a - 2b$ so $3(3) - 2(2) = 5$ (iii) $b^2 - 5$ which is $(2)^2 - 5 = -1$												
Inequalities ☺	$x > 2$ “ x is greater than 2” This just means the number must be bigger than 2 $x < 3$ “ x is less than 3” This just means the number must be smaller than 3 $x \geq 1$ “ x is 1 or greater” This means the number can either be 1 or bigger $x \leq 6$ “ x is 6 or less” This means the number can either be 6 or smaller	(a) State 3 integers that satisfy $x > 4$ You could have 5, 6 and 10234 (you can't have 4) (b) State 3 integers that satisfy $-2 < x \leq 4$ You could have -1, 3 and 4 (you can't have -2)												
Plotting Straight Line Graphs (Linear Graphs or Linear Functions) ☺	Method 1: Make a table of values using the method shown to the right. The graph of $y = 2x + 1$ is shown to the right for $-1 \leq x \leq 3$. All you have to do is substitute the values into the equation start with $x = -1$ and finishing with $x = 3$. Make sure the values are going up by 2 each time! Method 2: Use the gradient intercept method. The line $y = 2x + 1$ is in the form $y = mx + c$. The gradient is m and the y intercept is c . This line will pass through the y axis at the point $(0,1)$ and have a gradient of 2. That means it goes up 2 for every one it goes across. (Gradient shown below) Make sure your line is straight. Any ‘kinks’ suggests your coordinates are incorrect. The values will always be going up or down by a fixed amount. Don't worry if you get an equation such as $x + y = 5$. You can just use a table of values as before. You may even want to rearrange the equation to $y = 5 - x$ or $y = -x + 5$ if you find it easier. Finally! Make sure you draw a line!	<table border="1" data-bbox="1619 560 1955 635"> <tbody> <tr> <td>x</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>y</td> <td>-1</td> <td>1</td> <td>3</td> <td>5</td> <td>7</td> </tr> </tbody> </table>  <p data-bbox="1547 1031 2018 1062">The graph of $y = 2x + 1$ for $-1 \leq x \leq 3$</p>	x	-1	0	1	2	3	y	-1	1	3	5	7
x	-1	0	1	2	3									
y	-1	1	3	5	7									
Midpoint of a Line ☺	Method 1: Add the x coordinates and divide by 2, add the y coordinates & divide by 2. Method 2: Sketch the line (if you can) and find the values half between the two x 's and two y 's Another way of thinking about this is that the midpoint is the average of the x 's and the averages of the y 's. The numbers won't necessarily be integers.	Q: Find the midpoint of a line through 2,1 and 6,7 A: $\frac{2+6}{2} = 4$ and $\frac{1+7}{2} = 4$ so the midpoint is 4,4 												

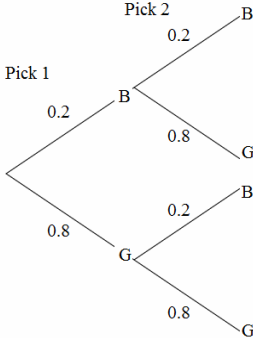
<p>The Gradient of a Line ☺</p>	<p>The gradient of a line is how steep it is. To find the gradient just divide the total distance up (or down) by the total distance across. In this unit you usually count squares and divide as shown to the right.</p> <p>The gradient can be positive (sloping upwards left to right) or negative (sloping downwards from left to right).</p> <p>Without a graph you could use the formula.</p> <p>The gradient (m) of the line passing through (1,2) and (11,6) would be</p> $m = \frac{6-2}{11-1} = \frac{4}{10} = \frac{2}{5}$ <p>This line goes up 2 units for every 5 across.</p>																			
<p>Quadratic Graphs ☺</p>	<p>Fill out the table as with the linear graphs. Be careful with negatives. Squaring a negative makes it positive! Subtracting a negative will mean you add it!</p> $y = x^2 - 3x + 1$ <table border="1" data-bbox="593 614 1176 694"> <tr> <td>x</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>y</td> <td>11</td> <td>6</td> <td>1</td> <td>-1</td> <td>-1</td> <td>1</td> <td>6</td> <td>11</td> </tr> </table> <p>This will be a parabola which is a sweeping curve & NOT a collection of lines.</p>	x	-2	-1	0	1	2	3	4	5	y	11	6	1	-1	-1	1	6	11	
x	-2	-1	0	1	2	3	4	5												
y	11	6	1	-1	-1	1	6	11												
<p>Graph Recognition ☺</p>	<p>Linear: A straight line graph in the form $y = mx + c$</p> <p>Quadratic: A parabola which is a sweeping curve in the form $y = ax^2 + bx + c$</p> <p>Cubic: A sweeping curve in the form $y = ax^3 + bx^2 + cx + d$</p> <p>Reciprocal: A curve in the form $y = \frac{a}{x}$.</p> <p>Don't be frightened of the a, b, c and d. These values will be given as numbers. You may have to fill out a table and plot the graphs. Use the method above.</p>																			
<p>Sequences ☺ (Number Patterns)</p>	<p>Look out for the Square Numbers</p> <p>Look out for the Cube Numbers</p> <p>Look out for the Fibonacci Sequence</p> <p>Look out for Linear (arithmetic) sequences such as 4, 10, 16, 22.....</p> <p>Look out Geometric sequences such as 2, 4, 8, 16, 32.....</p>	<p>1, 4, 9, 16, 25, 36, 49.....</p> <p>1, 8, 27, 64, 125, 216.....</p> <p>0, 1, 1, 2, 3, 5, 8, 13, 21, 34.....</p> <p>Increases or decreases by fixed amount. (+ or -)</p> <p>Has a common ratio (\times or \div)</p>																		
<p>nth Term Formula of a Linear (Arithmetic) Sequence ☺</p>	<p>Find the difference. (What's it increasing or decreasing by each time?)</p> <p>Multiply that by n (Be careful with negatives)</p> <p>Use values of n in the table starting with $n=1$ and substitute in.</p> <p>Find what number you need to add or subtract to find t. Check it for all of them! (This only works when there is a common difference). If your sequence doesn't increase or decrease by a fixed amount look at the possibilities shown above!</p>	<p>Find the nth term for: 3, 7, 11, 15</p> <table border="1" data-bbox="1601 1316 1960 1396"> <tr> <td>n</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>t</td> <td>3</td> <td>7</td> <td>11</td> <td>15</td> </tr> </table> <p>It's going up by 4 each time. Start with $4n$</p> <p>Take $n=1$ so $4 \times 1 = 4$ so we need to subtract 1 to get 3. The nth term for the sequence is $4n - 1$</p>	n	1	2	3	4	t	3	7	11	15								
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t	3	7	11	15																

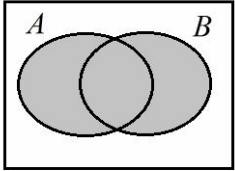
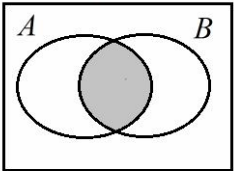
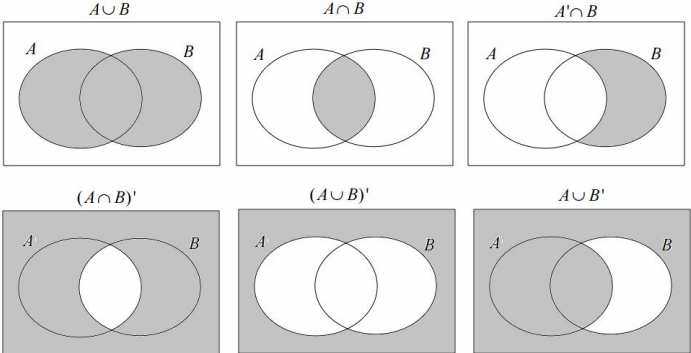
Statistics

Mean ☺ (Basic Average)	Add the values up (including 0s), divide by how many values there are. Tip! You might need to work this backwards to find missing values in the data.	The mean of 3,4,7,6,0,4,6 is $\frac{3+4+7+6+0+4+6}{7} = 5$																																						
Mean from a Table ☺ (Estimated and Actual)	When grouped data is used we get an estimated mean average. Find the midpoint of each class. Multiply Frequency by Midpoint. Divided the total by the sum of the frequency. Add these values up.	If the data is not grouped we get an Actual mean. Multiply the frequency by given values. Add them up. Divided by the sum of the frequency.																																						
	<table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>Height in cm</th> <th>Frequency</th> <th>Midpoint</th> <th>F × M</th> </tr> </thead> <tbody> <tr> <td>$0 < h \leq 10$</td> <td>8</td> <td>5</td> <td>$8 \times 5 = 40$</td> </tr> <tr> <td>$10 < h \leq 30$</td> <td>10</td> <td>20</td> <td>$10 \times 20 = 200$</td> </tr> <tr> <td>$30 < h \leq 40$</td> <td>6</td> <td>35</td> <td>$6 \times 35 = 210$</td> </tr> <tr> <td>Total</td> <td>24</td> <td>Ignore!</td> <td>450</td> </tr> </tbody> </table>	Height in cm	Frequency	Midpoint	F × M	$0 < h \leq 10$	8	5	$8 \times 5 = 40$	$10 < h \leq 30$	10	20	$10 \times 20 = 200$	$30 < h \leq 40$	6	35	$6 \times 35 = 210$	Total	24	Ignore!	450	<table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>Goals in Game</th> <th>Frequency</th> <th>F × Goals</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>3</td> <td>$3 \times 0 = 0$</td> </tr> <tr> <td>1</td> <td>5</td> <td>$5 \times 1 = 5$</td> </tr> <tr> <td>2</td> <td>4</td> <td>$4 \times 2 = 8$</td> </tr> <tr> <td>3</td> <td>1</td> <td>$1 \times 3 = 3$</td> </tr> <tr> <td>Total</td> <td>13</td> <td>16</td> </tr> </tbody> </table>	Goals in Game	Frequency	F × Goals	0	3	$3 \times 0 = 0$	1	5	$5 \times 1 = 5$	2	4	$4 \times 2 = 8$	3	1	$1 \times 3 = 3$	Total	13	16
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	Estimated Mean height: $450 \div 24 = 18.75\text{cm}$	Actual Mean goals scored per game: $16 \div 13 = 1.23$																																						
Median Value ☺	The middle value. Put them in order and find the middle one. If there are two values in the middle find the number half way between.	Find the median: 4,5,2,3,6,7,6 in order 2,3,4, <u>5</u> ,6,6,7. The Median = 5																																						
Mode / Modal # ☺	The number that appear most times in a list. 'Most Frequent'	4,5,2,3,6,4,7,8,4, Mode = 4. Check for multiple modes!																																						
Range ☺	Highest value subtract the lowest value. (A measure of spread)	3, 31, 26, 102, 37, 97, 4 Range: $102 - 3 = 99$																																						
Pie Charts ☺	A pie chart is a circle which means there are 360°. If you are drawing one divide 360 by the total frequency. This will tell you how many degrees to use for the frequency of each category you have.	If there are 40 people in a survey then each person will be worth 9° of the pie chart as $360/40 = 9$ Look out for right angles! $\frac{1}{4} = 90^\circ = 25\%$ of the data.																																						
Time Series Graphs and Frequency Polygons ☺	Time Series Graphs simply shows data collected over time. Time is plotted on the horizontal axis. Points are plotted and straight lines connect the points. These look similar to frequency polygons but are very different. Frequency Polygons are similar to histograms and show the frequency for grouped data. They are plotted at midpoints of the class.	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p>Time Series</p> </div> <div style="text-align: center;">  <p>Frequency Polygon</p> </div> </div>																																						
Scatter Graphs and Correlation ☺	Scatter Graphs plot data in pairs (bivariate). This might be the temperature and ice cream sales or the age of a car and the value of that car. Positive Correlation: As one value increases , the other increases . Negative Correlation: As one value increases the other decreases . No Correlation: There is no linear relationship between the two. If you are asked to find estimates from a scatter graph you must draw a line of best fit and read up and across from it.	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p>Positive Correlation</p> </div> <div style="text-align: center;">  <p>Negative Correlation</p> </div> <div style="text-align: center;">  <p>No Correlation</p> </div> </div>																																						
Pictograms ☺	Pictograms are a convenient visual way of representing data. Make sure you have a key. Some questions will require you to work backwards and find missing values instead of drawing them. Use the key to help you with this. The pictogram to the right shows 10 black 12 red, 2 green and 16 others.	<div style="text-align: center;"> <p>Black 🚗🚗🚗</p> <p>Red 🚗🚗🚗</p> <p>Green 🚗</p> <p>Others 🚗🚗🚗🚗</p> <p>🚗 = 4 cars</p> </div>																																						


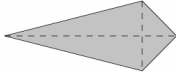
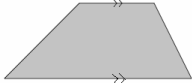
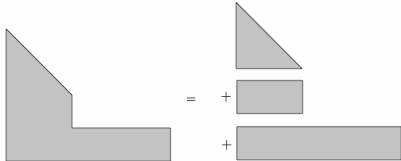
<p>Two Way Tables ☺</p>	<p>Two way tables allow us to model situations where there are two variables. In the example to the right we have gender and left/right handed. Just fill out the information step by step using the values given and make sure all of the totals add up for each row/column! Often one value is given in the question. Check this as you may think you are missing some information.</p> <p>You may be asked to work out some probability questions from the table.</p> <p>A question might be “One person is chosen at random. What is the probability that the person is left handed girl?”</p> <p>You would simply find left handed girls which is 6 and divide that by the total which is 100. The probability would be $\frac{6}{100}$ or you could simplify the fraction to give $\frac{3}{50}$.</p>	<p>Question: Complete the 2 way table below.</p> <table border="1" data-bbox="1424 108 2092 236"> <tr><td></td><td>Left Handed</td><td>Right Handed</td><td>Total</td></tr> <tr><td>Boys</td><td>10</td><td></td><td>58</td></tr> <tr><td>Girls</td><td></td><td></td><td></td></tr> <tr><td>Total</td><td></td><td>84</td><td>100</td></tr> </table> <p>Answer: Step 1, fill out the easy parts (the totals)</p> <table border="1" data-bbox="1424 268 2092 395"> <tr><td></td><td>Left Handed</td><td>Right Handed</td><td>Total</td></tr> <tr><td>Boys</td><td>10</td><td>48</td><td>58</td></tr> <tr><td>Girls</td><td></td><td></td><td>42</td></tr> <tr><td>Total</td><td>16</td><td>84</td><td>100</td></tr> </table> <p>Answer: Step 2, fill out the remaining parts</p> <table border="1" data-bbox="1424 427 2092 555"> <tr><td></td><td>Left Handed</td><td>Right Handed</td><td>Total</td></tr> <tr><td>Boys</td><td>10</td><td>48</td><td>58</td></tr> <tr><td>Girls</td><td>6</td><td>36</td><td>42</td></tr> <tr><td>Total</td><td>16</td><td>84</td><td>100</td></tr> </table>		Left Handed	Right Handed	Total	Boys	10		58	Girls				Total		84	100		Left Handed	Right Handed	Total	Boys	10	48	58	Girls			42	Total	16	84	100		Left Handed	Right Handed	Total	Boys	10	48	58	Girls	6	36	42	Total	16	84	100
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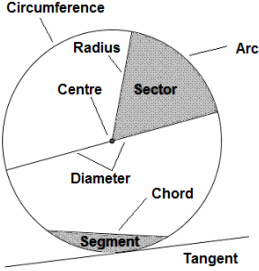
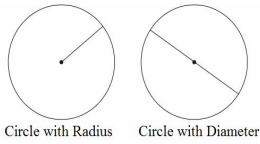
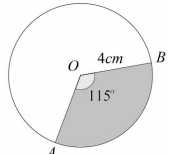
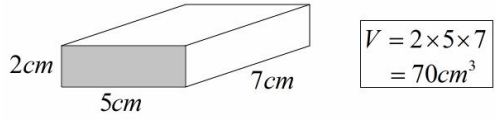
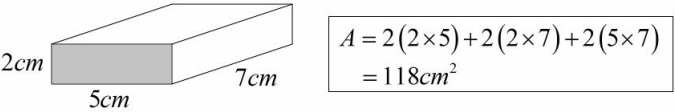
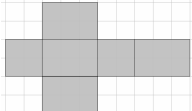
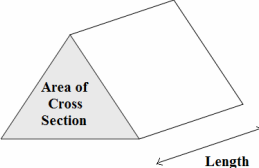
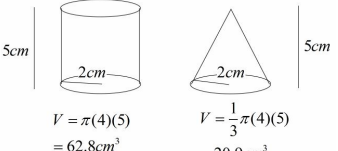
Probability


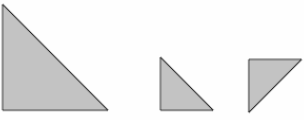
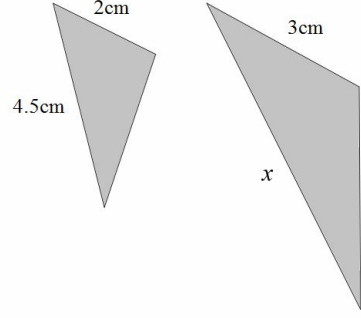
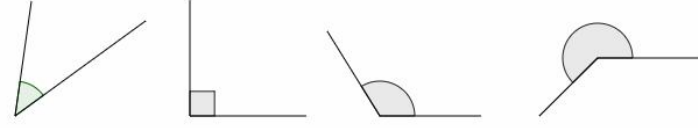

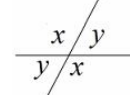
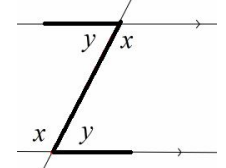
<p>Simple Probability (Theoretical) ☺</p>	<p>The number of things you want to happen divided by the number of things that could happen. 1 Head on a coin, two sides so the probability of head = $\frac{1}{2}$</p>	<p>Probability of rolling a 4 on a fair 6 sided die is $\frac{1}{6}$ There is one 4 and 6 different numbers.</p>
<p>Relative Frequency and Expected Outcomes ☺</p>	<p>This is how often something happens dividing by the number of trials that took place. To find the number of expected outcomes just multiply the probability by the number of trials.</p>	<p>Q: The probability a football team wins a game is 0.2. How many games would you expect them to win out of 40? A: $0.2 \times 40 = 8$, so about 8 games.</p>
<p>Tree Diagrams for Probability ☺</p>	<p>Use a tree diagram to help work out the probability of more than one event. All branches must sum to 1 when you add downwards ($0.2 + 0.8 = 1$ as shown in the example to the right). (The probability of something not happening is ‘1 minus the probability of it happening’)</p> <p>If you are modelling conditional probability check that your probabilities on the second branches reflects any changes. An example could be sweets in a bag. If you have 7 mints out of 10 in a bag of sweets on the first pick and you choose one then there will only be 6 mints left out of 9 sweets.</p>	
<p>Independent Events and Conditional Probability. ☺</p>	<p>If two or more events are said to be independent, the outcome of a previous event doesn't influence the next. The question may tell you that the events are independent. Anything ‘without replacement’ is conditional.</p>	<p>Independent for could be replacing a counter in a bag after picking it. Conditional would be where the counter wasn't replaced.</p>
<p>The AND and OR Rule in Probability ☺</p>	<p>OR means you need to Add the probabilities and AND means Multiply them. Be careful with the wording. Two chocolates for example is chocolate AND chocolate. Two mints or Two Toffee means you would have to add once you have multiplied! Check to see if the problem is independent or conditional!</p>	<p>Using the Tree Diagram above: $P(B \text{ and } B) = 0.2 \times 0.2 = 0.04$ $P(G \text{ and } G) = 0.8 \times 0.8 = 0.64$ $P(1 \text{ of each}) = (0.2 \times 0.8) + (0.2 \times 0.8) = 0.32$</p>

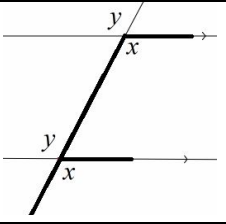
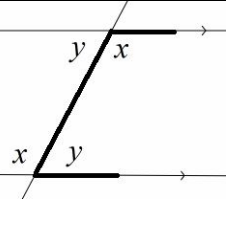
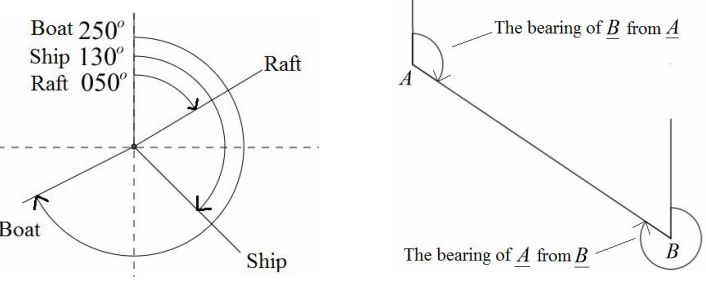
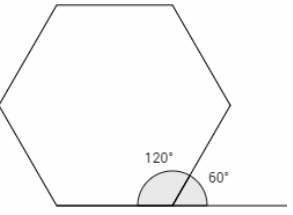
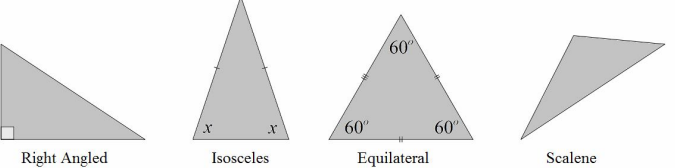
<p>Set Notation ☺</p>	<p>\in Element of set. (This is just a value in the set) $\{ \}$ Values in the set. (A collection of all the numbers in that set) ξ Universal Set (All values to consider even if they are not in A or B) A is an element of Set A A' is not an element of Set A (the compliment or NOT A) $A \cup B$ (The Union) is A or B or both. $A \cap B$ (The Intersection) is both A and B</p>	<p>Set A are the even numbers less than 10: $A = \{2, 4, 6, 8\}$ Set B are the prime numbers less than 10: $B = \{2, 3, 5, 7\}$ $4 \in A$ (4 is in A) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$ These are in either or both! $A \cap B = \{2\}$ This is the single value in both sets!</p>
<p>Venn Diagrams ☺</p>	<p>You can use Venn Diagrams to calculate probabilities. You may be asked to shade Venn Diagrams as shown below and left.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>$A \cup B$</p>  <p>The Union 'A or B or Both'</p> </div> <div style="text-align: center;"> <p>$A \cap B$</p>  <p>The Intersection 'A and B'</p> </div> </div> <p>You may also be asked to use Venn Diagrams for problem solving.</p>	


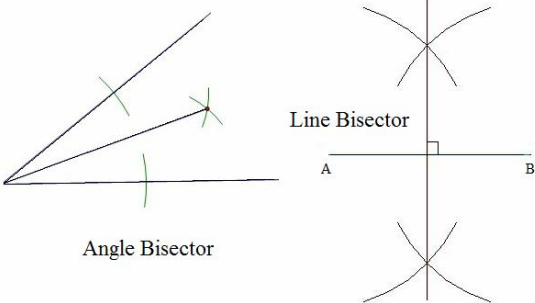
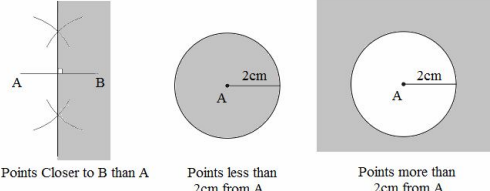
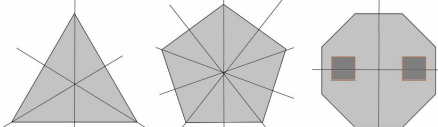
Geometry and Measures

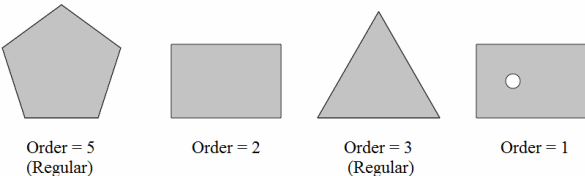
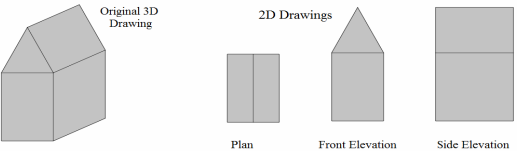

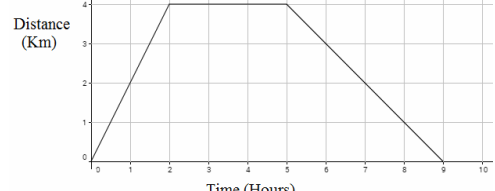
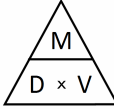
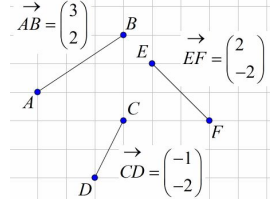
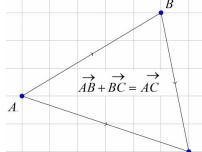
<p>Area of Rectangle ☺</p>	<p>Multiply the two side lengths. Look out for squares!</p>	<p>Area is the space inside a shape. Units are 'squared'</p>
<p>Perimeter of a Rectangle ☺</p>	<p>Add each side length! Some rectangles only show two lengths, make sure you are adding all four! Don't confuse area with perimeter. It's a length.</p>	<p>Distance around the outside. Answers are NOT squared as it's just a length.</p>
<p>Area of Parallelogram ☺</p>	<p>Treat these like a rectangle. Base \times Perpendicular Height. Not the slant height! It will be the same for a rhombus.</p>	
<p>Area of a Triangle ☺</p>	<p>Multiply the base by the height and half your answer. Please half the answer!</p>	<p>This is just half the area a rectangle of the same size!</p>
<p>Area of a Kite ☺</p>	<p>Use the same method as you do for a triangle. Height \times Width and half answer. Again this is half a rectangle of the same dimensions.</p>	
<p>Area of a Trapezium ☺</p>	<p>Add the two parallel sides, multiply it by the height and half your answer. If you can't remember this split it up into rectangles and triangles if you can.</p>	
<p>Area/Perimeter of Compound /Composite Shapes ☺</p>	<p>Simply split the shape into smaller shapes where there area or perimeter is easier to work out. Look out for missing lengths. You will need to find them! There are a number of different ways you can split most composite/compound shapes up. Use whichever method is easiest. Loads of possible splits on them!</p>	

<p>Circle Parts ☺</p>	<p>Diameter: Line through the centre from circumference to circumference Radius: Line from centre to circumference. (Half the length of a diameter) Centre: Middle of the circle. Tangent: A line that touches the circumference. (Doesn't go through circle) Chord: Line a diameter but doesn't go through the centre. Arc: Small part of the circumference. Sector: Part of the area of the circle. Enclosed by two radii. Segment: Area trapped between chord and circumference.</p>	
<p>Area and Circumference of a Circle ☺</p>	<p>Area = πr^2 which just means pi \times the radius squared. If the radius was 5cm then $A = \pi \times 5^2 = 78.5cm^2$ Circumference = πd which is just pi \times the diameter. If the radius was 5cm then $C = \pi \times 10 = 31.4cm$</p>	<p>Area = space inside. Just multiply the radius by the radius by π. The units will be 'squared'. Circumference = distance around the outside. The units are NOT squared. If you don't have π on your calculator use 3.142.</p> 
<p>Arc Length and Area of a sector ☺</p>	<p>The arc length is just part of the circumference. The sector is part of the area. Take the angle given as a fraction of 360° & \times formulae shown above for each. Arc: $AB = \frac{115}{360} \times \pi \times 8 = 8.03cm$ Sector Area: $OAB = \frac{115}{360} \times \pi \times 16 = 16.1cm^2$</p>	
<p>Volume of a Cuboid (Capacity) ☺</p>	<p>The volume is the amount of space that a 3D shape can hold. To find the volume you simply multiply the area of cross section by the length. You can choose any cross section. You <i>could</i> also just multiply each length! The units will be cubed such as cm^3, m^3, km^3</p>	
<p>Surface Area of a Cuboid ☺</p>	<p>The surface area is everything you can touch. Find the area of each face and add them up. Check if it is open or closed top. If it's open it will only have 5 faces. Think about a dice. You can touch all six faces. The total surface area would just be 6 times the area of each face. Units is always 'squared'</p>	
<p>Sketching the Net of a Cuboid ☺</p>	<p>Just think what the box would look like if you unfolded it. Don't forget the lid if it has one. Dimensions must be accurate and have a label.</p>	
<p>Volume of a Prism ☺</p>	<p>This is the same as the cuboid. Area of the cross section \times length. Be very careful with triangular prisms. Make sure you half your answer when finding the area of the cross section. For cylinders you will need the area of a circle. If you are already given the area simply multiply that by the length. Answer will be in something cubed such as cm^3</p>	
<p>Volume of a Cone ☺</p>	<p>A cone is not a prism as it doesn't have a constant cross section. The formula used is $V = \frac{1}{3} \pi r^2 h$. Just find the volume of a cylinder and divide by 3. A cylinder is just $V = \pi r^2 h$ which is the area of a circle \times the height. (A prism!)</p>	

Solids ☺	Faces = like faces of dice Edges = side lengths Vertices = corners	A Cube has 6 faces, 8 vertices and 12 edges
Congruent Shapes ☺	Identical (Same shape and same size). Some shapes will be rotated or reflected (their orientation changed) but still congruent to another shape shown. Do check!	
Similar Shapes ☺	Same shape, different size . The proportion is the same for each side length. Check each side length is a multiple of the other shape. Adding or subtracting a fixed value to each side length doesn't keep the proportion. Think scale factors	
Problem Solving with Similar Shapes ☺	To find a missing length look at the corresponding side lengths and then take one of two approaches: Method 1: The larger shape has side length 3 and the smaller side shape has side length of 2. Remember, these sides are in proportion as they are similar shapes . This gives a scale factor of $\frac{3}{2}$ or 1.5. To find x , multiply 4.5 by 1.5 as the sides are proportional. $x = 6.75\text{cm}$. Method 2: Set up an equation and look at the sides in proportion: $\frac{x}{3} = \frac{4.5}{2}$ Multiply both sides by 3 to give $x = \frac{3 \times 4.5}{2}$ which gives $x = 6.75\text{cm}$.	
Angle Types ☺	Acute angles are less than 90° , Right angles are 90° , (Often shown by a small square on no value) Obtuse angles are greater than 90° but less than 180° Reflex angles are greater than 180° and less than 360° If you are asked to draw a reflex angle it may be easier to draw an acute one and then mark the larger angle round the other side!	
Basic Angle Facts ☺	Angles on a straight line add to 180° . (You may see this written as 'sum to') Angles around a point add to 360° . Look out for right angles! These have a little square and often no numbers on! Some questions will need algebra to solve for an unknown. An example might be a line with an angle of $2x$ and an angle of $3x$. Just add them and solve: $2x + 3x = 180^\circ$ so $5x = 180^\circ$ and $x = 36^\circ$	
Opposite Angles ☺	Opposite angles are equal. Remember also that angles on a straight line add to 180° and angles around a point add to 360° . This will help you with some multi-step problems later on.	
Alternate Angles ☺	Alternate angles are equal. These look like the letter Z (Do not use the term 'Z angles' in an exam. You must use Alternate) Often you will be asked to state with a reason why you have given your answer. There is often more than one way to explain how you found the angle.	

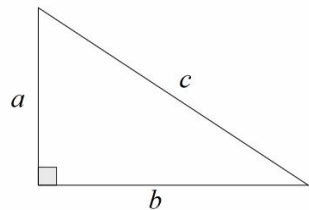
<p>Corresponding Angles ☺</p>	<p>Corresponding angles are equal. These look like the letter F (Do not use the term ‘F angles’ in an exam. You must use Corresponding) Often you will be asked to state with a reason why you have given your answer. There is often more than one way to explain how you found the angle. Some students may see two angles as corresponding instead of selecting alternate angles. As long as you show how you will get the marks.</p>	
<p>Co-interior Angles ☺</p>	<p>Co-interior angles add to 180°. $x + y = 180^\circ$ These look like the letter C (Do not use the term ‘C angles’ in an exam. You must use Co-interior) Often you will be asked to state with a reason why you have given your answer. Check your answer makes sense. Clearly $x + y$ are not the same size. Students often incorrectly just say ‘opposite angles’ if they haven’t learned them!</p>	
<p>Bearings ☺</p>	<p>Bearings are just angles! Here are the rules you must stick to: (1) Measure from North (Draw a north line at each point to help you) (2) Measure clockwise (Using a protractor or by using angle facts) (3) Your answer must have 3 digits (An angle of 40° has a bearing of 040°) The first diagram shows the bearing of 3 water vehicles from a harbour. The second diagram shows how to find the bearing of one place from another. B from A is measured at A and A from B is measured at B (Use north lines) and fill out all missing angles. Sometimes you can use angle facts above instead of measuring the angle. Co-interior angles add to 180° as shown above.</p>	
<p>Angles in a Polygons (Basic Method) ☺</p>	<p>Start off with a triangle. When you add a side to a polygon you add 180° to the sum of the interior angles. Drawing may help.</p>	<p>Angles in triangles add to 180°, quadrilaterals add to 360°, pentagons add to 540° hexagons add to 720° etc</p>
<p>Exterior Angles of a Regular Polygon ☺</p>	<p>For regular polygons divide 360 by the number of sides. This will give you the size of each exterior angle. This is shown on the straight line in the diagram. The regular hexagon shown has an exterior angle of 60° As $360^\circ \div 6 = 60^\circ$ The interior angles of 120° are also shown. This was found by subtracting the exterior angle from 180° as angles on a straight line add to 180° If you are still unsure, interior is inside and exterior is outside!</p>	
<p>Interior Angles of Regular Polygon ☺</p>	<p>To find an interior angle subtract the exterior angle from 180° as shown above. Add all the angles for the sum. The number of angles = number of sides.</p>	<p>Use the diagram above to help you! You could also use $(n - 2) \times 180$ to find the sum of the interior angles.</p>
<p>Types of Triangles ☺</p>	<p>Right Angle Triangles have a 90° angle in. Look out for the square in these! Isosceles have 2 equal sides and 2 equal base angle Equilateral have 3 equal sides and 3 equal angles (60° each). Scalene have different size sides and angles. You can spot isosceles and equilateral by the small lines on their sides.</p>	

Parallel and Perpendicular Lines ☺	<p>Parallel lines never meet and have a fixed distance between them.</p> <p>Perpendicular lines are at right angles. There is a 90° angle between them.</p> <p>If two straight lines are parallel the value of m in the equation $y = mx + c$ will be the same for both lines. $y = 3x - 1$ and $y = 3x + 2$ as their gradients are equal.</p>	 <p style="text-align: center;">Parallel Perpendicular</p>
Angle and Line Bisectors ☺	<p>Angle Bisector: Cuts the angle in half.</p> <p>Open the compass up. Place the sharp end on the vertex. Mark a point on each line Without changing the compass put the compass on each point and mark a centre point. Get a ruler and draw a line through the vertex and centre point.</p> <p>Line Bisector (Perpendicular Bisector): Cuts the line in half and at right angles</p> <p>Put the sharp end on Point A. Open the compass up past half way on the line. Mark a point above and below the line. Without changing the compass do the same from B. Draw a straight line through the points.</p> <p>You MUST leave your construction marks on!</p>	 <p style="text-align: center;">Angle Bisector Line Bisector</p>
Loci and Regions ☺	<p>A locus is just a path of points or region that follows a rule.</p> <p>For the locus of points closer to B than A you will create a perpendicular bisector as above and shade to the right of the line as shown to the right.</p> <p>For the locus of points less than or more than a fixed distance from A use a compass with the given radius to draw a circle. You may have to combine loci.</p>	 <p style="text-align: center;">Points Closer to B than A Points less than 2cm from A Points more than 2cm from A</p>
Translating a Shape ☺ (A Transformation)	<p>Translate means to move the shape. There is no change in its size or its orientation. Vectors are used to give information about the ‘movement’</p> <p>Top number tells you to move right or left. Right is + and left is - .</p> <p>Bottom tells you to move up or down. Up is + and down is - . If coordinates are used for the translation just treat them like vectors.</p>	<p>(a) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is right 2 and up 3 (b) $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ is left 1 and up 2</p> <p>(c) $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ is right 3 and down 5 (d) $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ is just up 4</p>
Rotating a Shape ☺ (A Transformation)	<p>The size of the shape doesn’t change. The shape is simply turned about a point.</p> <p>You will be given (i) A direction (ii) An angle and (iii) A centre of rotations.</p>	<p>(a) Rotate Shape A 90° clockwise about (0,1)</p> <p>(b) Rotate Shape B 270° anti clockwise about (0,0)</p>
Reflecting a Shape (A Transformation)	<p>Think about standing looking in a mirror. Learn lines such as $x = 2$ (vertical), $y = -1$ (horizontal) & $y = x$ (diagonal). Use a mirror if you are unsure.</p>	<p>(a) Reflect Shape A in the x axis.</p> <p>(b) Reflect Shape A in the line $x = 2$.</p>
Enlarging a Shape ☺ (A Transformation)	<p>You will be given a Scale Factor and centre. Multiply each side length by the scale factor. A scale factor of 2 is twice as big, not adding 2 to each side.</p>	<p>SF of 3 = 3 times larger (\times EACH side length by 3)</p> <p>SF of $\frac{1}{2}$ = half the size (\div EACH side length by 2)</p>
Naming Transformations ☺ (The 4 choices)	<p>Rotations will be the same size but often a different way around. (orientation)</p> <p>Translations have simply been moved. No change to size or orientation.</p> <p>Reflections will sometimes have the same orientation depending on the shape.</p> <p>Enlargements will be the same shape but either larger or smaller!</p>	<p>(Centre, direction and angle required for Rotations)</p> <p>(The vector is required for Translations)</p> <p>(The reflection line for Reflections) Look out for $y = x$</p> <p>(The scale factor is required for Enlargements)</p>
Line Symmetry ☺	<p>How many mirror lines can you draw on the shape? Regular shapes will have the same number of sides as they do symmetry lines and rotational symmetry.</p> <p>Be careful with patterns within shapes. This will change the symmetry!</p> <p>Parallelograms seem to catch people out too!</p>	

Rotational Symmetry ☺	<p>How many times does the shape look the same when you turn it through 360°? This gives us the order of rotational symmetry.</p> <p>Be careful with patterns. Regular shapes without patterns will have the same number of sides as their rotational symmetry. Use tracing paper if you need.</p> <p>A circle without a pattern will have an undefined number!</p>	
Plans and Elevations ☺	<p>This takes a 3D drawing and produces 3 different 2D drawings.</p> <p>Plan View: From above. Think ‘birds eye view’</p> <p>Side Elevation: A 2D shot from the side of the object.</p> <p>Front Elevation: A 2D shot from the side of the object.</p> <p>You will be told which is the front and/or side. Remember to put the units on!</p>	
Metric Units ☺	<p>Length: mm, cm, m and km. 1km = 1000m = 100'000cm = 1'000'000mm</p> <p>Mass: mg, g, kg, tonnes 1kg = 1000g</p> <p>Volume: ml, cl, l 1 litre = 1000ml</p>	<p>Mans height ~ 1.8-2m, credit card ~ 0.8mm thick</p> <p>Adults weight 70kg, a small cake = 150g</p> <p>Glass of coke is about 250ml</p>
Speed, Distance, Time ☺	<p>Speed = Distance ÷ Time</p> <p>Distance = Speed × Time</p> <p>Time = Distance ÷ Speed</p> <p>Remember the correct units!</p> 	<p>(a) Speed is 4mph, Time is 2 hours, Find the Distance. $D = S \times T$ so $4 \times 2 = 8$ miles.</p> <p>(b) Time is 5 hours, Distance = 12km, Find the Speed $S = D \div T$ so $12 \div 5 = 2.4$kph</p>
Distance/Time Graphs ☺	<p>Distance/Time graphs show the distance covered and the time taken as shown to the right. Distance is on the vertical axis and time is on the horizontal.</p> <p>You can find the speed from the gradient of the line (Distance ÷ Time).</p> <p>The steeper the line, the quicker the speed. If there is a flat line (horizontal to the time axis) the object is stationary. On the example to the right the speed on the first section is $4 \div 2 = 2$km/h, the second 0km/h and the third $4 \div 4 = 1$km/h</p>	
Density, Mass, Volume ☺	<p>Density = Mass ÷ Volume</p> <p>Mass = Density × Volume</p> <p>Volume = Mass ÷ Density</p> <p>Remember the correct units!</p> 	<p>(a) Density is $8 \text{kg} / \text{m}^3$, Mass is 2kg, Find the Volume. $V = M \div D$ so $2 \div 8 = \frac{1}{4} \text{m}^3$</p> <p>(b) Volume is 20cm^3, Mass is 30g, Find the Density $D = M \div V$ so $30 \div 20 = 1.5 \text{g} / \text{cm}^3$</p>
Vectors (Adding, Subtracting and Multiplying Vectors) ☺	<p>$\begin{pmatrix} a \\ b \end{pmatrix}$ is a column vector. a is right(+) or left(-) and b is up(+) or down(-)</p> <p>You can add $\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$ subtract $\begin{pmatrix} 8 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ & \times them $2 \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$</p>	 <p>Be Careful!</p> <p>$\vec{AB} \neq \vec{BA}$</p> <p>$\vec{AB} = -\vec{BA}$</p>
Vectors (Resultant Vectors) ☺	<p>The resultant vector is just a direct vector between two points. In the diagram we to get to C from A either via B or directly. The result is the same in terms of the units moved left and right and up and down. Resultant vectors are often used in physics for displacement, velocity and acceleration.</p>	

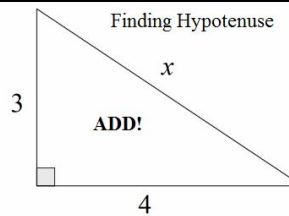
Pythagoras Theorem for Right Angle Triangles ☺

Used to find **missing lengths** in **right angled triangles** when 2 side lengths are given. The triangle must be a right angled triangle.

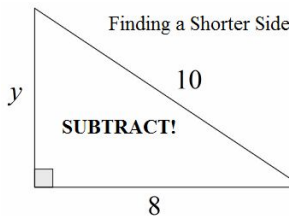


$$a^2 + b^2 = c^2$$

a & b are the **2 shorter sides** and c is the **hypotenuse** (longest). Make sure you label each correctly. Neither shorter side can be longer than the hypotenuse!



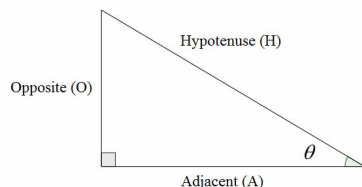
$$\begin{aligned} a &= 3, b = 4, c = x \\ a^2 + b^2 &= c^2 \\ 9 + 16 &= x^2 \\ 25 &= x^2 \end{aligned}$$



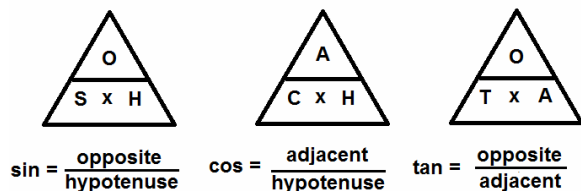
$$\begin{aligned} a &= y, b = 8, c = 10 \\ a^2 &= c^2 - b^2 \\ y^2 &= 100 - 64 \\ y^2 &= 36 \\ y &= 6 \end{aligned}$$

Trigonometric Ratios ☺

These are used to find **missing lengths and angles** in right angled triangles. You would use Pythagoras if you had 2 given sides and need to find the third. The triangle below shows each side length **relative** to the angle θ .

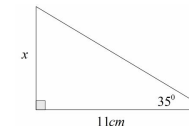


You can use the triangles below to help work out missing lengths and angles.



Use \sin, \cos & \tan for finding lengths and \sin^{-1}, \cos^{-1} & \tan^{-1} for finding angles. Press 'shift' on your Casio when you need the angle!

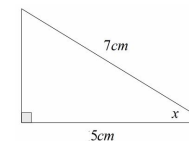
Q: (a) Find the value of x



A: We want the opposite (length) and we have the adjacent side to the given angle. We use \tan here.

$$x = 11 \times \tan(35^\circ) \text{ which gives } x = 7.70\text{cm}$$

Q: (b) Find the value of x



A: We want the angle. We have the adjacent side and the hypotenuse. We use \cos here.

$$\cos(x) = \frac{5}{7} \text{ which gives } x = \cos^{-1}\left(\frac{5}{7}\right) \text{ and } x = 44.4^\circ$$

Special Values for Angles in Trigonometry ☺

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

You can derive these values using the diagrams below.

